05 Scientific Notation (Standard Form of Number)

Scientific Notation is often used when dealing with either very large numbers or very small numbers. A number that is expressed in scientific notation is composed of two parts that are multiplied together. The two parts are:

- A decimal number whose size is between 1 and 10
- An integer power of 10

In general a number in the scientific notation is of the form $\boldsymbol{a} \times \mathbf{10}^{\boldsymbol{n}}$ where $\mathbf{1} \leq \boldsymbol{a} < \mathbf{10}$ and $\boldsymbol{n}$ is an integer.

Example:

$$\mathbf{236} = \mathbf{2.36} \times \mathbf{100} = \mathbf{2.36} \times \mathbf{10^2}$$

Note that for large numbers (bigger than 10) the power of 10 is a positive integer and for a small number (less than 1) the power of 10 is negative.

A similar example is, find x when $5^{3x+1} = 5^{x-7}$. The same base is raised to a power each side, therefore the powers must be the same, i.e. $3x + 1 = x - 7$.

We have reduced the problem from an exponent equation to a linear equation. The solution is then:

$$3x + 1 = x - 7$$

$$2x = -8$$

$$x = -4$$

A slightly more complicated example is $2^x = 4^2$. The solution is not $x = 2$ because different base values appear on each side. To solve this equation we must write 4 as a power of 2 and then simplify:

$$2^x = 4^2$$

$$2^x = (2^2)^2 = 2^4$$

$$x = 4$$

Rules for indices

The Product Rule: When multiplying powers of the same quantity add the indices.

$$a^m \times a^n = a^{m+n}$$

The Quotient Rule: To divide powers of the same quantity subtract the indices.

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

The Power Rule: When raising on power of a quantity to another power, multiply the indices.

$$(a^m)^n = a^{m \times n}$$

The Zero Index: Any quantity raised to the power zero equals 1.

$$a^0 = 1$$