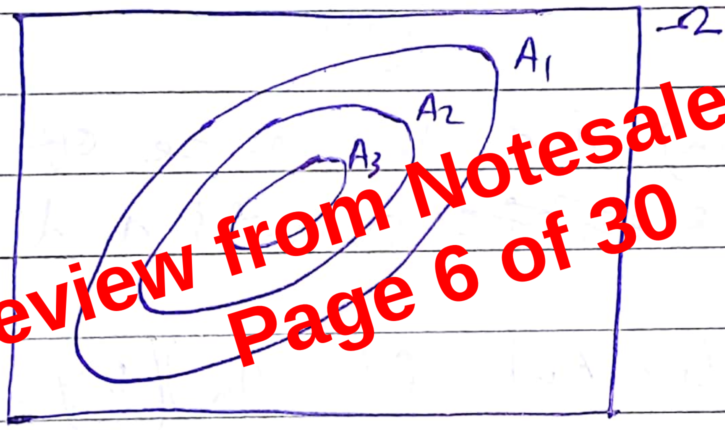
Here, we have $\lim A_n = \bigcup_{n=1}^{\infty} A_n$

The sequence A_n is said to be monotone non-increasing (or contracting) if

$$A_n \supseteq A_{n+1} \quad \forall n \geq 1$$



$$\lim A_n = \bigcap_{n=1}^{\infty} A_n$$

Theorem (Continuity theorem):-

Let $\{A_n\}$ be a monotone sequence of events defined on a prob. space $(\Omega, \mathcal{A}, P)$. Then, $P(\lim A_n) = \lim P(A_n)$

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conditional Probability

Let B be an event $\ni n(B) > 0$ where $n(B)$ is the nos of outcomes favourable to B. Here our objective is to find the probability of occurence of an event given the condition that B has already occured.

Since B has already occured it is known ~~that~~ one of the $n(B)$ outcomes must have occured and the other outcomes in the sample space of Ω must not have realised. In this situation for finding the probability of A one should consider $n(B)$ as the total number of possible outcomes ~~the~~ instead of N, the total number of outcoms in Ω. Again, since B has already occured if A has to occur, it must occur with B. Hence in this situation the outcomes favourable to A will be those which are favourable to B as well in other words in this case the number of outcomes favourable to A will be $n(A \cap B)$.

Thus, the conditional probability of the

1) Problem:- Suppose there are 3 machines in a factory which produce 5%, 10% and 7% defectives respectively. Suppose an item produced by the factory is chosen at random. What is the prob. that the item is defective? If the item is actually found to be defective, what is the prob. that it has been produced by the second machine?

Solⁿ: Let A_i be an event denoting that a randomly selected item is produced by the ith machine. Clearly the events A_i's are mutually exclusive and exhaustive. Also $P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$.

Let B be an event denoting that a randomly selected item is defective.

By the problem,

$$P(B|A_1) = 0.05$$
$$P(B|A_2) = 0.1$$
$$P(B|A_3) = 0.07$$

Thus, the probability that a randomly selected item is defective is given by

$$P(B) = P(A_1) \cdot P(B|A_1) + P(A_2)\, P(B|A_2) + P(A_3) \cdot P(B|A_3)$$

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$$= \frac{1}{3} \times 0.05 + \frac{1}{3} \times 0.1 + \frac{1}{3} \times 0.07$$

$$= \frac{0.22}{3}$$

Now, it is known that the item is defective. Hence, the prob. that it has been produced by the second machine is

$$P(A_2|B) = \frac{P(A_2) \cdot P(B|A_2)}{\sum_{i=1}^{3} P(A_i) P(B|A_i)}$$

$$= \frac{0.1}{0.22}$$

2) Show that conditional prob. satisfies the conditions of an ordinary prob. function

Sol^n:- Let B be an event such that $P(B) > 0$. The conditional prob. of an event A given that B has already occurred is defined as $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(i) To show that for any event A, $P(A|B) \geq 0$.

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$A = \{HH, HT\}$
$B = \{TH, TT\}$

Here $A \cap B = \phi \Rightarrow P(A \cap B) = 0$

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$\Rightarrow P(A) \cdot P(B) = \frac{1}{4}$

$\therefore P(A \cap B) \neq P(A) \cdot P(B)$

As such A and B are not mutually independent although they are mutually exclusive.

However, if at least one of the two events A and B is impossible, then

$P(A \cap B) = P(\phi) = 0$

Also $P(A) \cdot P(B) = 0$

Thus, two mutually exclusive events will be mutually independent if at least one of them is impossible.

2) Let A and B be two mutually independent events. Will they be mutually exclusive as well?

— Suppose $P(A) > 0$, $P(B) > 0$.