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Phase and Pure Substance

The term *phase* refers to a quantity of matter that is homogeneous throughout in both chemical composition and physical structure. Homogeneity in physical structure means that the matter is all *solid*, or all *liquid*, or all *vapor* (or, equivalently, all *gas*). A system can contain one or more phases. For example, a system of liquid water and water vapor (steam) contains *two* phases. A *pure substance* is one that is uniform and invariable in chemical composition. A pure substance can exist in more than one phase, but its chemical composition must be the same in each phase. For example, if liquid water and water vapor form a system with two phases, the system can be regarded as a pure substance because each phase has the same composition. The nature of phases that coexist in equilibrium is addressed by the *phase rule* (Section 1.3, Multicomponent Systems).

Equilibrium

Equilibrium means a condition of balance. In thermodynamics the concept includes not only a balance of forces, but also a balance of other influences. Each kind of influence refers to a particular aspect of thermodynamic (complete) equilibrium. *Thermal* equilibrium refers to an equality of temperature, *mechanical* equilibrium to an equality of pressure, and *phase* equilibrium to an equality of chemical potentials (Section 1.3, Multicomponent Systems). *Chemical* equilibrium is also established in terms of chemical potentials (Section 1.4, Reaction Equilibrium). For complete equilibrium, the several types of equilibrium must exist individually.

To determine if a system is in thermodynamic equilibrium, one may think of testing isolate the system from its surroundings and watch for changes in its object the property of t des. If there are no changes, it may be concluded that the system was in equilibrian 27 h. Moment it was isolated. The is with isolated, it cannot interact with its system can be said to be at an *equilibrium state*. Whe surroundings; however, its state can change as a b ence of spontan or s events occurring internally as its intensive properties, such as the practice and pressure tend toward wiferm values. When all such equine n.m. temperature and pressure are uniform changes cease, the system is in equilibrium. At with height can exist, as in a vertical column throughout ignificant, a pressure va riation Temperature

A scale of temperature independent of the *thermometric substance* is called a *thermodynamic* temperature scale. The Kelvin scale, a thermodynamic scale, can be elicited from the second law of thermodynamics (Section 1.1, The Second Law of Thermodynamics, Entropy). The definition of temperature following from the second law is valid over all temperature ranges and provides an essential connection between the several *empirical* measures of temperature. In particular, temperatures evaluated using a *constant-volume gas thermometer* are identical to those of the Kelvin scale over the range of temperatures where gas thermometry can be used.

The empirical gas scale is based on the experimental observations that (1) at a given temperature level all gases exhibit the same value of the product $p\overline{v}$ (p is pressure and \overline{v} the specific volume on a molar basis) if the pressure is low enough, and (2) the value of the product $p\overline{v}$ increases with the temperature level. On this basis the gas temperature scale is defined by

$$T = \frac{1}{\overline{R}} \lim_{p \to 0} (p\overline{\nu})$$

where *T* is temperature and \overline{R} is the *universal gas constant*. The absolute temperature at the *triple point* of water (Section 1.3, *P-v-T* Relations) is fixed by international agreement to be 273.16 K on the *Kelvin* temperature scale. \overline{R} is then evaluated experimentally as $\overline{R} = 8.314 \text{ kJ/kmol} \cdot \text{K} (1545 \text{ ft} \cdot \text{lbf/lbmol} \cdot ^{\circ}\text{R})$.

The *Celsius termperature scale* (also called the centigrade scale) uses the degree Celsius (°C), which has the same magnitude as the Kelvin. Thus, temperature *differences* are identical on both scales. However, the zero point on the Celsius scale is shifted to 273.15 K, as shown by the following relationship between the Celsius temperature and the Kelvin temperature:

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$$T(^{\circ}C) = T(K) - 273.15$$
 (1.1)

On the Celsius scale, the triple point of water is 0.01°C and 0 K corresponds to -273.15°C.

Two other temperature scales are commonly used in engineering in the U.S. By definition, the *Rankine scale*, the unit of which is the degree Rankine (°R), is proportional to the Kelvin temperature according to

$$T(^{\circ}R) = 1.8T(K)$$
 (1.2)

The Rankine scale is also an absolute thermodynamic scale with an absolute zero that coincides with the absolute zero of the Kelvin scale. In thermodynamic relationships, temperature is always in terms of the Kelvin or Rankine scale unless specifically stated otherwise.

A degree of the same size as that on the Rankine scale is used in the *Fahrenheit scale*, but the zero point is shifted according to the relation

$$T(^{\circ}F) = T(^{\circ}R) - 459.67 \tag{1.3}$$

Substituting Equations 1.1 and 1.2 into Equation 1.3 gives

$$T(^{\circ}\mathrm{F}) = 1.8T(^{\circ}\mathrm{C}) + 32$$

This equation shows that the Fahrenheit temperature of the *ice point* 0 to 7s 32°F and of the *steam point* (100°C) is 212°F. The 100 Celsius or Kelvin degree serve on the ice point and steam point corresponds to 180 Fahrenheit or Rankine degree.

To provide a standard for temperatur in exprement taking intra acounce of theoretical and practical considerations, the International Temperature Scale of 199 (ITS 90) is defined in such a way that the temperature mean receive it conforms with the Permit binamic temperature, the unit of which is the Kelvin a within the limits of accurace of meanment obtainable in 1990. Further discussion of ITSto it provided by Prester P. International (950).

The First Law of Thermodynamics, Energy

Energy is a fundamental concept of thermodynamics and one of the most significant aspects of engineering analysis. Energy can be *stored* within systems in various macroscopic forms: kinetic energy, gravitational potential energy, and internal energy. Energy can also be *transformed* from one form to another and *transferred* between systems. For closed systems, energy can be transferred by *work* and *heat transfer*. The total amount of energy is *conserved* in all transformations and transfers.

Work

In thermodynamics, the term *work* denotes a means for transferring energy. Work is an effect of one system on another that is identified and measured as follows: work is done by a system on its surroundings if the *sole effect* on everything external to the system *could have been* the raising of a weight. The test of whether a work interaction has taken place is not that the elevation of a weight is actually changed, nor that a force actually acted through a distance, but that the sole effect *could be* the change in elevation of a weight. The magnitude of the work is measured by the number of standard weights that could have been raised. Since the raising of a weight is in effect a force acting through a distance, the work concept of mechanics is preserved. This definition includes work effects such as is associated with rotating shafts, displacement of the boundary, and the flow of electricity.

Work done by a system is considered positive: W > 0. Work done on a system is considered negative: W < 0. The time rate of doing work, or *power*, is symbolized by \dot{W} and adheres to the same sign convention.

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$$\eta = \frac{W_{cycle}}{Q_A} = 1 - \frac{Q_R}{Q_A} \tag{1.9}$$

The thermal efficiency is strictly less than 100%. That is, some portion of the energy Q_A supplied is invariably rejected $Q_R \neq 0$.

The Second Law of Thermodynamics, Entropy

Many statements of the second law of thermodynamics have been proposed. Each of these can be called a statement of the second law *or* a corollary of the second law since, if one is invalid, all are invalid. In every instance where a consequence of the second law has been tested directly or indirectly by experiment it has been verified. Accordingly, the basis of the second law, like every other physical law, is experimental evidence.

Kelvin-Planck Statement

The Kelvin-Plank statement of the second law of thermodynamics refers to a *thermal reservoir*. A thermal reservoir is a system that remains at a constant temperature even though energy is added or removed by heat transfer. A reservoir is an idealization, of course, but such a system can be approximated in a number of ways — by the Earth's atmosphere, large bodies of water (lakes, oceans), and so on. Expressive properties of thermal reservoirs, such as internal energy, can change in interactions with other systems even though the reservoir temperature remains constant, however.

The Kelvin-Planck statement of the second law can be given as follow: the impossible for any system to operate in a thermodynamic cycle and deliver is not in an effective of energy by work to its surroundings while receiving energy by heat transfer from a shall mermal reservor: in other words, a perpetualmotion machine of the second kind is not sole. Expressed analytically, the Kelvin-Planck statement is

where the words *single reserver* explasize that the system communicates thermally only with a single reservoir as it executes the cycle. The "less than" sign applies when *internal irreversibilities* are present as the system of interest undergoes a cycle and the "equal to" sign applies only when no irreversibilities are present.

single reservoir)

Irreversibilities

A process is said to be *reversible* if it is possible for its effects to be eradicated in the sense that there is some way by which *both* the system and its surroundings can be *exactly restored* to their respective initial states. A process is *irreversible* if there is no way to undo it. That is, there is no means by which the system and its surroundings can be exactly restored to their respective initial states. A system that has undergone an irreversible process is not necessarily precluded from being restored to its initial state. However, were the system restored to its initial state, it would not also be possible to return the surroundings to their initial state.

There are many effects whose presence during a process renders it irreversible. These include, but are not limited to, the following: heat transfer through a finite temperature difference; unrestrained expansion of a gas or liquid to a lower pressure; spontaneous chemical reaction; mixing of matter at different compositions or states; friction (sliding friction as well as friction in the flow of fluids); electric current flow through a resistance; magnetization or polarization with hysteresis; and inelastic deformation. The term *irreversibility* is used to identify effects such as these.

Irreversibilities can be divided into two classes, *internal* and *external*. Internal irreversibilities are those that occur within the system, while external irreversibilities are those that occur within the surroundings, normally the immediate surroundings. As this division depends on the location of the boundary there is some arbitrariness in the classification (by locating the boundary to take in the

The Clausius Inequality

The Clausius inequality provides the basis for introducing two ideas instrumental for quantitative evaluations of processes of systems from a second law perspective: *entropy* and *entropy generation*. The Clausius inequality states that

$$\oint \left(\frac{\delta Q}{T}\right)_b \le 0 \tag{1.13a}$$

where δQ represents the heat transfer at a part of the system boundary during a portion of the cycle, and *T* is the absolute temperature at that part of the boundary. The symbol δ is used to distinguish the differentials of *nonproperties*, such as heat and work, from the differentials of properties, written with the symbol *d*. The subscript *b* indicates that the integrand is evaluated at the boundary of the system executing the cycle. The symbol \oint indicates that the integral is to be performed over all parts of the boundary and over the entire cycle. The Clausius inequality can be demonstrated using the Kelvin-Planck statement of the second law, and the significance of the inequality is the same: the equality applies when there are no internal irreversibilities as the system executes the cycle, and the inequality applies when internal irreversibilities are present.

The Clausius inequality can be expressed alternatively as

ssed alternatively as $\oint \left(\frac{\delta Q}{T}\right) = -S_{er} = 5$ (1.13b)

where S_{gen} can be viewed as representing the *strength* of the inequality the value of S_{gen} is positive when internal irreversibilities are present, zero when represent representing the value of S_{gen} is a measure of the irreversibilities present within the system never be negative. As a dingly, S_{gen} is a measure of the irreversibilities present within the system measure the cycle. In the next reprise S_{gen} is identified as the *entropy* generated (or *produced*) by internal irreversibilities in never explet.

Entropy and Entropy Generation

Entropy

Consider two cycles executed by a closed system. One cycle consists of an internally reversible process A from state 1 to state 2, followed by an internally reversible process C from state 2 to state 1. The other cycle consists of an internally reversible process B from state 1 to state 2, followed by the same process C from state 2 to state 1 as in the first cycle. For these cycles, Equation 1.13b takes the form

$$\left(\int_{1}^{2} \frac{\delta Q}{T}\right)_{A} + \left(\int_{2}^{1} \frac{\delta Q}{T}\right)_{C} = -S_{gen} = 0$$
$$\left(\int_{1}^{2} \frac{\delta Q}{T}\right)_{B} + \left(\int_{2}^{1} \frac{\delta Q}{T}\right)_{C} = -S_{gen} = 0$$

where S_{gen} has been set to zero since the cycles are composed of internally reversible processes. Subtracting these equations leaves

$$\left(\int_{1}^{2} \frac{\delta Q}{T}\right)_{A} = \left(\int_{1}^{2} \frac{\delta Q}{T}\right)_{B}$$

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Applying the definition of entropy change, the second integral of the foregoing equation can be expressed as

$$S_1 - S_2 = \int_2^1 \left(\frac{\delta Q}{T}\right)_{int}_{rev}$$

Introducing this and rearranging the equation, the *closed system entropy balance* results:

When the end states are fixed, the entropy change on the left side of Equation 1.16 can be evaluated independently of the details of the process from state 1 to state 2. However, the two terms on the right side depend explicitly on the nature of the process and cannot be determined solely from knowledge of the end states. The first term on the right side is associated with heat transfer to of them the system during the process. This term can be interpreted as the *entropy transfer associated with (or accompanying) heat transfer.* The direction of entropy transfer is the same as the first on of the heat transfer, and the same sign convention applies as for heat transfer appear with the means that entropy is transferred into the system, and a negative value means that entropy is transferred out. The entropy change of a system s of accounted for solely by entropy ansfer, but is also due to the

The entropy change of a sistem is a accounted for soluty by entropy ansfer, but is also due to the second term on the right ide of Equation 1.16 renote (b) U_{gen} . The term S_{gen} is positive when internal irreversibilities are observed during the process and vanishes when internal irreversibilities are absent. D is carried else right is a superposed by every transfer of the process and vanishes when internal irreversibilities are absent. D is carried else right is a superposed by every transfer of the process and vanishes when internal irreversibilities are absent. D is carried else right is a superposed of the process and vanishes when internal irreversibilities are absent. D is carried else right is a superposed of the process and vanishes when internal irreversibilities are absent. D is carried by irreversibilities and conserved only in the limit as irreversibilities are reduced to zero. Since S_{gen} measures the effect of irreversibilities present within a system during a process, its value depends on the nature of the process and not solely on the end states. Entropy generation is *not* a property.

When applying the entropy balance, the objective is often to evaluate the entropy generation term. However, the value of the entropy generation for a given process of a system usually does not have much significance by itself. The significance is normally determined through comparison. For example, the entropy generation within a given component might be compared to the entropy generation values of the other components included in an overall system formed by these components. By comparing entropy generation values, the components where appreciable irreversibilities occur can be identified and rank ordered. This allows attention to be focused on the components that contribute most heavily to inefficient operation of the overall system.

To evaluate the entropy transfer term of the entropy balance requires information regarding both the heat transfer and the temperature on the boundary where the heat transfer occurs. The entropy transfer term is not always subject to direct evaluation, however, because the required information is either unknown or undefined, such as when the system passes through states sufficiently far from equilibrium. In practical applications, it is often convenient, therefore, to enlarge the system to include enough of the immediate surroundings that the temperature on the boundary of the *enlarged system* corresponds to the ambient temperature, T_{amb} . The entropy transfer term is then simply Q/T_{amb} . However, as the irreversibilities present would not be just those for the system of interest but those for the enlarged system, the entropy generation term would account for the effects of internal irreversibilities within the

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FIGURE 1.1 One-inlet, one-outlet control volume at steady state.

transfers across the boundary. This may be the result of one or more of the following: (1) the outer surface of the control volume is insulated, (2) the outer surface area is too small for there to be effective heat transfer, (3) the temperature difference between the control volume and its surroundings is small enough that the heat transfer can be ignored, (4) the gas or liquid passes through the (or rely or une so quickly that there is not enough time for significant heat transfer to occur Techwork term \dot{W}_{cv} drops out of the energy rate balance when there are no rotating shafts distributed of the boundary, electrical effects, or other work mechanisms associated with the outer volume being considered. The changes in kinetic and potential energy of Equation 27a are frequently negligible relative to other terms in the equation.

The special forms of Equations 1.27a and 1.28a lifer in Table 1.1 are obtained as follows: when there is no heard under, Equation 1.28e gives

$$s_e - s_i = \frac{\dot{S}_{gen}}{\dot{m}} \ge 0 \tag{1.28b}$$

(no heat transfer)

Accordingly, when irreversibilities are present within the control volume, the specific entropy increases as mass flows from inlet to outlet. In the ideal case in which no internal irreversibilities are present, mass passes through the control volume with no change in its entropy — that is, *isentropically*.

For no heat transfer, Equation 1.27a gives

$$\dot{W}_{cv} = \dot{m} \left[\left(h_i - h_e \right) + \left(\frac{v_i^2 - v_e^2}{2} \right) + g(z_i - z_e) \right]$$
(1.27b)

A special form that is applicable, at least approximately, to *compressors*, *pumps*, and *turbines* results from dropping the kinetic and potential energy terms of Equation 1.27b, leaving

$$\dot{W}_{cv} = \dot{m} \left(h_i - h_e \right) \tag{1.27c}$$

(compressors, pumps, and turbines)

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$$\bar{c}_p = \sum_{i=1}^{J} y_i \bar{c}_{pi} \tag{1.61e}$$

When working on a mass basis, expressions similar in form to Equations 1.61 can be written using mass and mass fractions in place of moles and mole fractions, respectively, and using u, h, s, c_p , and c_y in place of \overline{u} , h, \overline{s} , \overline{c}_p , and \overline{c}_v , respectively.

The internal energy and enthalpy of an ideal gas depend only on temperature, and thus the \bar{u}_i and h_i terms appearing in Equations 1.61 are evaluated at the temperature of the mixture. Since entropy depends on two independent properties, the \bar{s}_i terms are evaluated either at the temperature and the partial pressure p_i of component *i*, or at the temperature and volume of the mixture. In the former case

$$S = \sum_{i=1}^{j} n_i \overline{s}_i (T, p_i)$$

$$= \sum_{i=1}^{j} n_i \overline{s}_i (T, x_i p)$$
(1.62)

(1.63)

Inserting the expressions for *H* and *S* given by Equations 1.61b and 1.61c into the Gibbs Juryson, = H - TS, G = H - TS,

 $G = \sum_{i=1}^{j} n_i \overline{h_i} \mathbf{V} \mathbf{V} \mathbf{O}_{i=1}^{j} \mathbf{k}_i s_i (T, p_i)$ $= \sum_{i=1}^{j} n_i t_i (\mathbf{A}_i) \mathbf{O}_{i=1}^{j}$ where the molar-specific ribbs runction of component *i* is $g_i(T, p_i) = h_i(T) - Ts_i(T, p_i)$. The Gibbs function of *i* can be expressed alternatively as

$$\overline{g}_i(T, p_i) = \overline{g}_i(T, p') + \overline{R}T\ln(p_i/p')$$

$$= \overline{g}_i(T, p') + \overline{R}T\ln(x_i p/p')$$
(1.64)

were p' is some specified pressure. Equation 1.64 is obtained by integrating Equation 1.32d at fixed temperature T from pressure p' to $p_{i'}$.

Moist Air

An ideal gas mixture of particular interest for many practical applications is moist air. Moist air refers to a mixture of dry air and water vapor in which the dry air is treated as if it were a pure component. Ideal gas mixture principles usually apply to moist air. In particular, the *Dalton model is* applicable, and so the mixture pressure p is the sum of the partial pressures p_a and p_y of the dry air and water vapor, respectively.

Saturated air is a mixture of dry air and saturated water vapor. For saturated air, the partial pressure of the water vapor equals $p_{sat}(T)$, which is the saturation pressure of water corresponding to the dry-bulb (mixture) temperature T. The makeup of moist air can be described in terms of the humidity ratio (specific humidity) and the relative humidity. The bulb of a wet-bulb thermometer is covered with a wick saturated with liquid water, and the *wet-bulb* temperature of an air-water vapor mixture is the temperature indicated by such a thermometer exposed to the mixture.

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When a sample of moist air is cooled at constant pressure, the temperature at which the sample becomes saturated is called the *dew point temperature*. Cooling below the dew point temperature results in the condensation of some of the water vapor initially present. When cooled to a final equilibrium state at a temperature below the dew point temperature, the original sample would consist of a gas phase of dry air and saturated water vapor in equilibrium with a liquid water phase.

Psychrometric charts are plotted with various moist air parameters, including the dry-bulb and wetbulb temperatures, the humidity ratio, and the relative humidity, usually for a specified value of the mixture pressure such as 1 atm.

Generalized Charts for Enthalpy, Entropy, and Fugacity

The changes in enthalpy and entropy between two states can be determined in principle by correcting the respective property change determined using the ideal gas model. The corrections can be obtained, at least approximately, by inspection of the generalized enthalpy correction and entropy correction charts, Figures 1.12 and 1.13, respectively. Such data are also available in tabular form (see, e.g., Reid and Sherwood, 1966) and calculable using a generalized equation for the compressibility factor (Reynolds, 1979). Using the superscript * to identify ideal gas property values, the changes in specific enthalpy and specific entropy between states 1 and 2 are

$$\overline{h}_{2} - \overline{h}_{1} = \underline{\overline{h}_{2}^{*}} - \overline{\overline{h}_{1}^{*}} - \overline{R}T_{c} \left[\left(\frac{\overline{h}^{*} - \overline{h}}{\overline{R}T_{c}} \right)_{2} - \left(\frac{\overline{h}^{*} - \overline{h}}{\overline{R}D_{1}} \right)_{1} \right] \qquad (1.65a)$$

$$\overline{P} = \overline{P} \left[\frac{\overline{P}}{\overline{P}} + \overline{S}_{1} - \overline{R} \left[\left(\frac{\overline{S}^{*} - \overline{S}}{\overline{P}} \right) - \left(\frac{\overline{A}^{*} - \overline{A}}{\overline{R}} \right)_{1} \right] \right] \qquad (1.65b)$$

Define modellined term on the registrice of each expression represents the respective property change assuming ideal gas behavior. The second underlined term is the correction that must be applied to the ideal gas value to obtain the actual value. The quantities $(\bar{h}^* - \bar{h})/\bar{R}T_c$ and $(\bar{s}^* - \bar{s})/\bar{R}$ at state 1 would be read from the respective correction chart or table or calculated, using the reduced temperature T_{R1} and reduced pressure p_{R1} corresponding to the temperature T_1 and pressure p_1 at state 1, respectively. Similarly, $(\bar{h}^* - \bar{h})/\bar{R}T_c$ and $(\bar{s}^* - \bar{s})/\bar{R}$ at state 2 would be obtained using T_{R2} and p_{R2} . Mixture values for T_c and p_c determined by applying Kay's rule or some other mixture rule also can be used to enter the generalized enthalpy correction and entropy correction charts.

Figure 1.14 gives the *fugacity* coefficient, f/p, as a function of reduced pressure and reduced temperature. The fugacity f plays a similar role in determining the specific Gibbs function for a real gas as pressure plays for the ideal gas. To develop this, consider the variation of the specific Gibbs function with pressure at fixed temperature (from Table 1.2)

$$\left.\frac{\partial g}{\partial p}\right|_T = v$$

For an ideal gas, integration at fixed temperature gives

$$g^* = RT \ln p + C(T)$$

where C(T) is a function of integration. To evaluate g for a real gas, fugacity replaces pressure,

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FIGURE 1.13 Generalized entropy correction chart. (*Source:* Adapted from Van Wylen, G. J. and Sonntag, R. E. 1986. *Fundamentals of Classical Thermodynamics*, 3rd ed., English/SI. Wiley, New York.)

$$g - g^* = RT \ln \frac{f}{p} \tag{1.66}$$

As pressure is reduced at fixed temperature, f/p tends to unity, and the specific Gibbs function is given by the ideal gas value.

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1.4 Combustion

The thermodynamic analysis of reactive systems is primarily an extension of principles presented in Sections 1.1 to 1.3. It is necessary, though, to modify the methods used to evaluate specific enthalpy and entropy.

Reaction Equations

In combustion reactions, rapid oxidation of combustible elements of the fuel results in energy release as combustion products are formed. The three major combustible chemical elements in most common fuels are carbon, hydrogen, and sulfur. Although sulfur is usually a relatively unimportant contributor to the energy released, it can be a significant cause of pollution and corrosion.

The emphasis in this section is on hydrocarbon fuels, which contain hydrogen, carbon, sulfur, and possibly other chemical substances. Hydrocarbon fuels may be liquids, gases, or solids such as coal. Liquid hydrocarbon fuels are commonly derived from crude oil through distillation and cracking processes. Examples are gasoline, diesel fuel, kerosene, and other types of fuel oils. The compositions of liquid fuels are commonly given in terms of mass fractions. For simplicity in combustion calculations, gasoline is often considered to be octane, C_8H_{18} , and diesel fuel is considered to be dodecane, $C_{12}H_{26}$. Gaseous hydrocarbon fuels are obtained from natural gas wells or are produced in certain chertical processes. Natural gas normally consists of several different hydrocarbons, with the major to soft the their methane, CH_4 . The compositions of gaseous fuels are commonly given in terms of gaseous fuels are commonly given in terms of soft are synthesized from eral of shale, and tar sands. The composition of coal varies considerably with the location from eral of shale. For combustion calculations, the makeup of coal is usually expressed as maltimate analysis. Giving the composition on a mass basis in terms of the relative end of the chemical elements (curbor) will, hydrogen, nitrogen, oxygen) and ash.

A fuel is said contrationed *completely* that if the carbon present in the fuel is burned to carbon dioxide that the hydrogen is burger to we'r, and all of the sulfur is burned to sulfur dioxide. In anacice, these condition the relative above fulfilled and combustion is *incomplete*. The presence of carbon monoxide (CO) in the products indicates incomplete combustion. The products of combustion of *actual* combustion reactions and the relative amounts of the products can be determined with certainty only by experimental means. Among several devices for the experimental determination of the composition of products of combustion are the *Orsat analyzer*, *gas chromatograph*, *infrared analyzer*, and *flame ionization detector*. Data from these devices can be used to determine the makeup of the gaseous products of combustion. Analyses are frequently reported on a "dry" basis: mole fractions are determined for all gaseous products as if no water vapor were present. Some experimental procedures give an analysis including the water vapor, however.

Since water is formed when hydrocarbon fuels are burned, the mole fraction of water vapor in the gaseous products of combustion can be significant. If the gaseous products of combustion are cooled at constant mixture pressure, the *dew point temperature* (Section 1.3, Ideal Gas Model) is reached when water vapor begins to condense. Corrosion of duct work, mufflers, and other metal parts can occur when water vapor in the combustion products condenses.

Oxygen is required in every combustion reaction. Pure oxygen is used only in special applications such as cutting and welding. In most combustion applications, air provides the needed oxygen. Idealizations are often used in combustion calculations involving air: (1) all components of air other than oxygen (O_2) *are* lumped with nitrogen (N_2). On a molar basis air is then considered to be 21% oxygen and 79% nitrogen. With this idealization the molar ratio of the nitrogen to the oxygen in combustion air is 3.76; (2) the water vapor present in air may be considered in writing the combustion equation or ignored. In the latter case the combustion air is regarded as *dry*; (3) additional simplicity results by regarding the nitrogen present in the combustion air as inert. However, if high-enough temperatures are attained, nitrogen can form compounds, often termed NO_x , such as nitric oxide and nitrogen dioxide.

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$\log_{10} K$										
								$\mathrm{CO}_2 + \mathrm{H}_2$		
T (T)	II 🛆 MI	$0 \Rightarrow 20$	$\mathbf{N} \leftrightarrow \mathbf{2N}$	$\frac{1}{2}O_2 + \frac{1}{2}N_2$	$H_2O \Leftrightarrow$	$H_2O \Leftrightarrow$	$CO_2 \Leftrightarrow$		Temp	
Temp (K)	$\mathbf{n}_2 \Leftrightarrow 2\mathbf{n}$	$0_2 \Leftrightarrow 20$	$\mathbb{N}_2 \Leftrightarrow 2\mathbb{N}$	⇔NU	$\mathbf{n}_{2} + \frac{1}{2}\mathbf{O}_{2}$	$\mathbf{OH} + 7_{2}\mathbf{H}_{2}$	$0 + 7_2 0_2$	$CO + H_2O$	(° K)	
298	-71.224	-81.208	-159.600	-15.171	-40.048	-46.054	-45.066	-5.018	537	
500	-40.316	-45.880	-92.672	-8.783	-22.886	-26.130	-25.025	-2.139	900	
1000	-17.292	-19.614	-43.056	-4.062	-10.062	-11.280	-10.221	-0.159	1800	
1200	-13.414	-15.208	-34.754	-3.275	-7.899	-8.811	-7.764	+0.135	2160	
1400	-10.630	-12.054	-28.812	-2.712	-6.347	-7.021	-6.014	+0.333	2520	
1600	-8.532	-9.684	-24.350	-2.290	-5.180	-5.677	-4.706	+0.474	2880	
1700	-7.666	-8.706	-22.512	-2.116	-4.699	-5.124	-4.169	+0.530	3060	
1800	-6.896	-7.836	-20.874	-1.962	-4.270	-4.613	-3.693	+0.577	3240	
1900	-6.204	-7.058	-19.410	-1.823	-3.886	-4.190	-3.267	+0.619	3420	
2000	-5.580	-6.356	-18.092	-1.699	-3.540	-3.776	-2.884	+0.656	3600	
2100	-5.016	-5.720	-16.898	-1.586	-3.227	-3.434	-2.539	+0.688	3780	
2200	-4.502	-5.142	-15.810	-1.484	-2.942	-3.091	-2.226	+0.716	3960	
2300	-4.032	-4.614	-14.818	-1.391	-2.682	-2.809	-1.940	+0.742	4140	
2400	-3.600	-4.130	-13.908	-1.305	-2.443	-2.520	-1.679	+0.764	4320	
2500	-3.202	-3.684	-13.070	-1.227	-2.224	-2.270	-1.440	+0.784	4500	
2600	-2.836	-3.272	-12.298	-1.154	-2.021	-2.038	-1.219	+0.802	1.80	
2700	-2.494	-2.892	-11.580	-1.087	-1.833	-1.823	-1.015	F0.818	4860	
2800	-2.178	-2.536	-10.914	-1.025	-1.658	-1.624	0.825	-0.833	5040	
2900	-1.882	-2.206	-10.294	-0.967	-1.495	-1.438	- 1 49	+0.846	5220	
3000	-1.606	-1.898	-9.716	-0.913	-1.341		-0.485	+0.858	5400	
3100	-1.348	-1.610	-9.174	-0.863	2.1	-1.103	2	+0.869	5580	
3200	-1.106	-1.340	-8.664	<i>o</i> . 15	-1.067	-0.951	0, 89	+0.878	5760	
3300	-0.878	-1.086		-).771	-0.942	-0 809	-0.054	+0.888	5940	
3400	-0.664	-0-845	-1 736	-0.729	-0/2	-0.574	+0.071	+0.895	6120	
3500	-0.462		-7.312	-0.9	-0. 1	-0.547	+0.190	+0.902	6300	
90 M e: 6	sed on data	a from the l	ANAI C	ur zhenneal T	Tables, NSR	DS-NBS-37,	1971.			

TABLE 1.11 Logarithms to the Base 10 of the Equilibrium Constant K

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1.6 Vapor and Gas Power Cycles

Vapor and gas power systems develop electrical or mechanical power from energy sources of chemical, solar, or nuclear origin. In *vapor* power systems the *working fluid*, normally water, undergoes a phase change from liquid to vapor, and conversely. In *gas* power systems, the working fluid remains a gas throughout, although the composition normally varies owing to the introduction of a fuel and subsequent combustion. The present section introduces vapor and gas power systems.

The processes taking place in power systems are sufficiently complicated that idealizations are typically employed to develop tractable thermodynamic models. The *air standard analysis* of gas power systems considered later in the present section is a noteworthy example. Depending on the degree of idealization, such models may provide only qualitative information about the performance of the corresponding real-world systems. Yet such information is frequently useful in gauging how changes in major operating parameters might affect actual performance. Elementary thermodynamic models can also provide simple settings to assess, at least approximately, the advantages and disadvantages of features proposed to improve thermodynamic performance.

Rankine and Brayton Cycles

In their simplest embodiments vapor power and gas turbine power plants are represented conventionally in terms of four components in series, forming, respectively, the Rankine cycle and the Brytt neycle shown schematically in Table 1.14. The thermodynamically ideal counterpart of these cycles are composed of four internally reversible processes in series: two isention conserves alternated with two constant pressure processes. Table 1.14 provides property dia Parco time actual and corresponding ideal cycles. Each actual cycle is denoted 1-2-3-4-1; in the exchangers are cot shown Invoking Equation 1.29 for the ideal cycles, the heat added per undoff mass flowing is expresented by the area *under* the isobar from state 2s to state 3 area added per undoff mass flowing is the area *under* the isobar from state 4s to state 1 area added be area 1 as 3-48 1 corresponds the net heat added per unit of mass flowing. For my power cycle, the net added equals the net work done.

• Expressions for the trincipal energy transfers shown on the schematics of Table 1.14 are provided by Equations 1 to 4 of the table. They are obtained by reducing Equation 1.27a with the assumptions of negligible heat loss and negligible changes in kinetic and potential energy from the inlet to the outlet of each component. All quantities are positive in the directions of the arrows on the figure. Using these expressions, the thermal efficiency is

$$\eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} \tag{1.94}$$

To obtain the thermal efficiency of the ideal cycle, h_{2s} replaces h_2 and h_{4s} replaces h_4 in Equation 1.94.

Decisions concerning cycle operating conditions normally recognize that the thermal efficiency tends to increase as the average temperature of heat addition increases and/or the temperature of heat rejection decreases. In the Rankine cycle, a high average temperature of heat addition can be achieved by superheating the vapor prior to entering the turbine, and/or by operating at an elevated steam-generator pressure. In the Brayton cycle an increase in the compressor pressure ratio p_2/p_1 tends to increase the average temperature of heat addition. Owing to materials limitations at elevated temperatures and pressures, the state of the working fluid at the turbine inlet must observe practical limits, however. The turbine inlet temperature of the Brayton cycle, for example, is controlled by providing air far in excess of what is required for combustion. In a Rankine cycle using water as the working fluid, a low temperature of heat rejection is typically achieved by operating the condenser at a pressure below 1 atm. To reduce



- For turbines, compressors, and motors, consider the most thermodynamically efficient options.
- · Minimize the use of throttling; check whether power recovery expanders are a cost-effective alternative for pressure reduction.
- Avoid processes using excessively large thermodynamic driving forces (differences in temperature, pressure, and chemical composition). In particular, minimize the mixing of streams differing significantly in temperature, pressure, or chemical composition.
- The greater the mass rate of flow, the greater the need to use the exergy of the stream effectively.
- The lower the temperature level, the greater the need to minimize friction.

Flowsheeting or process simulation software can assist efforts aimed at improving thermodynamic effectiveness by allowing engineers to readily model the behavior of an overall system, or system components, under specified conditions and do the required thermal analysis, sizing, costing, and optimization. Many of the more widely used flowsheeting programs: ASPEN PLUS, PROCESS, and CHEMCAD are of the sequential-modular type. SPEEDUP is a popular program of the equation-solver type. Since process simulation is a rapidly evolving field, vendors should be contacted for up-to-date information concerning the features of flowsheeting software, including optimization capabilities (if any). As background for further investigation of suitable software, see Biegler (1989) for a survey of the capabilities of 15 software products. .co.uk

Exergoeconomics 1.8

s exergy and cost analyses to Exergoeconomics is an exergy-aided cost-reduction method AL 01 19 70 with information not available through is a social product to tool for understanding provide the designer or operator of an energy conventional energy, exergy, or cost analysis Exerge conomics is a solar po of tor understanding the interconnections between ther notynamics and economics and thus, the behavior of an energy conversion plant from the cost viewpoint. For a more detailed presentation of exergoeconomics, readers may refer to the following references: Bejan at al. (1996), Tsatsaronis (1993), and Tsatsaronis et al. (1984, 1 8. (195). Sections 1.2 through we outlined the principles for conducting detailed thermodynamic evaluations of thermal ste of polarticular, techniques have been developed for evaluating the thermodynamic inefficiencies of these systems: exergy destructions and exergy losses. However, we often need to know the cost of such inefficiencies. Knowledge of these costs is very useful for improving the costeffectiveness of the system — that is, for reducing the costs of the final products produced by the system.

In addition, if a system has more than one product, as for example the net power and saturated vapor of the cogeneration system shown in Figure 1.19, we would want to know the production cost for each product. This is a common problem in chemical plants where electrical power, chilled water, compressed air, and steam at various pressure levels are generated in one department and sold to another. The plant operator wants to know the true cost at which each of the utilities is generated; these costs are then charged to the appropriate final products according to the type and amount of each utility used to generate a final product. In the design of a thermal system, such cost allocation assists in pinpointing costineffective processes and operations and in identifying technical options that might improve the costeffectiveness of the system.

Accordingly, the objective of an exergoeconomic analysis might be to (1) calculate separately the costs of each product generated by a system having more than one product, (2) understand the cost formation process and the flow of costs in the system, (3) optimize specific variables in a single component, or (4) minimize the costs associated with the overall system.

An exergoeconomic analysis must be preceded by an exergy analysis (see Section 1.5) and an economic analysis (see Bejan et al. [1996] and Section 1.10). Because of the variation of costs from year to year, when we evaluate the design of a thermal system from the cost viewpoint we must use the cost levelization approach. Therefore, the cost values used throughout the following discussion are levelized costs. For conciseness, the term levelized is omitted, however.

rational basis for assigning costs to the interactions a thermal system experiences with its surroundings and to the sources of inefficiencies within it. We refer to this approach as *exergy costing*.

In exergy costing, a cost rate is associated with each exergy transfer. Thus, for entering and exiting streams of matter with exergy transfers \dot{E}_i and \dot{E}_e , respectively, power \dot{W} , and exergy transfer associated with heat transfer, \dot{E}_q , we write, respectively,

$$\dot{C}_i = c_i \dot{E}_i, \quad \dot{C}_e = c_e \dot{E}_e \tag{1.99}$$

$$\dot{C}_w = c_w \dot{W}, \quad \dot{C}_q = c_q \dot{E}_q \tag{1.100}$$

where c_i , c_e , c_w , and c_g denote *average* costs per unit of exergy, for example, in dollars per gigajoule (\$/GJ).

Exergy costing does not necessarily imply that costs associated with streams of matter are related *only* to the exergy rate of each respective stream. Nonexergy-related costs also can affect the total cost rate associated with material streams. Examples include the cost rates associated with a treated water stream at the outlet of a water treatment unit, an oxygen or nitrogen stream at the outlet of an air separation unit, a limestone stream supplied to a gasifier or fluidized-bed reactor, iron feedstock supplied to a metallurgical process, and an inorganic chemical fed to a chemical reactor. Accordingly, when significant nonexergy-related costs occur, the total cost rate associated with the material stream *j*, depoted by $\dot{C_j}^{\text{TOT}}$, is given by

$$\dot{C}_{j}^{\text{TOT}} = \dot{C}_{j} + \dot{C}_{j}^{\text{NE}}$$
(1.101)

where \dot{C}_j is the cost rate directly related to the exercise of scream j (e.g., Equation 1.99) and \dot{C}_j^{NE} is the cost rate due to nonexergetic effects. That the \dot{C}_j^{NE} represents a component of \vec{v} where \vec{v}_j is the cost rate due to nonexergetic effects. That the \dot{C}_j^{NE} represents a component of \vec{v} with one component effects. More details about \dot{C}_j^{NE} are given as the probability of the cost of the co

Post balance Page

Exergy costing usually involves cost balances formulated for each component separately. A cost balance applied to the *k*th system component indicates that the sum of cost rates associated with all exiting exergy transfers equals the sum of cost rates of all entering exergy transfers plus the appropriate charges due to capital investment (\dot{Z}_k^{CL}) and operating and maintenance expenses (\dot{Z}_k^{OM}) . The sum of the last two terms is denoted by (\dot{Z}_k) . For example, for a component receiving a heat transfer (subscript q) and generating power (subscript w), we write

$$\sum_{e} \dot{C}_{e,k} + \dot{C}_{w,k} = \dot{C}_{q,k} + \sum_{i} \dot{C}_{i,k} + \dot{Z}_{k}$$
(1.102)

This equation simply states that the total cost of the exiting exergy transfers equals the total expenditure to obtain them: the cost of the entering exergy streams plus the capital and other costs. When a component receives power (as in a compressor or a pump) the term $\dot{C}_{w,k}$ would move with its positive sign to the right side of this expression. The term $\dot{C}_{q,k}$ would appear with its positive sign on the left side if there is a heat transfer *from* the component. Cost balances are generally written so that all terms are positive.

Introducing the cost rate expressions of Equations 1.99 and 1.100, Equation 1.102 becomes

$$\sum_{e} \left(c_{e} \dot{E}_{e} \right)_{k} + c_{w,k} \dot{W}_{k} = c_{q,k} \dot{E}_{q,k} + \sum_{i} \left(c_{i} \dot{E}_{i} \right)_{k} + \dot{Z}_{k}$$
(1.103)

Step 3: Auxiliary Equations

In general, if there are Ne exergy streams exiting the component being considered, we have Ne unknowns and only one equation, the costs balance (Equation 1.103). Therefore, we need to formulate $N_e - 1$ auxiliary equations. This is accomplished with the aid of the F and P rules presented next:

- The F rule refers to the *removal of exergy* from an exergy stream within the component being considered. The F rule states that the total cost associated with this removal of exergy must be equal to the average cost at which the removed exergy was supplied to the same stream in upstream components. The number of auxiliary equations provided by the F rule is always equal to the number (N_{e, F}) of exiting exergy streams that are considered in the definition of the fuel for the component.
- The P rule refers to the *supply of exergy* to an exergy stream within the component being considered and to the costing of streams associated with the product. The P rule states that each exergy unit is supplied to any stream associated with the product at the same average cost, c_p . This cost is calculated from the cost balance and the equations obtained by applying the F rule. The number of auxiliary equations provided by the P rule is always equal to $N_{e, P} - 1$, where $N_{e, P}$ is the number of exiting exergy streams that are included in the product definition.

Since the total number of exiting streams (N_e) is equal to the sum $(N_{e,F} + N_{e,P})$, the F and P rules together provide the required $N_e - 1$ auxiliary equations.



FIGURE 1.20 Schematic of a component in a thermal system to define fuel, product, and auxiliary equations.

Step 1: Referring to Figure 1.20, there are seven exergy streams (1, 2, and 5 through 9) entering the component (subscript i) and seven exergy streams (3 through 8 and 10) exiting the component (subscript e). The streams shown in this figure are selected to cover all situations that might be encountered. In an actual component, however, not all of the streams shown in Figure 1.20 exist.

Step 2: The exergy streams 1 through 4 are associated with the chemical exergy or total exergy of the corresponding material streams. The purpose of owning and operating the component dictates that the entering streams 1 and 2 should be part of the fuel, whereas the exiting streams 3 and 4 should be part of the product. For the exergy streams with the numbers 5, 6, 7, and 8, the purpose of the component dictates the consideration of the respective exergy differences between outlet and inlet. These are positive for streams 7 and 8 and negative for streams 5 and 6. Streams 9 at the inlet and 10 at the outlet represent exergy streams associated with the transport of mechanical, electrical, or thermal energy. We conclude that exergy streams 1, 2, 5, 6, and 9 are associated with the fuel, whereas streams 3, 4, 7, 8, and 10 are associated with the product of the component. Thus, the fuel and product of the component, respectively, are

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$$\dot{E}_{F} = \dot{E}_{1} + \dot{E}_{2} + \left(\dot{E}_{5i} - \dot{E}_{5e}\right) + \left(\dot{E}_{6i} - \dot{E}_{6e}\right) + \dot{E}_{9}$$
(1.104)

$$\dot{E}_{P} = \dot{E}_{3} + \dot{E}_{4} + \left(\dot{E}_{7e} - \dot{E}_{7i}\right) + \left(\dot{E}_{8e} - \dot{E}_{8i}\right) + \dot{E}_{10}$$
(1.105)

Step 3: In accordance with Equation 1.103, the cost balance for the component is

$$\dot{C}_3 + \dot{C}_4 + \dot{C}_{5e} + \dot{C}_{6e} + \dot{C}_{7e} + \dot{C}_{8e} + \dot{C}_{10} = \dot{C}_1 + \dot{C}_2 + \dot{C}_{5i} + \dot{C}_{6i} + \dot{C}_{7i} + \dot{C}_{8i} + \dot{C}_9 + \dot{Z}$$
(1.106)

By grouping the terms associated with fuel and product, we obtain

$$\dot{C}_{3} + \dot{C}_{4} + \left(\dot{C}_{7e} - \dot{C}_{7i}\right) + \left(\dot{C}_{8e} - \dot{C}_{8i}\right) + \dot{C}_{10} = \dot{C}_{1} + \dot{C}_{2} + \left(\dot{C}_{5e} - \dot{C}_{5i}\right) + \left(\dot{C}_{6e} - \dot{C}_{6i}\right) + \dot{C}_{9} + \dot{Z} \quad (1.107)$$

The F rule states that the total cost rate associated with the removal of exergy from stream 5: $(\dot{C}_{5i} - \dot{C}_{5e})$ must be equal to the *average* cost at which the removed exergy: $(\dot{E}_{5i} - \dot{E}_{5e})$ was supplied to that stream in upstream components. Since each exergy unit was supplied to stream 5 at the average cost of c_{5i} , the F rule for stream 5 becomes

$$\dot{c}_{5i} - \dot{C}_{5e} = c_{5i} \left(\dot{E}_{5i} - \dot{E}_{5e} \right) \qquad (1.108)$$
From this equation we obtain
$$c_{5e} = c_{5i} \qquad (1.109)$$
Similarly, for stream (1.109)
$$c_{6e} = c_{6i} \qquad (1.110)$$

The P rule states that each exergy unit is *supplied* to all streams associated with the product at the *same* average cost, c_p . This rule leads to the following equations:

$$c_{3} = c_{4} = \frac{\dot{C}_{7e} - \dot{C}_{7i}}{\dot{E}_{7e} - \dot{E}_{7i}} = \frac{\dot{C}_{8e} - \dot{C}_{8i}}{\dot{E}_{8e} - \dot{E}_{8i}} = c_{10} = c_{P}$$
(1.111)

Since we assume that the cost rate \dot{Z} and all costs associated with all entering streams are known, we can calculate the unknowns \dot{C}_3 , \dot{C}_4 , \dot{C}_{5e} , \dot{C}_{6e} , \dot{C}_{7e} , \dot{C}_{8e} , and \dot{C}_{10} by solving the system of Equations 1.107, 1.109, 1.110, and 1.111.

Note that Equation 1.107 may be written as

$$c_{P} \dot{E}_{P} = \dot{C}_{1} + \dot{C}_{2} + \left(\dot{C}_{5i} - \dot{C}_{5e}\right) + \left(\dot{C}_{6i} - \dot{C}_{6e}\right) + \dot{C}_{9} + \dot{Z}$$
(1.112)

or

$$c_P \dot{E}_P = c_F \dot{E}_F + \dot{Z} \tag{1.113}$$

where \dot{E}_F , \dot{E}_P , and c_P are given in Equations 1.104, 1.105 and 1.111, respectively. The variable c_F denotes the average cost at which each exergy unit of \dot{E}_F is supplied to the component. Equation 1.112 demonstrates that the average cost (c_P) at which each exergy unit is supplied to all streams associated with the

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not always possible to express the purchased-equipment costs as a function of the appropriate thermodynamic decision variables.

- A significant decrease in the product costs may be achievable only through changes in the structure of the system, but such changes are seldom elicited from conventional optimization techniques focusing on the optimization of a particular structure. Moreover, it is not always practical to develop a mathematical optimization model for every promising system structure.
- Even if all the required information is available, the complexity of the system might not allow a satisfactory mathematical model to be formulated and solved in a reasonable time.

In such cases the application of exergoeconomic techniques may provide significant benefits for the optimization process. The more complex the thermal system the larger are the expected benefits, particularly when chemical reactions are involved. The interactions of exergoeconomics with several other areas during the optimization procedure are shown schematically in Figure 1.21. Exergoeconomics uses results from the synthesis, cost analysis, and simulation of thermal systems and provides useful information for the evaluation and optimization of these systems as well as for the application of expert systems to improve the design and operation of such systems.



FIGURE 1.21 Interactions of exergoeconomics with other areas of engineering and optimization procedure.

The following section presents the main features of a general methodology that can be used to evaluate and iteratively optimize the design of a thermal system.

An Iterative Exergoeconomic Procedure for Optimizing the Design of a Thermal System

The conventional approach to optimization is to iteratively optimize subsystems and/or ignore the influence of some structural changes and decision variables. An effective alternative approach for the optimization of complex systems is the following iterative exergoeconomic optimization technique that consists of seven steps:

1. In the first step a workable design is developed. The guidelines presented in Section 1.7 and in Bejan et al. (1996), Lin and Tsatsaronis (1993), Linnhoff et al. (1982), Sama (1993), Tsatsaronis (1993), and Tsatsaronis and Pisa (1994) may assist in developing a workable design that is

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A cogeneration system studied by Frangopoulos (1994), Tsatsaronis and Pisa (1994), Valero et al. (1994), and von Spakovsky (1994) is used in the following case to demonstrate the application of steps 1, 2, 4, and 7. Examples of exergoeconomic evaluations and improvements of complex thermal systems are discussed by Tsatsaronis et al. (1991, 1992, 1994).

Case Study

Figure 1.19 presents the base-case design of a cogeneration system that develops a net power output of 30 MW and provides 14 kg/s of saturated water vapor at 20 bar. Not all data assumed for this system are realistic. The investment costs have been artificially increased to demonstrate the application of the exergoeconomic methodology to a variety of components. The optimization of an actual cogeneration system would be significantly easier because the components of a gas turbine system would not be optimized individually, as done here.

The first five columns of Table 1.17 show relevant thermodynamic and economic data. The second column of Table 1.18 shows the assumed purchased-equipment costs (PEC) for each component in the base-case design. These costs are obtained from the cost equations given in Appendix B of Bejan et al. (1996). The remaining direct costs, as well as the indirect costs, are estimated. The total capital investment of the cogeneration system in the base case is approximately \$46 million in mid-1994 dollars. Table 7.9 in Bejan et al. (1996) summarizes the parameters and assumptions used in the economic analysis which is based on the revenue-requirement method (EPRI Technical Assessment Guide, 1991)

TABLE 1.17	Mass Flow Rate,	Temperature,	Pressure,	Exergy	Rate,	and	is I	at a Lor	the S	Streams
of the Cogene	ration System				0	50		_		

or the	eogeneration bystem			ATE			
State	Stream	Mass Flow Rate n kg D	Temper in T (K)	p (Colf	Exergy 🔃 v Rate F (MW)	Cost Flow Rate Ċ (\$/h)	Cost per Exergy Unit c (\$/GJ)
1	Air Air	91.28	298 1	1.01	0.000	0	0
		91.28	60.5.7	10.13	27.538	2756	27.80
	A.	9.2	850.0	9.62	41.938	3835	25.40
4	Combustion produces	J2-15	1520.0	9.14	101.454	5301	14.51
5	Combustion products	92.92	1006.2	1.10	38.782	2026	14.51
6	Combustion products	92.92	779.8	1.07	21.752	1137	14.51
7	Combustion products	92.92	426.9	1.01	2.773	145	14.51
8	Water	14.00	298.1	20.00	0.062	0	0
9	Water	14.00	485.6	20.00	12.810	1256	27.23
10	Methane	1.64	298.1	12.00	84.994	1398	4.57
11	Power to air compressor	—	—	—	29.662	2003	18.76
12	Net power	_	_	_	30.000	2026	18.76

TABLE 1.18 Values of the Purchased-Equipment Costs (PEC) and the Thermoeconomic Variables for the Base Design Case ($T_3 = 850$ K; $T_4 = 1520$ K; $p_2/p_1 = 10$; $\eta_{sr} = \eta_{st} = 0.86$)^a

	0		,	-		, isc	•3t	,			
Component	PEC (10 ⁶ \$)	8 (%)	Ė (MW)	у _D (%)	c _F (\$/GJ)	с _Р (\$/GJ)	Ċ _D (\$/h)	Ż (\$/h)	Ċ _D +Ż (\$/h)	r (%)	f (%)
Combustion Chamber	0.34	80.37	25.48	29.98	11.45	14.51	1050	68	1118	26.7	6.1
Gas Turbine	3.74	95.20	3.01	3.54	14.51	18.76	157	753	910	29.2	82.7
Air Compressor	3.73	92.84	2.12	2.50	18.76	27.80	143	753	896	48.2	84.0
HRSG Air Preheater	1.31 0.94	67.17 84.55	6.23 2.63	7.33 3.09	14.51 14.51	27.36 20.81	326 137	264 189	590 326	88.5 43.4	44.8 57.9

 a For the overall plant, we have $\dot{C}_{P,tot}=\$3617/h$ and $\dot{C}_{L,tot}=\dot{C}_7=\$145/h.$

Component	8 (%)	Ė _D (MW)	у _D (%)	c _F (\$/GJ)	с _Р (\$/GJ)	Ċ _D (\$/h)	Ż (\$/h)	$\dot{C}_{D} + \dot{Z}$ (\$/h)	r (%)	f (%)
Combustion	80.3	25.93	29.77	10.50	13.26	980	72	1052	26.3	6.8
chamber	(81.3)	(27.47)	(29.92)	(9.42)	(11.71)	(931)	(55)	(986)	(24.4)	(5.5)
Gas turbine	94.9	3.18	3.66	13.26	16.97	152	647	799	28.0	81.0
	(94.3)	(3.69)	(4.01)	(11.71)	(13.75)	(155)	(296)	(451)	(17.5)	(65.6)
Air	92.1	2.34	2.69	16.97	23.96	143	546	689	41.2	79.2
compressor	(90.5)	(2.99)	(3.25)	(13.75)	(18.38)	(148)	(324)	(472)	(33.6)	(68.7)
HRSG	66.6	6.40	7.35	13.26	25.60	305	261	566	93.1	46.1
	(67.6)	(6.10)	(6.65)	(11.71)	(23.51)	(257)	(284)	(541)	(100.7)	(52.5)
Air preheater	84.7	3.15	3.62	13.26	18.94	150	206	356	42.9	57.8
•	(85.6)	(4.97)	(4.90)	(11.71)	(16.53)	(190)	(275)	(464)	(41.2)	(59.2)
Overall plant	49.1	41.01	47.09	4.57	21.80	675	1922	2597	377.0	74.0
I ····	(46.6)	(44.79)	(48.79)	(4.57)	(19.06)	(736)	(1424)	(2160)	(317.2)	(65.9)

TABLE 1.19 Component Data for the Second Iteration Case: $(T_3 = 870 \text{ K}; T_4 = 1520 \text{ K}; p_2/p_1 = 9; \eta_{sc} = 85\%; \eta_{sf} = 85\%)^a$. The Values Given in Parentheses Refer to the Third Iteration Case Considered Here $(T_3 = 910 \text{ K}; T_4 = 1480 \text{ K}; p_2/p_1 = 7; \eta_{sc} = 0.83)^b$

^a For the overall plant in the new design case, we have $\dot{C}_{P,tot} = $3355/h$ and $\dot{C}_{L,tot} = \dot{C}_7 = $157/h$.

^b For the overall plant in this design case, we have $\dot{C}_{P,tot} = $2934/h$ and $\dot{C}_{L,tot} = \dot{C}_7 = $167/h$.

The relatively high value of f in the air preheater suggests a reduction in the T_3 value. As near in the first iteration, however, changes suggested by the evaluation of this component hould only be considered if they do not contradict changes suggested by component with righer value of the sum $(\dot{C}_D + \dot{Z})$.

Summarizing the foregoing suggestions from us the procession of each component, the following changes in the decision variable rate expected to in prove the cost effectiveness of the cogeneration system:

Increase the pressure ratio σ₁ as suggested by the evaluation of the combustion chamber and HRSG.
 Decrease the pressure ratio σ₂ σ₁ and the isentropic efficiencies η_{sc} and η_{st}, as suggested by the evaluations of the air compressor and gas turbine.

• Decrease the temperature T_4 , as suggested from the evaluations of the gas turbine and the HRSG.

Third Design Case

To illustrate the effect of the suggested changes in the decision variables on the overall costs, we use the following new set of values for the design variables:

$$T_3 = 910 \ K, \ T_4 = 1480 \ K, \ p_2 / p_1 = 7, \ \eta_{sc} = 0.83, \ \text{and} \ \eta_{st} = 0.83$$

The results from the thermoeconomic analysis for the last set of values are summarized in Table 1.19 by the values given in parentheses. A comparison of the corresponding values shown in Table 1.19 demonstrates the improvement in the cost-effectiveness of the last design case. As a result of these changes, the value of the objective function $\dot{C}_{P,tot}$ is reduced from \$3355/h to \$2934/h. The \dot{C}_7 value has increased from \$157/h in the new case to \$167/h in the last case. This increase, however, is outweighed by the decreases in the values of $(\dot{C}_{D,k} + \dot{Z}_k)$.

Additional Iterations

Additional iterations conducted in a similar way are necessary to further decrease the value of the objective function and/or establish a nearly optimal design case. The cost-optimal values of the decision variables (Bejan et al., 1996) are

$$T_3 = 910 \ K, T_4 = 1463.0 \ K, p_2/p_1 = 5.77, \eta_{sc} = 0.811, \text{ and } \eta_{st} = 0.845.$$

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With these values we obtain the objective function $\dot{C}_{P,tot} =$ \$2870/h. For the cost-optimal case, the exergetic efficiency of the overall system is 45.0%, the cost rate associated with the exergy loss is \$205/h, and the pinch temperature difference in the heat-recovery steam generator is 49.7 K.

1.10 Economic Analysis of Thermal Systems

Generally, each company has a preferred approach for conducting an economic analysis and calculating the costs associated with the products generated in a thermal system. Here only a few general introductory comments are made. A detailed discussion of an economic analysis is given in Bejan et al. (1996).

A complete economic analysis consists of the following three steps:

- Step1. Estimate the required total capital investment.
- *Step 2.* Determine the economic, financial, operating, and market-associated parameters for the detailed cost calculation.
- Step 3. Calculate the costs of all products generated by the system.

These steps are now discussed in turn.

Estimation of Total Capital Investment

The capital needed to purchase the land, build all the necessary facilities, and purchase and in tall the required machinery and equipment for a system is called the *fixed-capital investment*. The fixed-capital investment represents the total system cost, assuming a zero-time design and construction period (overnight construction). The *total capital investment* is the solid of the fixed-capital investment and other outlays (e.g., startup costs, working capital costs at Beensing, research and development, as well as interest during construction).

The costs of all permanent equipment, materials abor, an other resources involved in the fabrication, erection, and instruction of the permanent facil de the the *direct costs*. The *indirect costs* (e.g., costs a social for which engineering, supervision, and construction, including contractor's profit and contingencies) do not become able metal and of the facilities but are required for the orderly completion of the project. The fixed-capital investment is the sum of all direct and indirect costs.

The *purchased equipment costs* are estimated with the aid of vendors' quotations, quotations from experienced professional cost estimators, calculations using extensive cost databases, or estimation charts. If necessary, the cost estimates of purchased equipment must be adjusted for size (with the aid of scaling exponents) and for time (with the aid of cost indices). The remaining direct costs are associated with equipment installation, piping, instrumentation, controls, electrical equipment and materials, land, civil structural and architectural work, and service facilities. These direct costs, the indirects costs, and the other outlays, if they cannot be estimated directly, are calculated as a percentage of the purchased equipment costs or alternatively as a percentage of the fixed-capital investment.

For more detailed information on the estimation of the total capital investment, readers should consult the references by Bejan et al. (1996), Chauvel et al. (1976), Douglas (1988), Garrett (1989), Guthrie (1969, 1970, 1974), Humphreys (1991), Perry and Chilton (1973), Peters and Timmerhaus (1991), Ulrich (1984), and Valle-Riestra (1983).

Principles of Economic Evaluation

Decisions about expenditures generally require consideration of the *time value of money*: if an amount of money P (*present value*) is deposited in an account earning *i* percent interest per time period and the interest is compounded at the end of each of *n* time periods, the amount will grow to a *future value* (F):

$$F = P(1+i)^{n}$$
(1.124)

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$$\boldsymbol{a}_{I} = \boldsymbol{a}_{R} + \frac{d^{2}\boldsymbol{R}}{dt^{2}} + 2\boldsymbol{\Omega} \times \boldsymbol{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}) + \frac{d\boldsymbol{\Omega}}{dt} \times \boldsymbol{r}$$
(2.2.8)

where R is the position vector of the origin of the noninertial frame with respect to that of the inertial frame, Ω is the angular velocity of the noninertial frame, and r and V the position and velocity vectors in the noninertial frame. The third term on the right-hand side of Equation (2.2.8) is the Coriolis acceleration, and the fourth term is the centripetal acceleration. For a noninertial frame Equation (2.2.7) is then

$$F_{\text{total}} - \iiint_{\text{system}} \left[\frac{d^2 \mathbf{R}}{dt^2} + 2\Omega \times \mathbf{V} + \Omega \times (\Omega \times \mathbf{r}) + \frac{d\Omega}{dt} \times \mathbf{r} \right] \rho \, d\upsilon = \frac{D}{Dt} \left(\iiint_{\text{system}} \rho \mathbf{V} \, d\upsilon \right)$$

$$= \frac{d}{dt} \left(\iiint_{\text{CV}} \rho \mathbf{V} \, d\upsilon \right) + \iint_{\text{CS}} \rho \mathbf{V} \cdot \left(\mathbf{V}_r \cdot \mathbf{n} \right) dA$$
(2.2.9)

where the frame acceleration terms of Equation (2.2.8) have been brought to the left-hand side because to an observer in the noninertial frame they act as "apparent" body forces.

For a fixed control volume in an inertial frame for steady flow it follows from the by

whis of many control volume analysis for fluid flow problems. or r, the position vector than espect to a convenient origin, with the momentum This expression is The cross or for an density particle of mass dm, noting that $(dr/dt) \times V = 0$, leads to the **P** u tic v (2.2.6) written ntegral moment of m ...en (b.) en ation

$$\sum \boldsymbol{M} - \boldsymbol{M}_{I} = \frac{D}{Dt} \iiint_{\text{system}} \rho(\boldsymbol{r} \times \boldsymbol{V}) \, d\boldsymbol{\upsilon}$$
(2.2.11)

where ΣM is the sum of the moments of all the external forces acting on the system about the origin of r, and M_I is the moment of the apparent body forces (see Equation (2.2.9)). The right-hand side can be written for a control volume using the appropriate form of the Reynolds transport theorem.

Conservation of Energy

The conservation of energy law follows from the first law of thermodynamics for a moving system

$$\dot{Q} - \dot{W} = \frac{D}{Dt} \left(\iiint_{\text{system}} \rho e \ d\nu \right)$$
(2.2.12)

where \hat{Q} is the rate at which heat is added to the system, \hat{W} the rate at which the system works on its surroundings, and e is the total energy per unit mass. For a particle of mass dm the contributions to the specific energy e are the internal energy u, the kinetic energy $V^2/2$, and the potential energy, which in the case of gravity, the only body force we shall consider, is gz, where z is the vertical displacement opposite to the direction of gravity. (We assume no energy transfer owing to chemical reaction as well

(2.2.10)

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using the fact that

$$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + \mathbf{V} \cdot \nabla\rho \qquad (2.2.17)$$

Special cases:

1. Steady flow $[(\partial/\partial t) () \equiv 0]$

$$\nabla \cdot (\rho V) = 0 \tag{2.2.18}$$

2. Incompressible flow $(D\rho/Dt \equiv 0)$

$$\nabla \cdot \boldsymbol{V} = 0 \tag{2.2.19}$$

Momentum Conservation

We note first, as a consequence of mass conservation for a system, that the right-hand side of Equation (2.2.6) can be written as

$$\frac{D}{Dt}\left(\iiint_{\text{system}} \rho V \, d\upsilon\right) \equiv \iiint_{\text{system}} \rho \frac{DV}{Dt} \, d\upsilon$$

The total force acting on the system which appears on the left f_{b} of Equation (2.2.6) is the sum of body forces F_{b} and surface forces F_{s} . The body force are often given as forces per unit mass (e.g., gravity), and so can be written

The surface forces are correspondent in terms of the second-order stress tensor^{*}
$$\underline{\sigma} = \{\sigma_{ij}\}$$
, where σ_{ij} is defined as the force per unit area in the *i* direction on a planar element whose normal lies in the *j* direction.^{**} From elementary angular momentum considerations for an infinitesimal volume it can be shown that σ_{ij} is a symmetric tensor, and therefore has only six independent components. The total

surface force exerted on the system by its surroundings is given by

$$F_{s} = \iint_{\substack{\text{system}\\\text{surface}}} \underline{\sigma} \cdot \boldsymbol{n} \, dA, \text{ with } i - \text{component } F_{s_{i}} = \iint_{ij} \sigma_{ij} n_{j} \, dA \qquad (2.2.22)$$

The integral momentum conservation law Equation (2.2.6) can then be written

$$\iiint_{\text{system}} \rho \frac{DV}{Dt} \, d\upsilon = \iiint_{\text{system}} \rho f \, d\upsilon + \iint_{\substack{\text{system} \\ \text{surface}}} \underline{\sigma} \cdot \boldsymbol{n} \, dA \tag{2.2.23}$$

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^{*}We shall assume the reader is familiar with elementary Cartesian tensor analysis and the associated subscript notation and conventions. The reader for whom this is not true should skip the details and concentrate on the final principal results and equations given at the ends of the next few subsections.

^{**}This assignment of roles to the first and second subscripts of the stress tensor is a convention and is far from universal. Frequently, their roles are reversed, the first subscript denoting the direction of the normal to the planar element, the second the direction of the force.

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angular velocity $\frac{1}{2}$ ($\nabla \times u$), and a velocity or rate of deformation. The coefficient of η in the last term in Equation (2.2.29) is defined as the rate-of-strain tensor and is denoted by \underline{e} , in subscript form

$$e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$
(2.2.30)

From e we can define a rate-of-strain central quadric, along the principal axes of which the deforming motion consists of a pure straining extension or contraction.

Relationship Between Forces and Rate of Deformation

We are now in a position to determine the required relationship between the stress tensor $\underline{\sigma}$ and the rate of deformation. Assuming that in a static fluid the stress reduces to a (negative) hydrostatic or thermodynamic pressure, equal in all directions, we can write

$$\underline{\boldsymbol{\sigma}} = -p\underline{\boldsymbol{I}} + \underline{\boldsymbol{\tau}} \quad \text{or} \quad \boldsymbol{\sigma}_{ii} = -p\boldsymbol{\delta}_{ii} + \boldsymbol{\tau}_{ii} \tag{2.2.31}$$

where $\underline{\tau}$ is the viscous part of the total stress and is called the deviatoric stress tensor $\underline{\tau}$ is deficiently tensor, and δ_{ij} is the corresponding Kronecker delta ($\delta_{ij} = 0$ if $i \neq j$; $\delta_{ij} = 1$ if $i \neq j$). We make further assumptions that (1) the fluid exhibits no preferred directions; (2) the stresses undependent of any previous history of distortion; and (3) that the stress depends on the arbit local thermodynamic state and the kinematic state of the immediate neighborhood. Therefore, we assume that (1) is linearly proportional to the first spatial derivatives of u, the coefficient of proportionality depending only on the local thermodynamic state. These assumptions and the relations below which follow from them are appropriate for a Newtonian fluid. The termonon fluids, such as all and water under most conditions, are Newtonian, is the earter many other fluids, including many which arise in industrial applications, which exhibit socalled non-Newtonian properties when study of such non-Newtonian fluids, such as viscoelastic fluids, is the subject of the field of theorogy.

With the Newtonian fluid assumptions above, and the symmetry of $\underline{\tau}$ which follows from the symmetry of $\underline{\sigma}$, one can show that the viscous part $\underline{\tau}$ of the total stress can be written as

$$\underline{\tau} = \lambda (\nabla \cdot \boldsymbol{u}) \underline{I} + 2\mu \underline{e} \tag{2.2.32}$$

so the total stress for a Newtonian fluid is

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \lambda (\nabla \cdot \boldsymbol{u})\boldsymbol{I} + 2\mu \boldsymbol{e} \tag{2.2.33}$$

or, in subscript notation

$$\sigma_{ij} = -p\delta_{ij} + \lambda \left(\frac{\partial u_k}{\partial x_k}\right) \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$
(2.2.34)

(the Einstein summation convention is assumed here, namely, that a repeated subscript, such as in the second term on the right-hand side above, is summed over; note also that $\nabla \cdot \boldsymbol{u} = \partial u_k / \partial x_k = e_{kk}$.) The coefficient λ is called the "second viscosity" and μ the "absolute viscosity," or more commonly the "dynamic viscosity," or simply the "viscosity." For a Newtonian fluid λ and μ depend only on local thermodynamic state, primarily on the temperature.

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i.e., *initially irrotational flows remain irrotational* (for inviscid flow). Similarly, it can be proved that $D\Gamma/Dt = 0$; i.e., the circulation around a material closed circuit remains constant, which is Kelvin's theorem.

If $v \neq 0$, Equation (2.2.67) shows that the vorticity generated, say, at solid boundaries, diffuses and stretches as it is convected.

We also note that for steady flow the Bernoulli equation reduces to

$$p + \frac{1}{2}\rho V^2 + \rho gz = \text{constant}$$
 (2.2.68)

valid for steady, irrotational, incompressible flow.

Stream Function

For two-dimensional planar, incompressible flows (V = (u, v)), the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(2.2.69)
can be identically satisfied by introducing a stream function ψ , defined by

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x}$$
(2.2.70)

Physically ψ is a measure of the flow between stree nlines. A tream functions can be similarly defined to satisfy identic, the continuity equations for incompressible cylindrical and spherical axisymmetric in ver, the flows, as well a case above planar flow, also when they are compressible, but only inensif they are steady. One of a given by

$$\boldsymbol{\zeta} = \left(0, 0, \zeta_z\right) = \left(0, 0, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$
(2.2.71)

With Equation (2.2.70)

$$\zeta_z = -\frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial y^2} = -\nabla^2 \Psi$$
(2.2.72)

For this two-dimensional flow Equation (2.2.67) reduces to

$$\frac{\partial \zeta_z}{\partial t} + u \frac{\partial \zeta_z}{\partial x} + v \frac{\partial \zeta_z}{\partial y} = v \left(\frac{\partial^2 \zeta_z}{\partial x^2} + \frac{\partial^2 \zeta_z}{\partial y^2} \right) = v \nabla^2 \zeta_z$$
(2.2.73)

Substitution of Equation (2.2.72) into Equation (2.2.73) yields an equation for the stream function

$$\frac{\partial \left(\nabla^2 \psi\right)}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \left(\nabla^2 \psi\right)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \left(\nabla^2 \psi\right) = \nu \nabla^4 \psi \qquad (2.2.74)$$

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More detail and additional information on the topics in this section may be found in more advanced books on fluid dynamics, such as

Batchelor, G.K. 1967. An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge, England.

Warsi, Z.U.A. 1999. *Fluid Dynamics. Theoretical and Computational Approaches*, 2nd ed., CRC Press, Boca Raton, FL.

Sherman, F.S. 1990. Viscous Flow, McGraw-Hill, New York.

Panton, R.L. 1996. Incompressible Flow, 2nd ed., John Wiley & Sons, New York.

Pozrikidis, C. 1997. *Introduction to Theoritical and Computational Fluid Dynamics*, Oxford University Press, New York.

temperature is at the expense of added complexity and is to be avoided. (Of course, mass could be replaced by force or temperature by energy as alternative fundamental dimensions.) In some rare cases *i* is actually greater than n - m. Then

$$i = n - k \tag{2.3.2}$$

where k is the maximum number of the chosen variables that cannot be combined to form a dimensionless group. Determination of the minimum number of dimensionless groups is helpful if the groups are to be chosen by inspection, but is unessential if the algebraic procedure described below is utilized to determine the groups themselves since the number is then obvious from the final result.

The *particular* minimal set of dimensionless groups is arbitrary in the sense that two or more of the groups may be multiplied together to any positive, negative, or fractional power as long as the number of independent groups is unchanged. For example, if the result of a dimensional analysis is

$$\phi\{X, Y, Z\} = 0 \tag{2.3.3}$$

where X, Y, and Z are independent dimensionless groups, an equally valid expression is

$$\phi \{XY^{1/2}, Z/Y^2, Z\} = 0$$
 (2.3.4)

Dimensional analysis itself does not provide anying glass to be best choice of equivalent dimensionless groupings, such as between those of Equations (20.5) and (2.3.4). How we, solation of each of the variables that are presumed to be the pastimportant in a separate group may be convenient in terms of interpretation and correction. Another possible criterion increasing between alternative groupings may be the relative maranee of a particular one. (h) functional relationship provided by Equation (2.3.3) m yiel arity well be expressed as

$$X = \phi\{Y, Z\} \tag{2.3.5}$$

where X is implied to be the dependent grouping and Y and Z to be independent or parametric groupings.

Three primary methods of determining a minimal set of dimensionless variables are (1) by inspection; (2) by combination of the residual variables, one at a time, with a set of chosen variables that cannot be combined to obtain a dimensionless group; and (3) by an algebraic procedure. These methods are illustrated in the examples that follow.

Example 2.3.1: Fully Developed Flow of Water Through a Smooth Round Pipe

Choice of Variables. The shear stress τ_w on the wall of the pipe may be postulated to be a function of the density ρ and the dynamic viscosity μ of the water, the inside diameter D of the pipe, and the spacemean of the time-mean velocity u_m . The limitation to fully developed flow is equivalent to a postulate of independence from distance x in the direction of flow, and the specification of a smooth pipe is equivalent to the postulate of independence from the roughness e of the wall. The choice of τ_w rather than the pressure drop per unit length -dP/dx avoids the need to include the acceleration due to gravity g and the elevation z as variables. The choice of u_m rather than the volumetric rate of flow V, the mass rate of flow w, or the mass rate of flow per unit area G is arbitrary but has some important consequences as noted below. The postulated dependence may be expressed functionally as $\phi{\{\tau_w, \rho, \mu, D, u_m\}} = 0$ or $\tau_w = \phi{\{\rho, \mu, D, u_m\}}$.

Speculative Reductions. Eliminating ρ from the original list of variables on speculative grounds leads to

$$\phi\left\{\frac{\tau_w D}{\mu u_m}\right\} = 0$$

or its exact equivalent:

dimensional analysis.

$$\frac{\tau_w D}{\mu u_m} = A$$

The latter expression with A = 8 is actually the exact solution for the laminar regime ($Du_m\rho/\mu < 1800$). A relationship that does not include ρ may alternatively be derived directly from the solution by the method of inspection as follows. First, ρ is eliminated from one group, say $\tau_w/\rho u_m^2$, by multiplying it with $Du_m\rho/\mu$ to obtain

$$\phi \left\{ \frac{\tau_w D}{\mu u_m}, \frac{D u_m \rho}{\mu} \right\} = 0$$

The remaining group containing ρ is now simply cooper that the original expression been composed of three independent groups each containing ρ , that variable would have to be diminated from two of them before dropping the third variable.

The relationships that is obtained by the soculative dimension of μ , D, and u_m , one at a time, do not appear to base any range of physical validity. Furthermore, if w or G had been chosen as the field bearent variable rather than u_m are limiting relationship for the laminar regime would not have been obtained by the chain at one ρ .

Alternative Forms. The solution may also be expressed in an infinity of other forms such as

$$\phi\left\{\frac{\tau_w D^2 \rho}{\mu^2}, \frac{D u_m \rho}{\mu}\right\} = 0$$

If τ_w is considered to be the principal dependent variable and u_m the principal independent variable, this latter form is preferable in that these two quantities do not then appear in the same grouping. On the other hand, if *D* is considered to be the principal independent variable, the original formulation is preferable. The variance of $\tau_w / \rho u_m^2$ is less than that of $\tau_w D / \mu u_m$ and $\tau_w D^2 \rho / \mu^2$ in the turbulent regime while that of $\tau_w D / \mu u_m$ is zero in the laminar regime. Such considerations may be important in devising convenient graphical correlations.

Alternative Notations. The several solutions above are more commonly expressed as

$$\phi\left\{\frac{f}{2}, \operatorname{Re}\right\} = 0$$
$$\phi\left\{\frac{f\operatorname{Re}}{2}, \operatorname{Re}\right\} = 0$$

or

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where g is the acceleration due to gravity, β is the volumetric coefficient of expansion with temperature, T_{∞} is the unperturbed temperature of the fluid, and x is the vertical distance along the plate. The corresponding tabulation is

	h	g	β	$T_w - T_\infty$	x	μ	ρ	c_p	k
М	1	0	0	0	0	1	1	0	1
L	0	1	0	0	1	-1	-3	2	1
θ	-3	-2	0	0	0	-1	0	-2	-3
Т	-1	0	$^{-1}$	1	0	0	0	-1	1

The minimal number of dimensionless groups indicated by both methods is 9 - 4 = 5. A satisfactory set of dimensionless groups, as found by any of the methods illustrated in Example 1, is

$$\frac{hx}{k} = \phi \left\{ \frac{\rho^2 g x^3}{\mu^2}, \frac{c_p \mu}{k}, \beta (T_w - T_w), c_p (T_w - T_w) \left(\frac{\rho x}{\mu} \right)^2 \right\}$$

It may be reasoned that the buoyant force which generates the convective motion must be proportional to $\rho g\beta(T_w - T_w)$, thus, g in the first term on the right-hand side must be multiplied by $\beta(T_w - T_w)$, the used in

$$\frac{hx}{k} = \phi \left\{ \frac{\rho^2 g \beta (T_w - T_w) x^3}{\mu^2}, \frac{c_p \mu}{k} \beta (T_y - T_w) c_p (T_w - T_w) \left(\frac{\rho x}{\mu}\right)^2 \right\}$$

The effect of expansion cherthan on the buoyancy is now e resented by $\beta(T_w - T_w)$, and the effect of viscous dissination $p(T_w - T_w)(\rho x/\mu)^2$. Both effects are negligible for all practical circumstances. In presented as expression may be required to

$$\frac{hx}{k} = \phi \left\{ \frac{\rho^2 g \beta (T_w - T_w) x^3}{\mu^2}, \frac{c_p \mu}{k} \right\}$$

or

$$\operatorname{Nu}_{x} = \phi \{\operatorname{Gr}_{x}, \operatorname{Pr}\}$$

where $Nu_x = hx/k$ and $Gr_x = \rho^2 g\beta (T_w - T_{\infty})x^3/\mu^2$ is the *Grashof number*. Elimination of x speculatively now results in

$$\frac{hx}{k} = \left(\frac{\rho^2 g\beta (T_w - T_w) x^3}{\mu^2}\right)^{1/3} \phi \{\Pr\}$$

or

$$\operatorname{Nu}_{r} = \operatorname{Gr}_{r}^{1/3} \phi \{\operatorname{Pr}\}$$

This expression appears to be a valid asymptote for $Gr_x \rightarrow \infty$ and a good approximation for the entire turbulent regime. Eliminating μ speculatively rather than x results in

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by the Burke–Plummer equation, $\Phi = 1.75$ (Re_p)², where $\Phi = \rho \epsilon^2 d_p (-dP_f/dx) \mu^2 (1 - \epsilon)$, Re_p = $d_p u_o \rho / \mu (1 - \epsilon)$, d_p = diameter of spherical particles, m, ϵ = void fraction of bed of spheres, dP_f/dx = dynamic pressure gradient (due to friction), Pa/m, and u_o = superficial velocity (in absence of the spheres), m/sec. For the origin of these two asymptotic expressions see Churchill (1988a). They both have a theoretical structure, but the numerical coefficients of 150 and 1.75 are basically empirical. These equations are both lower bounds and have one intersection. Experimental data are plotted in Figure 2.3.3, which has the form of Figure 2.3.1 with $Y = \Phi/150$ Re_p, $Y/Z = \Phi/(1.75 \text{ Re}_p)^2$ and Z = 1.75 Re_p²/150 Re_p = Re_p/85.7.

A value of n = 1 is seen to represent these data reasonably well on the mean, resulting in

$$\Phi = 150 \operatorname{Re}_p + 1.75 \left(\operatorname{Re}_p \right)^2$$

which was originally proposed as a correlating equation by Ergun (1952) on the conjecture that the volumetric fraction of the bed in "turbulent" flow is proportional to Re_p . The success of this expression in conventional coordinates is shown in Figure 2.3.4. The scatter, which is quite evident in the arithmetic split coordinates of Figure 2.3.3, is strongly suppressed in a visual sense in the logarithmic coordinates of Figure 2.3.4.



FIGURE 2.3.3 Arithmetic, split-coordinate plot of experimental data for the pressure drop in flow through a packed bed of spheres. (From Churchill, S.W. and Usagi, R. *AIChE J.* 18(6), 1123, 1972. With permission from the American Institute of Chemical Engineers.)

Example 2.3.5: The Friction Factor for Commercial Pipes for All Conditions

The serial application of the CUE is illustrated here by the construction of a correlating equation for both smooth and rough pipes in the turbulent regime followed by combination of that expression with ones for the laminar and transitional regimes.

The Turbulent Regime. The Fanning friction factor, f_F , for turbulent flow in a smooth round pipe for asymptotically large rates of flow (say $\text{Re}_D > 5000$) may be represented closely by the empirical expression:

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$$D_h = 4 \times \frac{\text{the cross sectional area}}{\text{the wetted perimeter}}$$

The analysis in this section can also be applied to gases and vapors, provided the Mach number in the duct does not exceed 0.3. For greater values of the Mach number, the compressibility effect becomes significant and the reader is referred to Section 2.7 on compressible flow.

Fluid Friction

The calculation of friction loss in pipes and ducts depends on whether the flow is laminar or turbulent. The Reynolds number is the ratio of inertia forces to viscous forces and is a convenient parameter for predicting if a flow condition will be laminar or turbulent. It is defined as

$$\operatorname{Re}_{D} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$
(2.4.6)

in which V is the mean flow velocity, D diameter, ρ fluid density, μ dynamic viscosity, and ν kinematic viscosity.

Friction loss (H_j) depends on pipe diameter (d), length (L), roughness (e), fluid density (ρ) or specific weight (γ), viscosity (ν), and flow velocity (V). Dimensional analysis can be used to provide a functional relationship between the friction loss H_j , pipe dimensions, fluid properties, and flow pirameters. The resulting equation is called the Darcy–Weisbach equation:

$$H = \frac{1}{2gd} \frac{1}{1.23gD^5} \frac{192}{192}$$
(2.4.7)

The friction face f(x) measure of piper ough easy the open evaluated experimentally for numerous pipes. The distance used to create the Model friction factor chart shown as Figure 2.4.1. For Re < 2000, the flow in a piper yill be larger and f is only a function of Re_D. It can be calculated by

$$f = \frac{64}{\text{Re}_D} \tag{2.4.8}$$

At Reynolds numbers between about 2000 and 4000 the flow is unstable as a result of the onset of turbulence (critical zone in Figure 2.4.1). In this range, friction loss calculations are difficult because it is impossible to determine a unique value of f. For Re > 4000 the flow becomes turbulent and f is a function of both Re and relative pipe roughness (e/d). At high Re, f eventually depends only on e/d; defining the region referred to as fully turbulent flow. This is the region in Figure 2.4.1 where the lines for different e/d become horizontal (e is the equivalent roughness height and d pipe diameter). The Re_D at which this occurs depends on the pipe roughness. Laminar flow in pipes is unusual. For example, for water flowing in a 0.3-m-diameter pipe, the velocity would have to be below 0.02 m/sec for laminar flow to exist. Therefore, most practical pipe flow problems are in the turbulent region.

Using the Moody chart in Figure 2.4.1 to get f requires that Re and e/d be known. Calculating Re is direct if the water temperature, velocity, and pipe diameter are known. The problem is obtaining a good value for e. Typical values of e are listed in Figure 2.4.1. These values should be considered as guides only and not used if more–exact values can be obtained from the pipe supplier.

Since roughness may vary with time due to buildup of solid deposits or organic growths, f is also time dependent. Manufacturing tolerances also cause variations in the pipe diameter and surface roughness. Because of these factors, the friction factor for any pipe can only be approximated. A designer is required to use good engineering judgment in selecting a design value for f so that proper allowance is made for these uncertainties.

is maximum for only one pump operating. One specific example (Tullis, 1989) showed that the torque on a butterfly valve was three times higher when one pump was operating compared with three pumps operating in parallel.

Cavitation. Cavitation is frequently an important consideration in selection and operation of control valves. It is necessary to determine if cavitation will exist, evaluate its intensity, and estimate its effect on the system and environment. Cavitation can cause noise, vibration, and erosion damage and can decrease performance. The analysis should consider the full range of operation. Some valves cavitate worst at small openings and others cavitate heavily near full open. It depends on both the system and the valve design. If cavitation is ignored in the design and selection of the valves, repairs and replacement of the valves may be necessary. Information for making a complete cavitation is contained in Tullis (1989, 1993).

The first step in a cavitation analysis is selecting the acceptable level of cavitation. Experimental data are available for four limits: incipient (light, intermittent noise), critical (light, steady noise), incipient damage (pitting damage begins), and choking (very heavy damage and performance drops off). Limited cavitation data are available for each of these limits (Tullis, 1989, 1993). Choosing a cavitation limit depends on several factors related to the operating requirements, expected life, location of the device, details of the design, and economics. For long-term operation of a control valve in a system where noise can be tolerated, the valve should never operate beyond incipient damage. In systems where noise is objectionable, critical cavitation would be a better operating limit.

Using a choking cavitation as a design limit is often misused. It is ease any appropriate as a design limit for valves that only operate for short periods of time sectors of pressure relief valve. The intensity of cavitation and the corresponding noise vibratine and errors on damage at the valve, are at their maximum just before a valve chokes. If the valve of artes leyond choking (connermes relief valve) are at their maximum just before a valve chokes. If the valve of artes leyond choking (connermes relief valve) are at their maximum just before a valve chokes. If the valve of artes leyond choking (connermes relief valve) are at their maximum just before a valve chokes are at the valve of the valv

A similar approach is to place multiple conventional valves in series or a valve in series with orifice plates. Proper spacing of valves and orifices placed in series is important. The spacing between valves depends upon the type. For butterfly valves it is necessary to have between five and eight diameters of pipe between valves to prevent flutter of the leaf of the downstream valve and to obtain the normal pressure drop at each valve. For globe, cone, and other types of valves, it is possible to bolt them flange to flange and have satisfactory operation.

For some applications another way to suppress cavitation is to use a free-discharge valve that is vented so cavitation cannot occur. There are valves specifically designed for this application. Some conventional valves can also be used for free discharge, if they can be adequately vented.

Cavitation damage can be suppressed by plating critical areas of the pipe and valve with cavitationresistant materials. Based on tests using a magnetostriction device, data show that there is a wide variation in the resistance of the various types of material. Limited testing has been done on the erosion resistance of different materials and coating to cavitation in flowing systems. The available data show that there is less variation in the damage resistance of materials in actual flowing systems. However, experience has shown the plating parts of the valve with the right material will extend valve life.

Injecting air to suppress cavitation is a technique which has been used for many years with varying degrees of success. The most common mistake is placing the air injection port in the wrong location so
will be pressurized to full system pressure. When the air valve is manually opened, releasing the pressurized air can cause rapid acceleration of the liquid and generate serious transients when the water is decelerated as it hits the air valve. If manual air valves are installed, they should be very small so the air release rate is controlled to a safe rate.

Locating air valves in a piping system depends on the pipe profile, pipe length, and flow rates. Preferably, pipes should be laid to grade with valves placed at the high points or at intervals if there are no high points. One should use engineering judgment when defining a high point. If the pipe has numerous high points that are close together, or if the high points are not pronounced, it will not be necessary to have an air valve at each high point. If the liquid flow velocity is above about 1 m/sec (3 ft/sec), the flowing water can move the entrained air past intermediate high points to a downstream air valve. Releasing the air through an air valve prevents any sizable air pockets under high pressure from forming in the pipe. Trapped air under high pressure is extremely dangerous.

Velocity of the flow during filling is important. A safe way to fill a pipe is to limit the initial fill rate to an average flow velocity of about 0.3 m/sec (1 ft/sec) until most of the air is released and the air/vacuum valves close. The next step is to flush the system at about 1 m/sec (3 ft/sec), at a low system pressure, to flush the remaining air to an air release valve. It is important that the system not be pressurized until the air has been removed. Allowing large quantities of air under high pressure to accumulate and move through the pipe can generate severe transients. This is especially true if the compressed air is allowed to pass through a control valve or manual air release valve. When pressurized air flows through a platfally open valve, the sudden acceleration and deceleration of the air and liquid can gene at his pressure otesale. transients.

Pump Selection

n, and maintenance of Optimizing the life of a water supr requires proper selection elector process, the designer multiple concerned about matching the pump n requirements and process a ticipate problems that will be encountered when the pumps. During the selecto performance to the st ned or stopped and when the pipe is filled and drained. The design should also consider ne effect of variation m fl. i vo in Ments, and also anticipate problems that will be encountered due to increased future der and and details of installation.

Selecting a pump for a particular service requires matching the system requirements to the capabilities of the pump. The process consists of developing a system equation by applying the energy equation to evaluate the pumping head required to overcome the elevation difference, friction, and minor losses. For a pump supplying water between two reservoirs, the pump head required to produce a given discharge can be expressed as

$$Hp = \Delta Z + H_f \tag{2.4.17}$$

or

$$Hp = \Delta Z + CQ^2 \tag{2.4.18}$$

in which the constant C is defined by Equation (2.4.13).

Figure 2.4.2 shows a system curve for a pipe having an 82-m elevation lift and moderate friction losses. If the elevation of either reservoir is a variable, then there is not a single curve but a family of curves corresponding to differential reservoir elevations.

The three pump curves shown in Figure 2.4.2 represent different impeller diameters. The intersections of the system curve with the pump curves identify the flow that each pump will supply if installed in that system. For this example both A and B pumps would be a good choice because they both operate at or near their best efficiency range. Figure 2.4.2 shows the head and flow that the B pump will produce

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FIGURE 2.4.2 Pump selection for a single pump.

when operating in that system are 97 m and 450 L/m. The net positive suction here (1) SH and brake horsepower (bhp) are obtained as shown in the figure.

The selection process is more complex when the system depend wirds, either due to variations in the water surface elevation or to changing flowered user out of the system must operate over a range of reservoir elevations, the pump should be elected to mat the system surface based on the mean (or the most frequently encountered, water level watersects the pump curve near the midpoint of the best efficiency range. If they atended a variation is not too grad, the pump may not be able to operate efficiently overall comprete flow range.

The orbit of pump selection are becomes more difficult when planning for future demands or if the pumps are required to stor by a rarying flow. If the flow range is large, multiple pumps or a variablespeed drive may be needed. Recent developments in variable-frequency drives for pumps make them a viable alternative for systems with varying flows. Selection of multiple pumps and the decision about installing them in parallel or in series depend on the amount of friction in the system. Parallel installations are most effective for low-friction systems. Series pumps work best in high-friction systems.

For parallel pump operation the combined two pump curve is constructed by adding the flow of each pump. Such a curve is shown in Figure 2.4.3 (labeled 2 pumps). The intersection of the two-pump curve with the system curve identifies the combined flow for the two pumps. The pump efficiency for each pump is determined by projecting horizontally to the left to intersect the single-pump curve. For this example, a C pump, when operating by itself, will be have an efficiency of 83%. With two pumps operating, the efficiency of each will be about 72%. For the two pumps to operate in the most efficient way, the selection should be made so the system curve intersects the single-pump curve to the right of its best efficiency point.

Starting a pump with the pipeline empty will result in filling at a very rapid rate because initially there is little friction to build backpressure. As a result, the pump will operate at a flow well above the design flow. This may cause the pump to cavitate, but the more serious problem is the possibility of high pressures generated by the rapid filling of the pipe. Provisions should be made to control the rate of filling to a safe rate. Start-up transients are often controlled by starting the pump against a partially open discharge valve located near the pump and using a bypass line around the pump. This allows the system to be filled slowly and safely. If the pipe remains full and no air is trapped, after the initial filling, subsequent start-up of the pumps generally does not create any serious problem. Adequate air release valves should be installed to release the air under low pressure.

Because δ^* and θ are defined in terms of integrals for which the integrand vanishes in the freestream, they are easier to evaluate experimentally than disturbance thickness δ .

Exact Solution of the Laminar Flat-Plate Boundary Layer

Blasius obtained an exact solution for laminar boundary layer flow on a flat plate. He assumed a thin boundary layer to simplify the streamwise momentum equation. He also assumed *similar* velocity profiles in the boundary layer, so that when written as $u/U = f(y/\delta)$, velocity profiles do not vary with x. He used a similarity variable to reduce the partial differential equations of motion and continuity to a single third-order ordinary differential equation.

Blasius used numerical methods to solve the ordinary differential equation. Unfortunately, the velocity profile must be expressed in tabular form. The principal results of the Blasius solution may be expressed as

$$\frac{\delta}{x} = \frac{5}{\sqrt{\operatorname{Re}_x}}$$
(2.6.4)

and

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{\text{Re}_x}}$$

These results characterize the laminar boundary layer on a flat pare dies snow that laminar boundary layer thickness varies as $x^{1/2}$ and wall shear stress pares at $x^{1/2}$.

Approximate Solutions

The Blasius solution cannot be expressed in closel formuld is inmited to laminar flow. Therefore, approximate mut cass a give solutions for only a mathematical and turbulent flow in closed form are desirable. Concast ²⁴ method is the *momentum megric equation* (MIE), which may be developed by integrating the boundary layer equation (Pox and McDonald, 1992). The result is the ordinary differential equation

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - \left(\frac{\delta^*}{\theta} + 2\right) \frac{\theta}{U} \frac{dU}{dx}$$
(2.6.6)

The first term on the right side of Equation (2.6.6) contains the influence of wall shear stress. Since τ_w is always positive, it always causes θ to increase. The second term on the right side contains the pressure gradient, which can have either sign. Therefore, the effect of the pressure gradient can be to either increase or decrease the rate of growth of boundary layer thickness.

Equation (2.6.6) is an ordinary differential equation that can be solved for θ as a function of x on a flat plate (zero pressure gradient), provided a reasonable shape is assumed for the boundary layer velocity profile and shear stress is expressed in terms of the other variables. Results for laminar and turbulent flat-plate boundary layer flows are discussed below.

Laminar Boundary Layers. A reasonable approximation to the laminar boundary layer velocity profile is to express *u* as a polynomial in *y*. The resulting solutions for δ and τ_w have the same dependence on *x* as the exact Blasius solution. Numerical results are presented in Table 2.6.1. Comparing the approximate and exact solutions shows remarkable agreement in view of the approximations used in the analysis. The trends are predicted correctly and the approximate values are within 10% of the exact values.

Turbulent Boundary Layers. The turbulent velocity profile may be expressed well using a power law, $u/U = (y/\delta)^{1/n}$, where *n* is an integer between 6 and 10 (frequently 7 is chosen). For turbulent flow it is

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flaps extended, tend to need final experimental confirmation and optimization. Many studies of vertical takeoff and vectored thrust aircraft require testing in wind tunnels.

Defining Terms

- Boundary layer: Thin layer of fluid adjacent to a surface where viscous effects are important; viscous effects are negligible outside the boundary layer.
- Drag coefficient: Force in the flow direction exerted on an object by the fluid flowing around it, divided by dynamic pressure and area.
- Lift coefficient: Force perpendicular to the flow direction exerted on an object by the fluid flowing around it, divided by dynamic pressure and area.
- Pressure gradient: Variation in pressure along the surface of an object. For a *favorable* pressure gradient, pressure *decreases* in the flow direction; for an *adverse* pressure gradient, pressure *increases* in the flow direction.
- Separation: Phenomenon that occurs when fluid layers adjacent to a solid surface are brought to rest and boundary layers depart from the surface contour, forming a low-pressure wake region. Separation can occur only in an adverse pressure gradient.
- Transition: Change from laminar to turbulent flow within the boundary layer. The location depends on distance over which the boundary layer has developed, pressure gradient, surface rou bress, sale.co. freestream disturbances, and heat transfer.

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Further Information

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Timely reviews of important topics are published in the Annual Review of Fluid Mechanics series (Annual Reviews, Inc., Palo Alto, CA.). Each volume contains a cumulative index.

ASME (American Society of Mechanical Engineers, New York, NY) publishes the Journal of Fluids Engineering quarterly. JFE contains fluid machinery and other engineering applications of fluid mechanics.

The monthly AIAA Journal and bimonthly Journal of Aircraft (American Institute for Aeronautics and Astronautics, New York) treat aerospace applications of fluid mechanics.

Let us now apply the entropy change relation across the shock using the stagnation conditions.

$$s_2 - s_1 = c_p \ln \frac{T_{02}}{T_{01}} - R \ln \frac{p_{02}}{p_{01}}$$

Note that entropy at stagnation conditions is the same as at the static conditions since to arrive at stagnation conditions, the gas is brought to rest isentropically. Since $T_{02} = T_{01}$,

$$s_{2} - s_{1} = -R \ln \frac{p_{02}}{p_{01}}$$

$$\frac{p_{02}}{p_{01}} = e^{-(s_{2} - s_{1})/R}$$
(2.7.11)

Since $s_2 > s_1$ across the normal shockwave, Equation (2.7.11) gives $P_{02} < P_{01}$ or, in other words, the total pressure decreases across a shock wave. co.uk

One-Dimensional Flow with Heat Addition

Consider one-dimensional flow through a control volume as she 2.7.3. Flow conditions going into this control volume are designated by 1 and control . A specified amount of heat per unit mass, q, is added to the control volume. The onditions 1 and 2 can a equations be written as

Preview page
$$2 A u_1 \Phi_{p_2} u_2$$

 $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$ (2.7.12)
 $h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$
P₁, T₁, ρ_1 , u_1 Control Volume p_2 , T₂, ρ_2 , u_2



The following relations can be derived from Equation (2.7.12) for a calorically perfect gas

$$q = c_p \left(T_{02} - T_{01} \right) \tag{2.7.13}$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{2.7.14}$$



where p_f is the value of p_E at which the normal shock stands right at the nozzle exit. However, by adding an additional diverging section, known as a diffuser, downstream of the test section as shown in Figure 2.7.10 it is possible to operate the tunnel at a lower pressure ratio than p_0/p_f . This happens because the diffuser can now decelerate the subsonic flow downstream of the shock isentropically to a stagnation pressure p'_0 . The pressure ratio required then is the ratio of stagnation pressures across a normal shock wave at the test section Mach number. In practice, the diffuser gives lower than expected recovery as a result of viscous losses caused by the interaction of shock wave and the boundary layer which are neglected here.

The operation of supersonic wind tunnels can be made even more efficient; i.e., they can be operated at even lower pressure ratios than p_0/p'_0 , by using the approach shown in Figure 2.7.7 where the diffuser has a second throat. It can slow down the flow to subsonic Mach numbers isentropically and, ideally, can provide complete recovery, giving $p'_0 = p_0$. However, due to other considerations, such as the starting process of the wind tunnel and viscous effects, it is not realized in real life.

Two-Dimensional Supersonic Flow

When supersonic flow goes over a wedge or an expansion corner, it goes through an oblique shock or expansion waves, respectively, to adjust to the change in surface geometry. Figure 2.7.11 shows the two

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2 \sin^2 \beta}{(\gamma - 1)M_1^2 \sin^2 \beta + 2}$$
(2.7.29)



FIGURE 2.7.12 Oblique shock on a wedge.

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = \left[1 + \frac{2\gamma}{\gamma+1} \left(M_1^2 \sin^2 \beta - 1\right)\right] \left[\frac{2 + (\gamma - 1)M_1^2 \sin^2 \beta}{(\gamma + 1)M_1^2 \sin^2 \beta}\right]$$
(2.7.30)

The Mach number M_2 (= w_2/a_2) can be obtained by using a Mach number corresponding to velocity u_2 (= $w_2 \sin(\beta - \theta)$) in the normal shock relation for the Mach mumber of other words,

$$M_2^2 \sin^2(\beta - \theta) = \frac{1 + \frac{\gamma - 1}{2} \Lambda c^2 \sin \beta}{\gamma \gamma_1^2 \sin^2 \beta - \frac{\gamma - 1}{2}}$$
(2.7.31)
To derive a relation between the order angle θ and the wave angle β , we have from Figure 2.7.12

$$\tan \beta = \frac{u_1}{v}$$
 and $\tan(\beta - \theta) = \frac{u_2}{v}$

so that

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{(\gamma-1)M_1^2\sin^2\beta + 2}{(\gamma+1)M_1^2\sin^2\beta}$$

This can be simplified to

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}$$
(2.7.32)

Dennard and Spencer (1964) have tabulated oblique shock properties as a function of M_1 . Let us now make some observations from the preceding relations.

From the normal shock relations, $M_1 \sin \beta \ge 1$. This defines a minimum wave angle for a given Mach number. The maximum wave angle, of course, corresponds to the normal shock or $\beta = \pi/2$. Therefore, the wave angle β has the following range



Equation 2.7.32 becomes zero at the two limits of for Figure 2713 shows a plot of θ against β for various values of M_1 . For each value of M_1 , there is a maximum value of θ . For $\theta < \theta_{max}$, there are two possible so throws having different value of β_2 . The targer value of β gives the stronger shock in which the flow becomes subsonic. A reason soul ions for which $M_2 = 1$ is also shown in the figure. It is seen from the figure that with weak shock solution, the flow remains supersonic except for a small range of θ slightly smaller than θ_{max} .

Let us now consider the limiting case of θ going to zero for the weak shock solution. As θ decreases to zero, β decreases to the limiting value μ , given by

$$M_1^2 \sin^2 \mu - 1 = 0$$

$$\mu = \sin^{-1} \frac{1}{M_1}$$
 (2.7.34)

For this angle, the oblique shock relations show no jump in flow quantities across the wave or, in other words, there is no disturbance generated in the flow. This angle μ is called the *Mach angle* and the lines at inclination μ are called *Mach lines*.

Thin-Airfoil Theory

For a small deflection angle $\Delta \theta$, it can be shown that the change in pressure in a flow at Mach M_1 is given approximately by

$$\frac{\Delta p}{P_1} \approx \frac{\gamma M_1^2}{\sqrt{M_1^2 - 1}} \Delta \theta \tag{2.7.35}$$



FIGURE 2.8.9 Flow characteristics in vertical pneumatic conveying. (From Zeng, F.A. and Othmer, D.F. 1960. *Fluidization and Fluid-Particle Systems*, Reinhold, New York. With permission.)

Thomas F. Irvine, Jr. and Massimo Capobianchi

Introduction

An important class of fluids exists which differ from Newtonian fluids in that the relationship between the shear stress and the flow field is more complicated. Such fluids are called non-Newtonian or rheological fluids. Examples include various suspensions such as coal–water or coal–oil slurries, food products, inks, glues, soaps, polymer solutions, etc.

An interesting characteristic of rheological fluids is their large "apparent viscosities". This results in laminar flow situations in many applications, and consequently the engineering literature is concentrated on laminar rather than turbulent flows. It should also be mentioned that knowledge of non-Newtonian fluid mechanics and heat transfer is still in an early stage and many aspects of the field remain to be clarified.

In the following sections, we will discuss the definition and classification of non-Newtonian fluids, the special problems of thermophysical properties, and the prediction of pressure drops in both laminar and turbulent flow in ducts of various cross-sectional shapes for different classes of non-Newtonian fluids.

Classification of Non-Newtonian Fluids

It is useful to first define a Newtonian fluid since all other fluids are a new artenian. Newtonian fluids possess a property called viscosity and follow a law antice is usine Hookian relation between the stress applied to a solid and its strain. For a one down we work a Newtonian factor how, the shear stress at a point is proportional to the rate of strain (called in the literature the stream day) which is the velocity gradient at that point. The constant of proportionality is the tynamic viscosity, i.e.,

Preview Page
$$2_{y,x}^{AO} = \mu \dot{d}u = \mu \dot{\gamma}$$
 (2.9.1)

where x refers to the direction of the shear stress y the direction of the velocity gradient, and $\dot{\gamma}$ is the shear rate. The important characteristic of a Newtonian fluid is that the dynamic viscosity is independent of the shear rate.

Equation (2.9.1) is called a constitutive equation, and if $\tau_{x,y}$ is plotted against $\dot{\gamma}$, the result is a linear relation whose slope is the dynamic viscosity. Such a graph is called a *flow curve* and is a convenient way to illustrate the viscous properties of various types of fluids.

Fluids which do not obey Equation (2.9.1) are called non-Newtonian. Their classifications are illustrated in Figure 2.9.1 where they are separated into various categories of purely viscous time-independent or time-dependent fluids and viscoelastic fluids. Viscoelastic fluids, which from their name possess both viscous and elastic properties (as well as memory), have received considerable attention because of their ability to reduce both drag and heat transfer in channel flows. They will be discussed in a later subsection.

Purely viscous time-independent fluids are those in which the shear stress in a function only of the shear rate but in a more complicated manner than that described in Equation (2.9.1). Figure 2.9.2 illustrates the characteristics of purely viscous time-independent fluids. In the figure, (a) and (b) are fluids where the shear stress depends only on the shear rate but in a nonlinear way. Fluid (a) is called pseudoplastic (or shear thinning), and fluid (b) is called dilatant (or shear thickening). Curve (c) is one which has an initial yield stress after which it acts as a Newtonian fluid, called Bingham plastic, and curve (d), called Hershel-Buckley, also has a yield stress after which it becomes pseudoplastic. Curve (e) depicts a Newtonian fluid.

Figure 2.9.3 shows flow curves for two common classes of purely viscous time-dependent non-Newtonian fluids. It is seen that such fluids have a hysteresis loop or memory whose shape depends ranging from topics related to the cooling of the earth to the current-carrying capacities of wires. The general analyses given there have been applied to a range of modern-day problems from laser heating to temperature control systems.

Today, conduction heat transfer is still an active area of research and application. A great deal of interest has been shown in recent years in topics like contact resistance — where a temperature difference develops between two solids that do not have perfect contact with each other. Additional issues of current interest include non-Fourier conduction, where the processes occur so fast that the equations described below do not apply. Also, the problems related to transport in minaturized systems are garnering a great deal of interest. Increased interest has also been focused on ways of handling composite materials, where the ability to conduct heat is very directional.

Much of the work in conduction analysis is now accomplished by the use of sophisticated computer codes. These tools have given the heat-transfer analyst the capability to solve problems in inhomogenous media with very complicated geometries and with very involved boundary conditions. It is still important to understand analytical methods for determining the performance of conducting systems. At the minimum these can be used as calibrations for numerical codes.

Fourier's Law

The basis of conduction heat transfer is **Fourier's Law**. This law involves the idea that the heat firx is proportional to the temperature gradient in any direction n. Thermal conductivity h a property of materials that is temperature dependent, is the constant of proportionality.

In many systems the area x is a brachon of the distance in the direction n. One important extension is that this can be obtained with the First Law of Theorem Manness to yield the **Heat Conduction** Fquation For constant thermal conjectivity rule is given as:

 ∇^2

$$T + \frac{\dot{q}_G}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$
(3.1.2)

In this equation, α is the thermal diffusivity, and \dot{q}_G is the internal heat generation per unit volume. Some problems, typically steady-state, one-dimensional formulations where only the heat flux is desired, can be easily solved using Equation (3.1.1). Most conduction analyses are performed with Equation (3.1.2). In the latter — a more general approach — the temperature distribution is found from this equation and the appropriate boundary conditions. Then the heat flux, if desired, is found at any location using Equation (3.1.1). Normally, it is the temperature distribution that is of most importance. For example, it may be desirable to determine through analysis if a material will reach some critical temperature, like its melting point. Less frequently, the heat flux is the desired objective.

While there are times when one needs only to understand the temperature response of a structure, often the engineer is faced with a need to increase or decrease heat transfer to some specific level. Examination of the thermal conductivity of materials gives some insight to the range of possibilities that exist through simple conduction.

Of the more common engineering materials, pure copper exhibits one of the highest abilities to conduct heat, with a thermal conductivity approaching 400 W/m² K. Aluminum, also considered to be a good conductor, has a thermal conductivity a little over half that of copper. In order to increase the heat transfer above the values possible through simple conduction, more sophisticated designs are necessary that incorporate a variety of other heat transfer modes like convection and phase change.

Decreasing the rate of heat transfer is accomplished with the use of insulation. A discussion of this follows.

Here the notations I and K denote Bessel functions that are given in Abramowitz and Stegun (1964).

Fin effectiveness can be used as one indication whether or not fins should be added. A rule of thumb indicates that if the effectiveness is less than about three, fins should not be added to the surface.

Transient Systems

Negligible Internal Resistance

Consider the transient cooling or heating of a body with surface area A and volume V. This is taking place by convection through a heat transfer coefficient h_c to an ambient temperature of T_{∞} . Assume the thermal resistance to conduction inside the body is significantly less than the thermal resistance to convection (as represented by Newton's law of cooling) on the surface of the body. This ratio is denoted by the **Biot Number**, Bi:

$$\operatorname{Bi} = \frac{R_k}{R_c} = \frac{\overline{h_c}(V/A)}{k}$$
(3.1.26)

The temperature (which will be uniform throughout the body at any time for this situation) response ES3 E.CO.U (3.1.27 Fabour 0.1. with time for this system is given by the following relationship. Note that the shape of the body is not important — only the ratio of its volume to its area matters.

nter being less than about 0.1 Typically this will hold for the the Bodies with Similarit Internal Resistance

when Code is being heated or exceptrationity in a convective environment, but the internal thermal resistance of the bod Convector neglected, the analysis becomes more complicated. Only simple geometries (a symmetrical plane wall, a long cylinder, a composite of geometrical intersections of these geometries, or a sphere) with an imposed step change in ambient temperature are addressed here.

The first geometry considered is a large slab of minor dimension 2 l. If the temperature is initially uniform at T_{o} , and at time 0+ it begins convecting through a heat transfer coefficient to a fluid at T_{∞} , the temperature response is given by:

$$\theta = 2\sum_{n=1}^{\infty} \left(\frac{\sin \lambda_n L}{\lambda_n L + \sin \lambda_n L \cos \lambda_n L} \right) \exp(-\lambda_n^2 L^2 \operatorname{Fo}) \cos(\lambda_n x)$$
(3.1.28)

and the λ_n are the roots of the transcendental equation: $\lambda_n L \tan \lambda_n L = Bi$. The following definitions hold:

Bi
$$\equiv \frac{\overline{h_c}L}{k}$$
 Fo $\equiv \frac{\alpha t}{L^2}$ $\theta \equiv \frac{T - T_{\infty}}{T_o - T_{\infty}}$

The second geometry considered is a very long cylinder of diameter 2R. The temperature response for this situation is

$$\theta = 2\mathrm{Bi} \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 R^2 \mathrm{Fo}) J_o(\lambda_n r)}{(\lambda_n^2 R^2 + \mathrm{Bi}^2) J_o(\lambda_n R)}$$
(3.1.29)

(3.1.27)

Finite Difference Analysis of Conduction

Today, numerical solution of conduction problems is the most-used analysis approach. Two general techniques are applied for this: those based upon finite difference ideas and those based upon finite element concepts. General numerical formulations are introduced in Chapter 5. Here, we briefly introduce some finite difference concepts as they might be applied in heat conduction analysis.

First, consider formulation ideas. Consider the two-dimensional form of Equation (3.1.2), neglecting heat generation. This is given by:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$
(3.1.32)

By using finite approximations to the first and second derivatives, the following results:

$$\frac{T_{i+1,j} - 2T_{i,j} - T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} - T_{i,j-1}}{\Delta y^2} = \frac{T_{i,j}' - T_{i,j}}{\alpha \Delta t}$$
(3.1.33)

In this notation, *i* denotes node center locations in the *x* direction, *j* indicates node center locations in the *y* direction, and $T_{i,j} = T(i, j, t)$, and $T_{i,j} = T(i, j, t+\Delta t)$. As written, the form is denoted as the *aplicit* formulation because the forward difference is used for the time derivative (a new time superature is found in terms of all of the old time-step temperatures). If the time derivative is taken as the backward difference, then the new (and, hence, unknown) temperatures at a superature nodes are used with the known (old) temperature at each node to find the time and wat the new time. Because each equation contains a number of unknowns that manabe found simultaneously, one could is termed the *implicit* formulation. This is shown black or a general interior nodes

$$Pie^{\frac{T'_{i+1,j}-2T'_{i}}{2}} = \frac{T'_{i-1}}{2} + \frac{T'_{i,j}-2T'_{i,j}-T'_{i,j-1}}{\Delta y^2} = \frac{T'_{i,j}-T_{i,j}}{\alpha \Delta t}$$
(3.1.34)

A formal differencing approach, where finite differences are generated directly from partial derivatives, can be carried out for each application. However, the possibility of unusual geometries and mixing of boundary conditions often results in situations that are not easily differenced. For this reason, an energy balance method often yields an approach that can be more easily applied physically.

Attention is drawn to a one-dimensional slab (very large in two directions compared to the thickness). The slab is divided across the thickness into smaller subslabs (denoted as nodes in what follows), and this is shown in Figure 3.1.5. All nodes are of thickness Δx except for the two boundaries where the thickness is $\Delta x/2$. A characteristic temperature for each node is assumed to be represented by the temperature at the node center. Of course this assumption becomes more accurate as the size of the node becomes smaller. On the periphery of the object being analyzed the location representing the temperature of the node is located on the boundary of the object, and this results in fractionally sized nodes at these positions. Hence, with the two boundary node centers located exactly on the boundary, a total of *n* nodes are used (n - 2 full, interior nodes and half-nodes on each of the two boundaries).

In the analysis, a general interior node *i* (this applies to all nodes 2 through n - 1) is considered for an overall energy balance. Conduction in from node i - 1 and from node i + 1, as well as any heat generation present, is assumed to be energy per unit time flowing into the node. This is then equated to the time rate of change of energy within the node. A backward difference on the time derivative is applied here, and the notation $T'_i \equiv T_i(t+\Delta t)$ is used. The balance gives the following on a per-unit-area basis: (3.1.38)

The time is then advanced by Δt and the temperatures are again found by simultaneous solution.

The finite difference approach just outlined using the backward difference for the time derivative is termed the *implicit* technique, and it results in an $n \ge n$ system of linear simultaneous equations. If the forward difference is used for the time derivative, then only one unknown will exist in each equation. This gives rise to what is called an *explicit* or "marching" solution. While this type of system is more straightforward to solve because it deals with only one equation at a time with one unknown, a stability criterion must be considered which limits the time step relative to the distance step.

Two- and three-dimensional problems are handled in conceptually the same manner. One-dimensional heat fluxes between adjoining nodes are again considered. Now there are contributions from each of the dimensions represented. Details are outlined in the book by Jaluria and Torrance (1986). e.co.

Defining Terms

- Biot Number: Ratio of the internal (conductive) resistance on al (convective) resistance from a solid exchanging heat with a fluid. Fin: Additions of material to a surface to increase area and thus degrease the xiernal thermal resistance
- from convecting and/or radiating solids.
- Fin effectiveness Plate of the actual heat transfer from Om to the heat transfer from the same cross-
- sectional area of the wall without the final
 m final area of the wall without the final
 m final area of the wall without transfer from a fin to the heat transfer from a fin with the same geometry but completely when base temperature.
- Fourier's law: The fundamental law of heat conduction. Relates the local temperature gradient to the local heat flux, both in the same direction.
- Heat conduction equation: A partial differential equation in temperature, spatial variables, time, and properties that, when solved with appropriate boundary and initial conditions, describes the variation of temperature in a conducting medium.
- Overall heat transfer coefficient: The analogous quantity to the heat transfer coefficient found in convection (Newton's law of cooling) that represents the overall combination of several thermal resistances, both conductive and convective.
- Thermal conductivity: The property of a material that relates a temperature gradient to a heat flux. Dependent upon temperature.

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FIGURE 3.2.2 Nomenclature for heat transfer from planar surfaces of different shapes.



$$C_t^V = 0.13 \Pr^{0.22} / (1 + 0.61 \Pr^{0.81})^{0.42}$$
 (3.2.4)

$$C_t^H = 0.14 \left(\frac{1+0.0107 \text{Pr}}{1+0.01 \text{Pr}} \right)$$
 (3.2.5)

The superscripts V and H refer to the vertical and horizontal surface orientation.

The Nusselt numbers for fully laminar and fully turbulent heat transfer are denoted by Nu_{ℓ} and Nu_{i} , respectively. Once obtained, these are blended (Churchill and Usagi, 1972) as follows to obtain the equation for Nu:

$$\mathbf{N}\mathbf{u} = \left(\left(\mathbf{N}\mathbf{u}_{\ell}\right)^{m} + \left(\mathbf{N}\mathbf{u}_{\ell}\right)^{m}\right)^{1/m}$$
(3.2.6)

The blending parameter m depends on the body shape and orientation.

The equation for Nu_{ℓ} in this section is usually expressed in terms of Nu^T , the Nusselt number that would be valid if the thermal boundary layer were thin. The difference between Nu_{ℓ} and Nu^T accounts for the effect of the large boundary layer thicknesses encountered in natural convection.

It is assumed that the wall temperature of a body exceeds the ambient fluid temperature $(T_s > T_{\infty})$. For $T_s < T_{\infty}$ the same correlations apply with $(T_{\infty} - T_s)$ replacing $(T_s - T_{\infty})$ for a geometry that is rotated

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Further Information

There are several excellent heat transfer textbooks that provide fundamental information and correlations for natural convection heat transfer (e.g., Kreith and Bohn, 1993; Incropera and DeWitt, 1990). The correlations in this section closely follow the recommendations of Raithby and Hollands (1985), but that reference considers many more problems. Alternative equations are provided by Churchill (1983).

3.2.2 Forced Convection — External Flows

N.V. Suryanarayana

Introduction

In this section we consider heat transfer between a solid surface and an adjacent fluid which is in motion relative to the solid surface. If the surface temperature is different from that of the fluid, heat is transferred as forced convection. If the bulk motion of the fluid results solely from the difference in temperature of the solid surface and the fluid, the mechanism is natural convection. The velocity and temperature of the fluid far away from the solid surface are the free-stream velocity and free-stream temperature. Both

are usually known or specified. We are then required to find the heat flux from or to the surface with specified surface temperature or the surface temperature if the heat flux is specified. The specified temperature or heat flux either may be uniform or may vary. The convective heat transfer coefficient h is defined by

$$q'' = h(T_s - T_{\infty}) \tag{3.2.29}$$

In Equation (3.2.29) with the local heat flux, we obtain the local heat transfer coefficient, and with the average heat flux with a uniform surface temperature we get the average heat transfer coefficient. For a specified heat flux the local surface temperature is obtained by employing the local convective heat transfer coefficient.

Many correlations for finding the convective heat transfer coefficient are based on experimental data which have some uncertainty, although the experiments are performed under carefully controlled conditions. The causes of the uncertainty are many. Actual situations rarely conform completely to the experimental situations for which the correlations are applicable. Hence, one should not expect the actual value of the heat transfer coefficient to be within better than $\pm 10\%$ of the predicted value.

Many different correlations to determine the convective heat transfer coefficient have been developed. In this section only one or two correlations are given. For other correlations and more details, refer to the books given in the bibliography at the end of this section.

Flat Plate

With a fluid flowing parallel to a flat plate, changes in velocity at the menture of the fluid are confined to a thin region adjacent to the solid boundary the optime of have. Several cases arise:

- 1. Flows without or with pressure trail
- 2. Laminar or turbulent toundary layer
- 3. Negligible of syndicant viscous discipition effect of frictional heating)
- 4. 72 07 gases and most liquids) or 27 21 (liquid metals)

Flows with Zero Preside C.a lient and Negligible Viscous Dissipation

When the free-stream pressure is uniform, the free-stream velocity is also uniform. Whether the boundary layer is laminar or turbulent depends on the Reynolds number $\text{Re}_x (\rho U_{\infty} x/\mu)$ and the shape of the solid at entrance. With a sharp edge at the leading edge (Figure 3.2.7) the boundary layer is initially laminar but at some distance downstream there is a transition region where the boundary layer is neither totally laminar nor totally turbulent. Farther downstream of the transition region the boundary layer becomes turbulent. For engineering applications the existence of the transition region is usually neglected and it is assumed that the boundary layer becomes turbulent if the Reynolds number, Re_x , is greater than the critical Reynolds number, Re_{cr} . A typical value of 5×10^5 for the critical Reynolds number is generally accepted, but it can be greater if the free-stream turbulence is low and lower if the free-stream turbulence is high, the surface is rough, or the surface does not have a sharp edge at entrance. If the entrance is blunt, the boundary layer may be turbulent from the leading edge.



FIGURE 3.2.7 Flow of a fluid over a flat plate with laminar, transition, and turbulent boundary layers.

		Nu _d	
	$0.15 \leq Pr \leq 3$	$0.715 \text{Re}_{4}^{1/2} \text{Pr}^{0.4}$	(3.2.83)
<i>r/d</i> < 0.787	Pr > 3	$0.797 \text{Re}_{d}^{1/2} \text{Pr}^{1/3}$	(3.2.84)
$0.787 < r/d < r_v/d$		$0.632 \operatorname{Re}_{d}^{1/2} \operatorname{Pr}^{1/3} \left(\frac{d}{r}\right)^{1/2}$	(3.2.85)
$r_v/d < r/d < r_t/d$		$0.407 \operatorname{Re}_{d}^{1/3} \operatorname{Pr}^{1/3} (d/r)^{2/3}$	(3.2.86)
		$\overline{\left[\frac{0.1713}{(r/d)^2} + \frac{5.147}{\text{Re}_d}\frac{r}{d}\right]^{2/3}} \left[\frac{(r/d)^2}{2} + C\right]^{1/3}$	
		where	
		$C = -5.051 \times 10^{-5} \mathrm{Re}_{d}^{2/3}$	
		$\frac{r_{i}}{d} = \left\{ -\frac{s}{2} + \left[\left(\frac{s}{2}\right)^{2} + \left(\frac{p}{3}\right)^{3} \right]^{1/2} \right\}^{1/3}$	N
		+ $\left\{-\frac{s}{2}+\left[\left(\frac{s}{2}\right)^2-\left(\frac{p}{3}\right)^3\right]^{1/2}\right\}^{1/3}$	un
$r > r_t$	Pr < 3.9	$p = \frac{-2C}{0.2058PI} + 2 \frac{3}{0.2058Pr} + \frac{10006C}{0.2058Pr} + \frac{1000}{0.2058Pr} + \frac{1000}{0.25} + \frac{1000}{$	(3.2.87)
ivon	ew T	$\frac{1}{\operatorname{Re}_{d}} \prod_{r=0}^{d} \left[1 + \binom{r}{0} \frac{d}{d} \right] = \frac{1}{2} \sum_{r=0}^{d} \frac{h_{r}}{d} + 0.0371 \frac{h_{r}}{d}$	
pre	Da	$v = h_t = h$ at r_t and	
-		$h = 0.1713 = 5.147(r)^2$	

TABLE 3.2.5 Correlations for Free-Surface Jets $r_y/d = 0.1773 \text{ Re}_{d}^{1/3}$

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 $\overline{\operatorname{Re}_{d}}\left(\frac{1}{d}\right)$

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d

r/d

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transfer coefficient in the fully developed region set d/L = 0. A simpler correlation (fully developed region) is the Dittus–Boelter (1930) equation. Evaluate properties at T_b .

$$0.7 \le \Pr \le 160$$
 $\operatorname{Re}_{d} > 10,000$ $d/L > 10$

$$Nu_{d} = 0.023 Re_{d}^{4/5} Pr^{n}$$
(3.2.99)

where n = 0.4 for heating $(T_s > T_b)$ and n = 0.3 for cooling $(T_s < T_b)$.

For liquid metals with $Pr \ll 1$ the correlations due to Sleicher and Rouse (1976) are Uniform surface temperature:

$$Nu_{db} = 4.8 + 0.0156 \operatorname{Re}_{df}^{0.85} \operatorname{Pr}_{s}^{0.93}$$
(3.2.100)

Uniform heat flux:

$$Nu_{d,b} = 6.3 + 0.0167 Re_{d,f}^{0.85} Pr_s^{0.93}$$
(3.2.101)

Subscripts b, f, and s indicate that the variables are to be evaluated at the bulk temperature tilm temperature (arithmetic mean of the bulk and surface temperatures), and surface temperature, respectively.

In the computations of the Nusselt number the properties (called the bulk temperature) vary in the direction of flow and hence give differency lues proved different locations. In many cases a representative average value of the convertine hert cansfer coefficients increase. Such an average value can be obtained either by theights ariannetic average of the convective heat transfer coefficients evaluated at the inletta dexit bulk temperature of the convective heat transfer coefficient evaluated at the arithmetic me not me inlet and exit bulk temperatures. If the variation of the convective heat transfer of elicencies large, it may be apply instended to divide the tube into shorter lengths with smaller variation in the bulk temperature and vant ting the average heat transfer coefficient in each section.

Uniform Surface Temperature — Relation between the Convective Heat Transfer Coefficient and the <u>Total Heat Transfer Rate</u>: With a uniform surface temperature, employing an average value of the convective heat transfer coefficient the local convective heat flux varies in the direction of flow. To relate the convective heat transfer coefficient to the temperatures and the surface area, we have, for the elemental length Δz (Figure 3.2.22).



FIGURE 3.2.22 Elemental length of a tube for determining heat transfer rate.

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$$\ln \frac{T_s - T_{be}}{T_s - T_{bi}} = -\frac{hA_s}{\dot{m}C_p}$$
(3.2.103)

Equation (3.2.103) gives the exit temperature. For a constant-density fluid or an ideal gas, the heat transfer rate is determined from

$$q = \dot{m}C_{p}(T_{be} - T_{bi}) \tag{3.2.104}$$

Equation (3.2.103) was derived on the basis of uniform convective heat transfer coefficient. However, if the functional relationship between h and T_b is known, Equation (3.2.102) can be integrated by substituting the relationship. The convective heat transfer coefficient variation with T_b for water in two tubes of different diameters for two different flow rates is shown in Figure 3.2.23. From the figure it is clear that h can be very well approximated as a linear function of T. By substituting such a linear function relationship into Equation (3.2.102), it can be shown that

$$\ln \frac{h_i}{h_e} \frac{T_s - T_{be}}{T_s - T_{bi}} = -\frac{h_s A_s}{\dot{m} C_p}$$
(3.2.105)

where h_i , h_e , and h_s are the values of the convectively as the new coefficient evaluated at bulk temperatures of T_{bi} , T_{be} , and T_s , respectively. Although it has the demonstrated that h wave reproximately linearly with the bulk temperature with twa er is the fluid, the variation of h with air or oil as the fluid is much smaller and is very very proximated by a linear relationship. For other fluids it is suggested that the relationship the relation employing Equation (3.2.105). [Note: It is tempting to determine the heat in inference for the relation

$$q = hA_s \frac{\left(T_s - T_{be}\right) + \left(T_s - T_{bi}\right)}{2}$$

Replacing q by Equation (3.2.104) and solving for T_{be} for defined values of the mass flow rate and tube surface area, the second law of thermodynamics will be violated if $hA_s/\dot{m}C_p > 2$. Use of Equation (3.2.103) or (3.2.105) ensures that no violation of the second law occurs however large A_s is.]

<u>Uniform Surface Heat Flux</u>: If the imposed heat flux is known, the total heat transfer rate for a defined length of the tube is also known. From Equation (3.2.104) the exit temperature of the fluid is determined. The fluid temperature at any location in the pipe is known from the heat transfer rate up to that location ($q = q''A_s$) and Equation (3.2.104). The convective heat transfer coefficient is used to find the surface temperature of the tube.

Temperature Variation of the Fluid with Uniform Surface Temperature and Uniform Heat Flux: The fluid temperature variations in the two cases are different. With the assumption of uniform heat transfer coefficient, with a uniform surface temperature the heat flux decreases in the direction of flow leading to a progressively decreasing rate of temperature change in the fluid with axial distance. With uniform heat flux, the surface and fluid temperatures vary linearly except in the entrance region where the higher heat transfer coefficient leads to a smaller difference between the surface and fluid temperatures. The variation of the fluid temperature in the two cases is shown in Figure 3.2.24.

Convective Heat Transfer in Noncircular Tubes

Laminar Flows: The Nusselt numbers for laminar flows have been analytically determined for different noncircular ducts. Some of them can be found in Kakac et al. (1987), Kays and Crawford (1993), and



FIGURE 3.2.23 Variation of h with T_b in 1-, 2-, and 4-cm-diameter tubes with water flow rates of 0.2 kg/sec and 0.4 kg/sec with uniform surface temperature.



Infinite parallel plates: a = spacing between plates, $d_h = 2a$

Both plates maintained at uniform and equal temperatures: Nu = 7.54Both plates with imposed uniform and equal heat fluxes: Nu = 8.24

Rectangular ducts: a = longer side, b = shorter side, $d_h = \frac{2ab}{(a + b)}$

b/a	1	0.7	0.5	0.2 5	0.12 5
Uniform surface temperature	2.98	3.08	3.39	4.44	5.6
Uniform heat flux*	3.61	3.73	4.12	5.33	6.49

Equilateral triangle: $d_h = a/3^{1/2}$, a = length of each side

Uniform surface temperature: Nu = 2.35Uniform surface heat flux:* Nu = 3.0

^{*} Uniform axial heat flux but circumferentially uniform surface temperature.

All other properties at the average bulk temperature of the fluid

$$Gz = \frac{\dot{m}C_{p}}{kL} \qquad Gr = g\beta\Delta T d^{3}/\nu^{2}$$

$$Nu_{d} = 1.75 \left[Gz + 0.12 \left(GzGr^{1/3}Pr^{0.36}\right)^{0.88}\right]^{1/3} \left(\mu_{b}/\mu_{s}\right)^{0.14} \qquad (3.2.107)$$

Uniform Heat Flux. Properties at $(T_{s+}T_b)/2$: $3 \times 10^4 < \text{Ra} < 10^6$, 4 < Pr < 175, $2 < hd^2/(k_w t) < 66$, $k_w = 10^{-10}$ tube wall thermal conductivity, t = tube wall thickness.

$$Gr_{d}^{*} = g\beta d^{4} q_{w}'' / (v^{2}k) \qquad P_{w} = kd/(k_{w}t) \qquad Ra_{d} = g\beta\Delta T d^{3}Pr/v^{2}$$
$$Nu_{d} = \left\{ 4.36^{2} + \left[0.145 \left(\frac{Gr_{d}^{*}Pr^{1.35}}{P_{w}^{0.25}} \right)^{0.265} \right]^{2} \right\}^{0.5}$$
(3.2.108)

In Equation (3.2.107) and (3.2.108) evaluate fluid properties at the arithmetic mean of the buck in vall temperatures.

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Further Information

Other sources which may be consulted for more detailed information are Cho and Hartnett (1982), Shenoy and Mashelkar (1982), Irvine and Karni (1987), and Hartnett and Kostic (1989).

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FIGURE 3.4.6 Flow reg mes and boiling mechanisms for upflow convective boiling in a vertical tube at moderate wall superheat.

(1989) which has been fit to a broad spectrum of data for both horizontal and vertical tubes. For this method the heat transfer coefficient for a tube of diameter D is given by

$$h = h_l \Big[C_1 C o^{C_2} \Big(25 \mathrm{Fr}_{l_e} \Big)^{C_5} + C_3 B o^{C_4} F_K \Big]$$
(3.4.8)

where

$$Co = \left(\frac{1-x}{x}\right)^{0.8} \left(\frac{\rho_{\nu}}{\rho_l}\right)^{0.5}$$
(3.4.9)

$$Bo = q''/Gh_{fg} \tag{3.4.10}$$

$$Fr_{le} = G^2 / \rho_l^2 gD \tag{3.4.11}$$

and h_l is the single-phase heat transfer coefficient for the liquid phase flowing alone in the tube computed as

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FIGURE 3.4.8 Flow regimes during horizontal cocurrent flow with condensation.

The vapor velocity is generally high at the inlet end of the condenser tube, and the liquid film is driven along the tube by strong vapor shear on the film. At low vapor flow rates, some stratification may occur and the film may be thicker on the bottom of the horizontal tube. At high vapor flow rates, turbulent stresses acting on the liquid may tend to keep the thickness of the liquid film nominally uniform over the perimeter of the tube.

In most condenser applications, shear-dominated annular flow persists to very low Qualities and the overwhelming majority of the heat transfer occurs in this regime. The very diversities of the condensation process, corresponding to qualities less than a few percentum accuracy in obbly two-phase flow. Generally these regimes represent such a strat port of of the overall beat in nsfer in the condenser that some inaccuracy in estimating the best transfer coefficient for them is operated. As a first estimate, the heat transfer coefficient has the tradited using a correction for pare single-phase liquid flow in the tube at the same tetal flow rate, or a correlation for multiple condensation may simply be extrapolated to the beat due of the annular flow regime, accurate prediction of the overall heat transfer performance of the others in the annular flow regime, accurate prediction of the overall heat transfer performance of the others in the form of most correlation methods for predicting local

transport in this regime. For this reason, the form of most correlation methods for predicting local convective condensation heat transfer coefficients are optimized to match data in the annular flow regime. One example of such a correlation is the following relation for the local heat transfer coefficient for annular flow condensation proposed by Traviss et al. (1973):

$$\frac{hD}{k_l} = \frac{0.15 \operatorname{Pr}_l \operatorname{Re}_l^{0.9}}{F_T} \left[\frac{1}{X_u} + \frac{2.85}{X_u^{0.476}} \right]$$
(3.4.20)

where

$$\operatorname{Re}_{l} = \frac{G(1-x)D}{\mu_{l}}, \qquad X_{ll} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_{v}}{\rho_{l}}\right)^{0.5} \left(\frac{\mu_{l}}{\mu_{v}}\right)^{0.1}$$

and F_T is given by

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- **Nucleate boiling:** Generation of vapor at a hot surface by formation of bubbles at discrete nucleation sites with full liquid wetting of the surface.
- **Polar molecules:** Molecules which have a permanent electric dipole moment. Examples include water and ammonia.
- **Pool boiling:** Generation of vapor at the surface of a hot body immersed in an extensive liquid pool.
- **Transition boiling:** Generation of vapor at a hot surface with intermittent or partial liquid wetting of the surface.

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FIGURE 3.4.14 A liquid-solid phase diagram of a binary mixture.

The presence of a two-phase mixture zone with temperature-dependent concentration and phase proportion obviously complicates heat transfer analysis, and requires the simultaneous plut prof both the heat and mass transfer equations. Furthermore, the liquid usually does not olicify on a simple planar surface. Crystals of the solid phase are formed at some preferred locations in the liquid, or on colder solid surfaces immersed in the liquid, and as freezing process the crystals grow in the form of intricately shaped fingers, called dendrites. This complicates are geometry significantly and makes mathematical modeling of the process very different An antroduction to such problems and further references are available in Hayashi and Kun me (1992) and Pomekak (\$ 1194).

Flow of the field hase often has an inportation of in the inception of, and during, melting and in converse incropera and Viskerta (1992), the flow may be forced, such as in the freezing of a liquid flowing through or actors are one pipe, and/or may be due to natural convection that arises whenever there are density gradients in the liquid, here generated by temperature and possibly concentration gradients. It is noteworthy that the change in phase usually affects the original flow, such as when the liquid flowing in a cooled pipe gradually freezes and the frozen solid thus reduces the flow passage, or when the evolving dendritic structure gradually changes the geometry of the solid surfaces that are in contact with the liquid. Under such circumstances, strong coupling may exist between the heat transfer and fluid mechanics, and also with mass transfer when more than a single species is present. The process must then be modeled by an appropriate set of continuity, momentum, energy, mass conservation, and state equations, which need to be solved simultaneously.

More-detailed information about melting and freezing can be found in the monograph by Alexiades and Solomon (1993) and in the comprehensive reviews by Fukusako and Seki (1987) and Yao and Prusa (1989).

Melting and Freezing of Pure Materials

Thorough mathematical treatment of melting and freezing is beyond the scope of this section, but examination of the simplified one-dimensional case for a pure material and without flow effects provides important insights into the phenomena, identifies the key parameters, and allows analytical solutions and thus qualitative predictive capability for at least this class of problems.

In the freezing model, described in Figure 3.4.15, a liquid of infinite extent is to the right (x > 0) of the infinite surface at x = 0, initially at a temperature T_i higher than the fusion temperature T_f . At time t = 0 the liquid surface temperature at x = 0 is suddenly lowered to a temperature $T_0 < T_f$, and maintained at that temperature for t > 0. Consequently, the liquid starts to freeze at x = 0, and the freezing interface (separating in Figure 3.4.15 the solid to its left from the liquid on its right) located



FIGURE 3.4.15 Freezing of semi-inifinite liquid with heat conduction in both phases.

at the position x = X(t) moves gradually to the right (in the positive x direction). We note that in this problem heat is conducted in both phases.

Assuming for simplification that heat transfer is by conduction only — ethough at least natural convection (Incropera and Viskanta, 1992) and schemmes forced cover than and radiation also take place — the governing equation are In the liquid; The transient heat conduction entration is

$$PIP \qquad \qquad PIP \qquad \qquad PIP$$

$$T_i(x,t) = T_i$$
 in $x > 0$, at $t = 0$ (3.4.30)

where α_{ℓ} is the thermal diffusivity of the liquid, with the initial condition and the boundary condition

$$T_{\ell}(x \to \infty, t) \to T_i \quad \text{for} \quad t > 0$$
 (3.4.31)

In the solid: The transient heat conduction equation is

$$\frac{\partial T_s(x,t)}{\partial t} = \alpha_s \frac{\partial^2 T_s(x,t)}{\partial x^2} \quad \text{in} \quad 0 < x < X(t) \quad \text{for} \quad t > 0$$
(3.4.32)

where α_s is the thermal diffusivity of the solid, with the boundary condition

$$T_{c}(0,t) = T_{0}$$
 for $t > 0$ (3.4.33)

The remaining boundary conditions are those of temperature continuity and heat balance at the solid–liquid phase-change interface X(t),

$$T_{\ell}[X(t)] = T_{s}[X(t)] = T_{f} \quad \text{for} \quad t > 0$$
(3.4.34)

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hytical solution for helding, consider the semi-For a simple one-dimensional melting example e T_f . For time t > 0 the fusion temperature T_f . For time t > 0 the 0 is raised to $T \sim T_f$, in the solid consequently starts to melt there. infinite solid described in Figure temperature of the surfect (at x case the tender three in the solid remains constant, $T_s = T_f$, so the temperature distribution needs the need only in the liquid phase of is assumed that the liquid formed by melting remains In this case the n notionless and in place /es sur arry to the above-described freezing case, the equations describing this problem are the heat conduction equation

$$\frac{\partial T_{\ell}(x,t)}{\partial t} = \alpha_{\ell} \frac{\partial^2 T_{\ell}(x,t)}{\partial x^2} \quad \text{in} \quad 0 < x < X(t) \quad \text{for} \quad t > 0$$
(3.4.42)

with the initial condition

$$T_{\ell}(x,t) = T_{f}$$
 in $x > 0$, at $t = 0$ (3.4.43)

the boundary condition

$$T_{\ell}(0,t) = T_0 \quad \text{for} \quad t > 0$$
 (3.4.44)

and the liquid-solid interfacial temperature and heat flux continuity conditions

$$T_{\ell}[X(t)] = T_{f} \text{ for } t > 0$$
 (3.4.45)

$$-k_{\ell} \left(\frac{\partial T_{\ell}}{\partial x}\right)_{[X(t)]} = \rho h_{\ell s} \frac{dX(t)}{dt} \quad \text{for} \quad t > 0$$
(3.4.46)

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$$\operatorname{Ste}_{\ell} = \frac{(2.14 \text{ kJ/kg}^{\circ}\text{C})(58^{\circ}\text{C} - 28^{\circ}\text{C})}{241.2 \text{ kJ/kg}} = 0.266$$

The solution of Equation (3.4.48) as a function of Ste_{ℓ} is given in Figure 3.4.17, yielding $\lambda \approx 0.4$. By using Equation (3.4.50), the time of interest is calculated by

$$t = \frac{\left[X(t)\right]^2}{4\lambda^2 \alpha_{\ell}} = \frac{(0.1 \text{ m})^2}{4(0.4)^2 \left[(1.09)10^7 \text{ m}^2/\text{sec}\right]} = (1.43)10^5 \text{ sec} = 39.8 \text{ hr}$$

The axisymmetric energy equation in *cylindrical coordinates*, applicable to both the solid phase and immobile liquid phase (with appropriate assignment of the properties) is

$$\frac{\partial T(r,t)}{\partial t} = \frac{1}{\rho c} \frac{\partial}{\partial r} \left(\frac{k}{r} \frac{\partial T(r,t)}{\partial r} \right) \quad \text{for} \quad t > 0$$
(3.4.51)

and the temperature and heat balance conditions at the solid–liquid phase-change interface r = R(t) are

$$T_{\ell}[R(t)] = T_{s}[R(t)] \quad \text{for} \quad t > 0$$

$$k_{s}\left(\frac{\partial T_{s}}{\partial r}\right)_{R(t)} - k_{s}\left(\frac{\partial T_{\ell}}{\partial r}\right)_{\ell} = c \frac{\sigma_{s}(t)}{dt} \qquad (3.4.53)$$

Because of the nature of the differential equations describing nonplanar and multidimensional geometries, analytical celulible are available for out a lew cores, such as line heat sources in cylindrical coordinary writers or point heat sources in spin deal ones, which have very limited practical application. Other phase-change producing it phase and numerical methods (Yao and Prusa, 1989; Alexiades and Solomon, 1993).

Some Approximate Solutions

Two prominent approximate methods used for the solution of melting and freezing problems are the integral method and the *quasi-static* approximation. The integral method is described in Goodman (1964), and only the quasi-static approximation is described here.

To obtain rough estimates of melting and freezing processes quickly, in cases where heat transfer takes place in only one phase, it is assumed in this approximation that effects of sensible heat are negligible relative to those of latent heat (Ste \rightarrow 0), thus eliminating the sensible-heat left-hand side of the energy equations (such as (3.4.29), (3.4.32), and (3.4.51)). This is a significant simplification, since the energy equation then becomes independent of time, and solutions to the steady-state heat conduction problem are much easier to obtain. At the same time, the transient phase-change interface condition (such as Equations (3.4.35) and (3.4.53)) 5 is retained, allowing the estimation of the transient interface position and velocity. This is hence a quasi-static approximation, and its use is shown below.

We emphasize that these are just approximations, without full information on the effect of specific problem conditions on the magnitude of the error incurred when using them. In fact, in some cases, especially with a convective boundary condition, they may produce incorrect results. It is thus necessary to examine the physical viability of the results, such as overall energy balances, when using these approximations.

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$$t_m = \frac{L^2}{2\alpha_\ell (1+\omega) \operatorname{Ste}_\ell} \left[1 + \frac{2k_\ell}{\bar{h}L} + \left(0.25 + 0.17\omega^{0.7} \right) \operatorname{Ste}_\ell \right]$$
(3.4.78)

valid for $0 \le \text{Ste}_{\ell} \le 4$ and $\overline{h}L/k_{\ell} \ge 0.1$, and the temperature, T(0,t), of the surface across which the heat is supplied can be estimated from the implicit time-temperature relationship:

$$t = \frac{\rho c_{\ell} k_{\ell}}{2\bar{h}^2 \operatorname{Ste}_{\ell}} \left[1.18 \operatorname{Ste}_{\ell} \left(\frac{T(0,t) - T_f}{T_a - T(0,t)} \right)^{1.83} + \left(\frac{T_a - T_f}{T_a - T(0,t)} \right)^2 - 1 \right]$$
(3.4.79)

Both equations (3.4.78) and (3.4.79) are claimed to be accurate within 10%.

The suitability of using several simplified analytical solutions for the estimation of freezing and melting times for more-realistic problems was assessed by Dilley and Lior (1986).

Defining Terms

- **Eutectic concentration:** A concentration of a component of a multicomponent liquid at which the liquid would upon freezing form a solid containing the same concentration, and at which the freezing process is completed at a single temperature.
- **Mushy zone:** The zone composed of both liquid and solid, bounded by the liquid **r** and relians curves, in a freezing or melting process.

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Solute	Sc	M
Helium	120	4.003
Hydrogen	190	2.016
Nitrogen	280	28.02
Water	340	18.016
Nitric oxide	350	30.01
Carbon monoxide	360	28.01
Oxygen	400	32.00
Ammonia	410	17.03
Carbon dioxide	420	44.01
Hydrogen sulfide	430	34.08
Ethylene	450	28.05
Methane	490	16.04
Nitrous oxide	490	44.02
Sulfur dioxide	520	64.06
Sodium chloride	540	58.45
Sodium hydroxide	490	40.00
Acetic acid	620	60.05
Acetone	630	58.08
Methanol	640	32.04
Ethanol	640	46.07
Chlorine	670	70.0
Benzene	720	78 11
Ethylene glycol	720	62.07
n-Propanol		6009
<i>i</i> -Propanol		67.09
Propane	750	44.09
Aniline	800	93.13
Beuzois add		122.12
(y e u	1010	92.09
Nicrose A	1670	342.3

TABLE 3.5.4 Schmidt Numbers for Dilute Solution in Water at 300 K^a

^a Schmidt number $\Omega_{\mu\nu}$; since the solutions are dilute, μ and ρ can be taken as pure water values. For other temperatures use Sc/Sc_{300 K} $\simeq (\mu^2/\rho T)/(\mu^2 \rho T)_{300 K}$, where μ and ρ are for water, and *T* is absolute temperature. For chemically similar solutes of different molecular weights use Sc₂/Sc₁ $\simeq (M_2/M_1)^{0.4}$. A table of $(\mu^2/\rho T)/(\mu^2 \rho T)_{300 K}$ for water follows.

<i>T</i> (K)	$(\mu^2/\rho T)/(\mu^2/\rho T)_{300 \text{ K}}$
290	1.66
300	1.00
310	0.623
320	0.429
330	0.296
340	0.221
350	0.167
360	0.123
370	0.097

From Spalding, D.B. 1963. Convective Mass Transfer, McGraw-Hill, New York. With permission.

$$\rho \frac{Dm_i}{Dt} = \nabla \cdot \boldsymbol{j}_i + \dot{r}_i^{\prime\prime\prime\prime} \tag{3.5.31}$$

where D/Dt is the substantial derivative operator.

If we consider a binary system of species 1 and 2 and introduce Fick's law, Equation (3.5.24a) into Equation (3.5.31), then

P

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FIGURE 3.5.3 Steady diffusion across a plane wall.

where the *u*- and *u*'-surfaces are shown in Figure 3.5.3. Solubility data are required to relate the *u*- and *u*'-surface concentrations to *s*- and *s*'-surface concentrations. Alternatively for systems that obey Henry's law, a solubility \mathcal{G} can be defined as the volume of solute gas (at STP of 0°C and 1 atm) dissolved in unit volume when the gas is at a partial pressure of 1 atm. Then, defining permeability P as the product $D_{12}\mathcal{G}$, the volume flow of species 1 is

$$\dot{V}_{1} = \frac{P_{12}A}{I} (P_{1,s}, \mathbf{v}_{s}, \mathbf{p}^{3}, (STP)/sec$$
 (3.5.36)

where the partial pressures P_1 are in at the pressure of the SI units for permeability are m³ (STP)/m²sec(that w) rermeability and solutions data are given in Table 3.5.6. For example, consider the inmatril of Pa contained in a veb callest vessel with a 1-mm-thick wall at 680 K. For a surface area of 0.01 m², the leakage attention at bient air is

$$\dot{V} = \frac{(1.0 \times 10^{-12})(0.01)}{(0.001)} (10^5 - 0) = 1.0 \times 10^{-6} \text{ m}^3 \text{ (STP)/sec}$$

where the value P_{12} was obtained from Table 3.5.6.

In general, mass fractions are discontinuous across phase interfaces. Hence, Equation (3.5.35) cannot be generalized to a number of walls in series by simply adding diffusion resistances. However, equilibrium partial pressures P_1 are continuous, and for two walls A and B, Equation (3.5.36) becomes

$$\dot{V}_{1} = \frac{P_{1,s} - P_{1,s'}}{\frac{L_{A}}{P_{1A}A} + \frac{L_{B}}{P_{1B}A}} \text{ m}^{3} \text{ (STP)/sec}$$
(3.5.37)

Transient Diffusion in a Semi-Infinite Solid

The typically low diffusion coefficients characterizing solids result in many situations where concentration changes are limited to a thin region near the surface (of thickness $\delta_c \sim (D_{12}t)^{1/2}$). Examples include case-hardening of mild steel and coloring of clear sapphires. Details of the geometry are then unimportant



In an automobile catalytic convertor, Equation 3.5.47 applies to the catalyst washcoat. However, the mass transfer problem also involves a convective process for transport of reactants from the bulk flow. Referring to Figure 3.5.6 there are two mass transfer resistances in series, and the consumption rate of species 1 per unit surface area of the washcoat is

$$J_{1,s} = \frac{-x_{1,e}}{\frac{1}{L\eta_p k'' c} + \frac{1}{G_{m1}}} \,\mathrm{kmol/m^2 \,sec}$$
(3.5.48)

where G_{m1} is the mole transfer conductance describing convective transport to the washcoat surface (see the section on mass and mole transfer conductances). Notice that when $G_{m1} \ll L\eta_p k''c$ the reaction rate is controlled by mass transfer from the gas stream to the washcoat surface; when $L\eta_p k''c \ll G_{m1}$, the reaction rate is controlled by diffusion within the washcoat and the kinetics of the reaction.



FIGURE 3.5.13 The surface energy balance for evaporation of water into an air stream.


FIGURE 3.5.14 Wet bulb of a wet- and dry-bulb psychrometer.

Also, h_{fg} (305 K) = 2.425 × 10⁶ J/kg, and $c_{p \text{ air}}$ = 1005 J/kg K; thus



High Mass Transfer Rate Theory

enough for engineering purposes.

When there is net mass transfer across a phase interface, there is a convective component of the absolute flux of a species across the *s*-surface. From Equation (3.5.23a) for species 1,

$$n_{1s} = \rho_{1s} v_s + j_{1s} kg/m^2 sec$$
 (3.5.67)

During evaporation the convection is directed in the gas phase, with a velocity normal to the surface v_s . When the convective component cannot be neglected, we say that the mass transfer rate is *high*. There are two issues to consider when mass transfer rates are high. First, the rate at which species 1 is transferred across the *s*-surface is not simply the diffusive component $j_{1,s}$ as assumed in low mass transfer rate theory, but is the sum of the convective and diffusive components shown in Equation (3.5.67). Second, the normal velocity component v_s has a *blowing* effect on the concentration profiles, and hence on the Sherwood number. The Sherwood number is no longer analogous to the Nusselt number of conventional heat transfer correlations, because those Nusselt numbers are for situations involving impermeable surfaces, e.g., a metal wall, for which $v_s = 0$.

plants the cost of water is about \$1.4 to \$2/m³, dropping to less than \$1/m³ for desalting brackish water. A methodology for assessing the economic viability of desalination in comparison with other water supply methods is described by Kasper and Lior (1979). Desalination plants are relatively simple to operate, and progress toward advanced controls and automation is gradually reducing operation expenses. The relative effect of the cost of the energy on the cost of the fresh water produced depends on local conditions, and is up to one half of the total.

The boiling point of a salt solution is elevated as the concentration is increased, and the **boiling point** elevation is a measure of the energy needed for separation. Thermodynamically reversible separation defines the minimal energy requirement for that process. The minimal energy of separation W_{\min} in such a process is the change in the Gibbs free energy between the beginning and end of the process, ΔG . The minimal work when the number of moles of the solution changes from n_1 to n_2 is thus

$$W_{\min} = \int_{n_1}^{n_2} (\Delta G) \, dn_W \tag{4.1.1}$$

The minimal energy of separation of water from seawater containing 3.45 wt.% salt, at 25°C, is 2.55 kJ/(kg fresh water) for the case of zero fresh water recovery (infinitesimal concentration change) and 2.91 kJ/(kg fresh water) for the case of 25% freshwater recovery. W_{min} is, however, severalfold amaller than the energy necessary for water desalination in practice. Improved energy economy can be builtined when desalination plants are integrated with power generation plants (Aschner, 1987), the duar-purpose plants save energy but also increase the capital cost and complexity of orderator.

Two aspects of the basically simple desalination process require **Main attention.** One is the highcorrosivity of seawater, especially pronounced to be in the inperature destination processes, which e sive materials. Typical material is use are copper-nickel requires the use of corrosion-resistant alloys, stainless steel, titanium at 1, it lower temperatures, fiber-temperatures, fib 1975). Another aspect the scale formation (Glorer Stal., 1980, Heitman, 1990). Salts in saline water, particularly culling surfate, magnesium haro de, and calcium carbonate, tend to precipitate when a or thin emperature and concent at corre exceeded. The precipitate, often mixed with dirt entering with he seawater and with or solution of the second seco transfer surfaces reduces heat transfer rates and thus impairs plant performance. While the ambienttemperature operation of membrane processes reduces scaling, membranes are much more susceptible not only to minute amounts of scaling or even dirt, but also to the presence of certain salts and other compounds that reduce their ability to separate salt from water. To reduce corrosion, scaling, and other problems, the water to be desalted is pretreated. The pretreatment consists of filtration, and may inlude removal of air (deaeration), removal of CO_2 (decarbonation), and selective removal of scale-forming salts (softening). It also includes the addition of chemicals that allow operation at higher temperatures without scale deposition, or which retard scale deposition and/or cause the precipitation of scale which does not adhere to solid surfaces, and that prevent foam formation during the desalination process.

Saline waters, including seawater, contain, besides a variety of inorganic salts, also organic materials and various particles. They differ in composition from site to site, and also change with time due to both natural and person-made causes. Design and operation of desalination plants requires good knowledge of the saline water composition and properties (Fabuss, 1980; Heitman, 1991).

The major water desalination processes that are currently in use or in advanced research stages are concisely described below. Information on detailed modeling can be found in the references.

Distillation Processes

Multistage Flash Evaporation (MSF)

Almost all of the large desalination plants use the MSF process shown schematically in Figure 4.1.1. A photo of an operating plant is shown in Figure 4.1.2. The seawater feed is preheated by internal heat recovery from condensing water vapor during passage through a series of stages, and then heated to its



FIGURE 4.1.3 Two HTME desalination units, each producing 5000 m³/day, in St. Croix, P. Virgin Islands. (Courtesy of I.D.E. Technologies Ltd.)

the exterior of the tube, causing it to evaporate. The vapor government thereby flows into the tubes in the next effect, and the process is repeated from effect to effect.

In the VTE the saline water typically in we do ynward inside vertical tub evaporates as a result of condensation of vapor coming from a higher temperature ffect on the tube exterior. While internal heat recovery is relative common to both MS2 and Mc processes, there are at least three important them. One is that wap better in the ME process occurs on the heat transfer surfaces differer s v hile in the We place in the free stream. This makes the ME process much SS more susceptible to so the formation. At the same time, the heat transfer coefficient between the vapor and the preheated brine is lower in the MSF process because the heated brine does not boil. In the ME process it does boil, and it is well known that boiling heat transfer coefficients are significantly higher than those where the heating does not result in boiling. In using direct transfer of latent heat of condensation to latent heat of evaporation, instead of sensible heat reduction to latent heat of evaporation as in MSF, the ME process requires a much smaller brine flow than the MSF. Limiting brine concentration in the last effect to about three times that of the entering seawater, for example, requires a brine flow of only about 1.5 times that of the distillate produced. At the same time, a pump (although much smaller than the two pumps needed in MSF) is needed for each effect.

The PR of ME plants is just sightly lower than the number of effects, which is determined as an optimized compromise between energy efficiency and capital cost. Six effects are typical, although plants with as many as 18 effects have been built.

Further detail about ME desalination can be found in Steinbruchel and Rhinesmith (1980) and Standiford, (1986a).

Vapor Compression Distillation (VC)

As stated earlier, the vapor pressure of saline water is lower than that of pure water at the same temperature, with the pressure difference proportional to the boiling point elevation of the saline water. Desalination is attained here by evaporating the saline water and condensing the vapor on the pure water. Therefore, the pressure of the saline water vapor must be raised by the magnitude of that pressure difference, plus some additional amount to compensate for various losses. This is the principle of the vapor compression desalination method. Furthermore, as shown in Figure 4.1.5, the heat of condensation

future climate because it is the most stable climate parameter. The period for averaging generally involves decades or longer. Consequently, the average global temperature does not vary significantly on a yearto-year basis. Typically, the surface air temperature can change by as much as 20°C from day to night. Averaging over the entire globe eliminates the effect of the day-night cycle, since the same total area of the globe is always under illumination. The global average temperature is a good indicator of the total energy that Earth has absorbed from the sun. The atmosphere, land, and oceans have enormous capacities to store heat. Accordingly, these reservoirs of heat maintain the average temperature over long time intervals despite fluctuations in the global heat input or loss. The temperature changes are caused by variations in the sun's energy reaching Earth and movements of air masses with different meteorological characteristics that include temperature, moisture, clouds, and precipitation. If the temperature over a much larger region were averaged, then much less variability would be observed. In a similar vein, periodic disruptions of oceanic flow and temperature in the southern Pacific, known as El Nino/Southern Oscillation (ENSO) or La Nina, cause major weather disruptions due to buildup of hot or cold water masses, respectively, during these relatively local occurrences. The atmosphere and tropical Pacific Ocean interact in such a way that wind and water currents periodically create large pools of warm surface water over the eastern Pacific Ocean. This is a major factor contributing to the important El Nino Southern Oscillation (ENSO) phenomenon, which periodically triggers anomalous and destructive weather around the Pacific Ocean basin.

Climate studies focus on the global balance of energy. The source of energy is the sun. The absorption of solar energy by Earth, flow of energy through Earth reservoirs, and eventual loss of energy back o space are the processes that are of concern in environmental heat transfer. There is a number of other concerns that can be attributed to environmental heat transfer on a more logarized even, such as the effect of:

- Fluorochlorocarbons (CFCs), ideal heat item for this used in refrict and air conditioning, on the stratospheric ozone layer
- Deposition of ammon in more and ammonium out it is on boiler heat transfer surfaces due to the reaction of the nomin (added for NO (control)) with SO₂ and NO₂.

These potential areas is en transmittal heat transfer will not be discussed at this time.

The sun generates energy by the process of nuclear fusion. The atoms are fused together to form heavier nuclei. When nuclei fuse, huge amounts of energy are released. The nuclear fusion of hydrogen atoms (H) to form helium atoms (He), for example, is the basic process that powers the sun. An empirical way of explaining this process is through the concept of binding energy or mass defect per nuclear particle. Figure 4.2.1 relates the binding energies of all nuclear particles to their atomic mass. Iron ($_{26}Fe^{56}$) is considered the most stable element and the fusion of two atoms lighter than iron, e.g., $_1H^2 + _1H^3$ (the two heavy isotopes of hydrogen, deuterium, and tritium) give up energies on the order of 17 MeV. Nuclear fission, which is the process operating in all nuclear reactors, is similarly explained as uranium ($_{92}U^{235}$) splitting due to the capture of a thermal neutron. The uranium fragments or fission products have higher binding energies because they have lower mass, producing 200 MeV per fission. Nuclear processes will be discussed later as alternatives to fossil fuel combustion for producing power without emitting infrared absorbing gases.

Average Temperature of Earth

The balance of energy held in various heat reservoirs determines the average temperature of Earth at any moment. Two overall processes control the total heat content of these reservoirs, viz., the absorption of energy from incoming sunlight and the emission of thermal radiation back into space.

Figure 4.2.2a compares the spectra of sunlight and terrestrial heat radiation. The sun is a blackbody with an emission temperature of about 6000 K. The emission peaks at a wavelength of about 0.55 μ m (as predicted by Wien's law, $\lambda_{max} = 2987/T$ in μ m·K). The absolute intensity of the radiation reaching Earth is controlled by the distance from the sun and the size of the sun itself. The size of the sun is

Reservoir	Volume (km ³) ^a	Mass (Gt)	Temp. (K)	Heat Capacity (J/g • K)	Energy Content (EJ)
Atmosphere ^b	5.0×10^{9}	5.2×10^{6}	250	1.0	1.3×10^{6}
Land					
Surface ^c	3.0×10^4	3.3×10^4	290	3.7	3.5×10^{4}
Subsurface ^c	1.5×10^{5}	1.6×10^{5}	280	3.7	1.7×10^{5}
Oceans					
Surfaced	3.3×10^{7}	3.3×10^{7}	280	4.2	4.0×10^{7}
Thermocline ^d	1.7×10^{8}	1.7×10^{8}	275	4.2	1.9×10^{8}
Deep ^d	1.2×10^{9}	1.2×10^{9}	270	4.2	1.4×10^{9}
Cryosphere	5.1×10^7	4.7×10^7	265	2.1	2.6×10^7

TABLE 4.2.4 Earth's Energy Reservoirs

^a Volume is specified in order to estimate the reservoir's heat contents.

^b The atmosphere is assumed to be 10 km thick (approximately the Troposphere) at density of 1.05 kg/m³.

 $^{\rm c}$ The land surface depth is taken to be 0.2 m thick for diurnal response, with a density of 1100 kg/m³, based on saturated sandy and clay soil with 80% saturated pore space. The deep soil layer is taken as 1.0 m thick, for seasonal variations.

^d The oceans have a density of 1025 kg/m³ and average depths of about 100 m for the surface, 0.5 km for the thermocline, and 3.7 km for the deep ocean e^a The ice and snow reservoir has a density of 920 kg/m³ and an a^{ba}ge core for the surface.

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of 2 km.

Source: Modified from Turco, R.P. Ecotoria of State, Oxford University Press, New York, 1987.

Required ace Ocean Surface Land Atmosphere IR emission 50 20 860 Net IR tran fer to atmosphere 160 60 50 Sensible heat transfer to atmosphere 50 Latent heat transfer to atmosphere 200 50 Total 460 180 860 70000 6000 Time, seconds 3×10^{7} 1 year 1 day 2 h

Data taken from Table 4.2.4 and Platt (1976) can be used with Equation (4.2.7) to estimate the time required to heat or cool the surface of the ocean, land, and atmosphere by 10 K.

The estimated time of cooling the ocean surface reservoir is about 1 year. With a small percentage reduction in insolation, the ocean surface is therefore likely to cool several degrees per decade. Land surface is much more susceptible to rapid temperature change. In the absence of sunlight, the time for land to cool by 10 K is on the order of a day. The atmosphere can cool even faster, i.e., on the order of hours. We know empirically that the ground and surface air can indeed cool overnight.

The Greenhouse Effect

As indicated above, molecules in the atmosphere can absorb radiation at some wavelengths and scatter radiation at all wavelengths by means of Rayleigh scattering (intensity of scattered light is proportional to the inverse of wavelength to the fourth power). Also, molecules experience vibrational and rotational motions that lead to absorption and emission of radiation in the infrared portion of the electromagnetic spectrum. Absorption spectroscopy resolves their spectra into a series of sharp lines, concentrated in broader spectral bands. Molecules composed of different atoms, like carbon dioxide (CO₂) and water vapor (H₂O), tend to have very strong absorption bands. Most of the other molecules, such as oxygen (O₂), nitrogen (N₂), and hydrogen (H₂), have very weak bands.

Figures 4.2.2 and 4.2.3 show the ability of the atmosphere to absorb incoming solar radiation as a function of wavelength throughout the solar and infrared spectrum. Sunlight at a visible wavelength of 0.5 μ m is nearly unaffected by atmospheric absorption, whereas at an ultraviolet wavelength of 0.2 μ m it is completely absorbed. Surface infrared radiation at a wavelength of about 8 to 9 μ m passes unaffected through the atmosphere to space, but at 6 μ m escape is not possible.

The longer the path of a photon through the atmosphere, the more likely it will be absorbed. The probability that a photon will be absorbed along the path is expressed as the negative exponential of the optical depth by the Beer–Bouger–Lambert law. Optical depth, at a specific wavelength, is the product of the absorption coefficient, concentration of absorbing molecules, and path length. Consequently, radiation traversing the atmosphere at an angle is more likely to be absorbed than if it were travelling normal to Earth.

Figure 4.2.2 shows that atmospheric absorption in the thermal wavelength regions is dominated by the presence of H_2O and CO_2 absorption bands. Water vapor absorbs both solar near-infrared and thermal longwave radiation. The important absorption bands are in the near-infrared bands, the 6.3-µm vibration-rotation band, and the pure rotation band at wavelengths longer than 15 µm. CO_2 is active mainly in the 15-µm vibration-rotation band. The ozone (O_3) 9.6-µm vibration-rotation band appears in a vigion without other strong absorptions.

The atmospheric absorption spectrum has two regions through which malia can travel easily. Outside these regions, the air is quite opaque. The first region come ultraviolet, visible, and nearinfrared spectrum, from about 0.3 to 0.7 µm in watel for second region in the infrared region, rhad region is the operator of to by Luther and from about 8 to 13 µm. In climate studies, this th Ellingson (1985) as the atmosphere γ how. The window only has a strong absorption of O₃ (9.6 µm band) and a weaker back round absorption due to the water whor continuum. The greenhouse effect is caused by the inlare ase with which solar radii tion can reach Earth's surface through the UV visible re time the spectrum, and the diff every may thermal radiation has in escaping from the surface. Both H_2O and CO_2 behave a subscript H_2O and CO_2 behave a subscript H_2O and CO_2 behave a subscript H_2O and H_2O an house gases because they are transparent at visible and near-infrared wavelengths, and are on aque at longer infrared wavelengths. As will be discussed later, many other gases have similar radiative properties and can act as effective greenhouse gases.

The emission spectrum of Earth is compared with blackbody radiation at various temperatures in Figure 4.2.5. The actual emission spectrum does not resemble a perfect blackbody spectrum. There is no reason to believe that all the thermal radiation leaving Earth is emitted from the same atmospheric level. Atmospheric temperature varies substantially with height. It follows that the radiation emitted by gases near the surface has a different emission temperature than the radiation emitted by gases at the tropopause, or the upper thermosphere. Figure 4.2.6 shows the structure of the atmosphere up to 100 km. It should be noted that 90% of the mass of the atmosphere is found in the lower 10 km.

The blackbody emission spectra in Figure 4.2.5 has the same intensity as the CO_2 band at 15 µm emission at close to 210 K. Comparing this temperature with the temperature profile in Figure 4.2.6, we see that temperatures in this range can be found in the lower stratosphere and middle mesosphere. Although either is possible, it has been established to be from the mesosphere. Similarly, the region from about 10 to 13 µm falls close to a temperature of 290 K, based on comparisons with the blackbody curves. Such a temperature is found only near the surface. This is consistent with the atmospheric window.

The mechanism of the greenhouse effect can now be explained. At constant solar energy input, the thermal emissions must also remain constant. Thus, the area under the emission curve in Figure 4.2.5 must remain constant. Greenhouse gases partially close the atmospheric window and trap heat at the surface and in the lower atmosphere. To balance this effect, the surface and lower atmosphere must warm and emit thermal radiation at a greater intensity.

The greenhouse effect occurs when the atmosphere absorbs thermal emissions. In accordance with the principle of conservation of energy, the amount of heat absorbed equals the amount reemitted.



FIGURE 4.2.5 Comparison of satellite-measured infrared emissions of Earth's with blackbody temperature (Clark, 1982). Sample spectra from the Iris satellite, indicating the effective radiation temperature for different waves of thermal infrared radiation. (Adapted from Hanel et al., *J. Geophys. Res.*, 77, 2629-2641 19 2.)

Roughly half of the reemitted heat travels in the same direction as the original thermal radiation, but the other half travels in the opposite direction. Thus, makingly or currences of absorption and reradiation inhibit the escape of thermal energy to space. The travelsphere absorbs at the miss radiation with nearly the same efficiency as determined by kirchfoff's law, i.e., at sorptivity and emissivity of air are equal. Consequently, the primitive dinate model is modified by representing the entire atmosphere as an absorber and emissivity of the same average emissivity at all wavelengths. This result was derived by anthradim (1978).

$$T_{s} = T_{e} (2/(2-\epsilon))^{1/4}$$
(4.2.8)

where: $T_s =$ the actual average temperature of the surface, K

 T_e = the effective blackbody radiation temperature of Earth, K

 $\epsilon = \text{emissivity} \approx 0.75$

The emissivity is about three-quarters of the heat radiated by Earth's surface which is effectively absorbed by the atmosphere. Using this value in Equation (4.2.8) yields an average planetary surface temperature of 287 K, in accord with the measurements. The greenhouse effect of the atmosphere, therefore, explains the difference between the effective blackbody emission temperature of Earth and its surface temperature. Furthermore, most of the radiation leaving Earth originates in the upper atmosphere and not at the surface.

The absorption of trace components in the atmospheric window is explained in great detail in Figure 4.2.7. It should be noted that ozone and the CFCs (referred to as Freons, or F11, F12, F13, and F22) have strong absorptions in the window region, while methane and nitrous oxide absorb near the short-wavelength edge of the window. CO_2 tends to narrow the window from the long-wavelength side. As these greenhouse gases become more concentrated, their absorption bands overlap, narrowing the window. Earth's radiation where absorptions are strong is colder and therefore less intense. The surface and lower atmosphere respond by warming up, intensifying the emission in the more transparent regions of the spectrum to compensate for the loss of intensity in the window.

The greenhouse power of the trace atmospheric components is extremely large. This can be seen even for CFC concentrations of about 1 ppb(v) which produce the same global warming as roughly 300 ppm(v)



arG (RL 4.2.7 The about the orbit of the constituents in the atmosphere window (Watson et al., 1986). Spectral locations of the absorption relatives covarious trace gases. The spectrum between 7 and 13 μ m is referred to as the atmospheric "window."

TABLE 4.2.6 Effects of Clouds

Туре	Description	Area Covered	Effects on Shortwave	Longwave
Stratus	Low-lying, dense, surface to 2 km	Large	Efficient reflector, albedo = 0.5	Absorb/reradiate, blackbody radiation
Cumulus	Convectively active, separate puffs, forms storm clouds, 2 to 7 km	Localized	Efficient reflector, albedo = 0.5	Absorb/reradiate blackbody radiation
Cirrus	High, thin ice, 5 to 13 km	Large	Inefficient reflector, albedo = 0	Absorb/reradiate blackbody radiation

by the cloud bottom. The cloud bottom emits a flux of the same intensity back to the surface, preventing heat loss from the system. However, the top of the cloud also emits heat out to space. From space, the cloud tops look like the surface. The net effect is a cooling of local climate. In the case of cirrus clouds, the solar albedo is small. The large ice crystals in these clouds are not efficient in reflecting sunlight back to space. However, these clouds remain opaque to longwave radiation and behave as blackbodies. The thermal radiation emitted by the surface and lower atmosphere is absorbed at the cloud bottom. The cloud bottom reradiates the heat back toward the surface. The top of the cloud also radiates heat towards space. Since the cloud top is much colder than the surface, the emission to space is less intense than





beyond the normal fluctuations (noise location of CO₂ and the outer increase absorbing trace gases in order to estimate future global verage temperatures of A centric is based on data presented by:

2 giver (1986) on regional erenzy consumption

- Global energy 100 in a exist by Häfele (1981) with modifications from various industrial projections
 - Growth projections of CH₄, N₂O, CFC 11, CFC 12, and other CFCs by Ramanathan (1985) and Wigley (1987), as summarized by Krause et al. (IPSEP, 1989)
 - Energy efficiency improvement and conservation measures from various DOE sources

It should be noted that the scenario discussed here is similar to the business-as-usual scenario IS 92A in the IPCC study (Pepper et al., 1992).

Methodology

The impact of per capita energy growth patterns on future energy demand and growth of atmospheric carbon dioxide was evaluated by subdividing the globe into 6 regions of similar sociopolitical background and population growth rate. The 6 key global regions specifically considered are

- 1. North America (U.S. and Canada) [NA].
- 2. Middle East (North Africa and Persian Gulf States) [ME].
- 3. Commonwealth of Independent States and Eastern Europe [CISEE].
- 4. China and other centrally planned Asiatic economies [CPAE].
- 5. Industrialized countries (including Western Europe, Australia, New Zealand, Israel, Japan, and South Africa) [IC].
- 6. Less-developed countries (including all of South and Central America, Central Africa, and the rest of Asia) [LDC].

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4.3 Heat Exchangers

Ramesh K. Shah and Kenneth J. Bell

The two major categories of heat exchangers are shell-and-tube exchangers and compact exchangers. Basic constructions of gas-to-gas compact heat exchangers are plate-fin, tube-fin and all prime surface recuperators (including polymer film and laminar flow exchangers), and compact regenerators. Basic constructions of liquid-to-liquid and liquid-to-phase-change compact heat exchangers are gasketed and welded plate-and-frame, welded stacked plate (without frames), spiral plate, printed circuit, and dimple plate heat exchangers.

Shell-and-tube exchangers are custom designed for virtually any capacity and operating condition, from high vacuums to ultrahigh pressures, from cryogenics to high temperatures, and for any temperature and pressure differences between the fluids, limited only by the materials of construction. They can be designed for special operating conditions: vibration, heavy fouling, highly viscous fluids, erosion, corrosion, toxicity, radioactivity, multicomponent mixtures, etc. They are made from a variety of metal and nonmetal materials, and in surface areas from less than 0.1 to 100,000 m² (1 to over 1,000,000 ft²). They have generally an order of magnitude less surface area per unit volume than the compact exchangers, and require considerable space, weight, support structure, and footprint.

Compact heat exchangers have a large heat transfer surface area per unit volume of the exchanger, resulting in reduced space, weight, support structure and footprint, energy requirement end cast at well as improved process design, plant layout and processing conditions, together with low lund inventory compared with shell-and-tube exchangers. From the operating condition are praintenance point of view, compact heat exchangers of different constructions are used to exchange applications, such as for high-temperature applications (up to about 850°C or 0.51%) and pressure applications, such as for high-temperature applications. However, a price of the involve both heat temperature and pressure simultaneously. Plate-fin exchange core generally brazer, will the largest size currently manufactured is $1.2 \times 1.2 \times 6$ (m/4a/s) $\times 20$ ft). Fouling is one of the major potential problems in many compact exchanges of exchangers compared with shell-and-tube exchangers. With a large frontal area exchanger, flow maldistribution could be another problem or such as of short transient times, a careful design of controls is required for startup of some compart heat exchangers compared with shell-and-tube exchangers. No industry standards or recognized practice for compact heat exchangers is yet available.

This section is divided into two parts: Compact Heat Exchangers and Shell-and-Tube Exchangers, written by R. K. Shah and K. J. Bell, respectively. In the compact heat exchangers section, the following topics are covered: definition and description of exchangers, heat transfer and pressure drop analyses, heat transfer and flow friction correlations, exchanger design (rating and sizing) methodology, flow maldistribution, and fouling. In the shell-and-tube heat exchangers section, the following topics are covered: construction features, principles of design, and an approximate design method with an example.

Compact Heat Exchangers

Ramesh K. Shah

Introduction

A heat exchanger is a device to provide for transfer of internal thermal energy (enthalpy) between two or more fluids, between a solid surface and a fluid, or between solid particulates and a fluid, in thermal contact without external heat and work interactions. The fluids may be single compounds or mixtures. Typical applications involve heating or cooling of a fluid stream of concern, evaporation or condensation of single or multicomponent fluid stream, and heat recovery or heat rejection from a system. In other applications, the objective may be to sterilize, pasteurize, fractionate, distill, concentrate, crystallize, or control process fluid. In some heat exchangers, the fluids transferring heat are in direct contact. In other heat exchangers, heat transfer between fluids takes place through a separating wall or into and out of a



FIGURE 4.3.10 A section of a printed circuit heat exchanger. (Courtesy of Heatric Ltd., Dorset, U.K.)

Printed Circuit Heat Exchangers. This exchanger, as shown in Figure 4.3.10, has only primary heat transfer surfaces as PHEs. Fine grooves are made in the plate by using the same techniques as those employed for making printed electrical circuits. High surface area densities (650 to 1350 m²/m³ or 200 to 400 ft²/ft³ for operating pressures of 500 to 100 bar respectively) are achievable. A variety of materials including stainless steel, nickel, and titanium alloys can be used. It has been successfully user with relatively clean gases, liquids and phase-change fluids in chemical processing, free processing, waste heat recovery, and refrigeration industries. Again, this exchanger is a recovery with limited special applications currently.

Exchanger Heat Transfer and Pressure Drip Lharysis

In this subsection, starting with the mal-circuit associated with a two-fluid exchanger, ε-NTU, P-NTU, and mean tenne plure linerence (MTD) met ods us defor an exchanger analysis are presented, followed by the methiciency concept and at observersions. Finally, pressure drop expressions are of the over various single-phase with angers.

Two energy conservation d its ential equations for a two-fluid exchanger with any flow arrangement are (see Figure 4.3.11 for counterflow)

$$dq = q'' \, dA = -C_h \, dT_h = \pm C_c \, dT_c \tag{4.3.1}$$

where the \pm sign depends upon whether dT_c is increasing or decreasing with increasing dA or dx. The local overall rate equation is

$$dq = q'' dA = U(T_h - T_c)_{\text{local}} dA = U\Delta T dA$$
(4.3.2)

Integration of Equations (4.3.1) and (4.3.2) across the exchanger surface area results in

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i})$$
(4.3.3)

and

$$q = UA\Delta T_m = \Delta T_m / R_o \tag{4.3.4}$$

where ΔT_m is the true mean temperature difference (or MTD) that depends upon the exchanger flow arrangement and degree of fluid mixing within each fluid stream. The inverse of the overall thermal conductance *UA* is referred to as the overall thermal resistance R_a as follows (see Figure 4.3.12).

$$R_{o} = R_{h} + R_{s,h} + R_{w} + R_{s,c} + R_{c}$$
(4.3.5)

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 TABLE 4.3.5
 Fin Efficiency Expressions for Plate-Fin and Tube-Fin Geometries of Uniform Fin Thickness

For individually finned tubes as shown in Figure 4.3.5a, flow expansion and contraction take place along each tube row, and the magnitude is of the same order as that at the entrance and exit. Hence, the entrance and exit losses are generally lumped into the core friction factor. Equation (4.3.29) then reduces for individually finned tubes to

$$\frac{\Delta p}{p_i} = \frac{G^2}{2g_c} \frac{1}{p_i \rho_i} \left[f \frac{L}{r_h} \rho_i \left(\frac{1}{\rho} \right)_m + 2 \left(\frac{\rho_i}{\rho_o} - 1 \right) \right]$$
(4.3.35)

Regenerators. For regenerator matrices having cylindrical passages, the pressure drop is computed using Equation (4.3.29) with appropriate values of f, K_c , and K_e . For regenerator matrices made up of any porous material (such as checkerwork, wire, mesh, spheres, copper wools, etc.), the pressure drop is calculated using Equation (4.3.35) in which the entrance and exit losses are included in the friction factor f.

Plate heat exchangers. Pressure drop in a PHE consists of three components: (1) pressure drop associated with the inlet and outlet manifolds and ports, (2) pressure drop within the core (plate passages), and (3) pressure drop due to the elevation change. The pressure drop in the manifolds and ports should be kept as low as possible (generally < 10%, but it is found as high as 25 to 30% of higher in some designs). Empirically, it is calculated as approximately 1.5 times the inlet velocity head per pass tince the entrance and exit losses in the core (plate passages) cannot be determined experimentally, hey are included in the friction factor for the given plate geometry. The pressure loop (rise) caused by the elevation change for liquids is given by Equation (4.3.28). Hence the pressure drop on one fluid side in a PHE is given by

$$\Delta p = \frac{4}{2} \frac{G}{g_c} \frac{Q}{\rho_i} + \frac{4}{2} \frac{fLG^2}{g_c} \frac{1}{Q} + \frac{1}{Q_o} \frac{1}{\rho_i} \frac{1}{\rho_c} \frac{G^2}{g_c} \pm \frac{\rho_m gL}{g_c}$$
(4.3.36)

where N_p is the number of rate with given fluid side and D_e is the equivalent diameter of flow passages (usually twicture plate pacing). Note that the third term on the right-hand side of the equality sign of Equation (4.3.36) is for the momentum effect which is generally negligible in liquids.

Heat Transfer and Flow Friction Correlations

Accurate and reliable surface heat transfer and flow friction characteristics are a key input to the exchanger heat transfer and pressure drop analyses or to the rating and sizing problems (Shah, 1985). Some important analytical solutions and empirical correlations are presented next for selected exchanger geometries.

The heat transfer rate in laminar duct flow is very sensitive to the thermal boundary condition. Hence, it is essential to identify carefully the thermal boundary condition in laminar flow. The heat transfer rate in turbulent duct flow is insensitive to the thermal boundary condition for most common fluids (Pr > 0.7); the exception is liquid metals (Pr < 0.03). Hence, there is generally no need to identify the thermal boundary condition in turbulent flow for all fluids except liquid metals.

Fully developed laminar flow analytical solutions for some duct shapes of interest in compact heat exchangers are presented in Table 4.3.6 for three important thermal boundary conditions denoted by the subscripts H1, H2, and T (Shah and London, 1978; Shah and Bhatti, 1987). Here, H1 denotes constant axial wall heat flux with constant peripheral wall temperature, H2 denotes constant axial and peripheral wall heat flux, and T denotes constant wall temperature. The entrance effects, flow maldistribution, free convection, property variation, fouling, and surface roughness all affect fully developed analytical solutions. In order to account for these effects in real plate-fin plain fin geometries having fully developed flows, it is best to reduce the magnitude of the analytical Nu by at least 10% and to increase the value of the analytical fRe by 10% for design purposes.

Here we have neglected the wall and fouling thermal resistances. This equation in nondimensional form is given by

$$\frac{1}{\text{NTU}} = \frac{1}{\text{ntu}_h(C_h/C_{\min})} + \frac{1}{\text{ntu}_c(C_c/C_{\min})}$$
(4.3.64)

$$G_i = \left[\frac{2g_c \Delta p}{\text{Deno}}\right]_i^{1/2}$$
 $i = 1,2$ (4.3.65)

where

Deno_i =
$$\left[\frac{f}{j}\frac{\text{ntu}}{\eta_o} \Pr^{2/3}\left(\frac{1}{\rho}\right)_m + 2\left(\frac{1}{\rho_o} - \frac{1}{\rho_i}\right) + \left(1 - \sigma^2 + K_c\right)\frac{1}{\rho_i} - \left(1 - \sigma^2 - K_e\right)\frac{1}{\rho_o}\right]_i$$
 (4.3.66)

$$V = \frac{A_1}{\alpha_1} = \frac{A_2}{\alpha_2} \tag{4.3.67}$$

In the iterative solutions, the first time one needs ntu_h and ntu_c to start the iterations. These can be either determined from the past experience or by estimations. If both fluids are gase or both nuids are liquid, one could consider that the design is "balanced," i.e., that the the null resistances are distributed approximately equally on the hot and cold sides. It that is $C_h = C_c$, and

$$\approx$$
 ntu ≈ 2 NTU (4.3.68)

Alternatively invertice liquid on one sident has on the other side, consider 10% thermal resistance the varia-side, i.e., $0.10 \left(\frac{1}{UA}\right) = \frac{1}{\left(\eta_o hA\right)_{\text{line}}}$ (4.3.69)

(1.5.07)

Then, from Equations (4.3.63) and (4.3.64) with $C_{gas} = C_{min}$, we can determine the ntu values on each side as follows:

$$ntu_{gas} = 1.11NTU, \quad ntu_{lig} = 10C^*NTU$$
 (4.3.70)

Also note that initial guesses of η_o and *j/f* are needed for the first iteration to solve Equation (4.3.66). For a good design, consider $\eta_o = 0.80$ and determine an approximate value of *j/f* from the plot of *j/f* vs. Re curve for the known *j* and *f* vs. Re characteristics of each fluid side surface. The specific step-bystep design procedure is as follows:

1. In order to compute the fluid bulk mean temperature and the fluid thermophysical properties on each fluid side, determine the fluid outlet temperatures from the specified heat duty

$$q = \left(\dot{m}c_{p}\right)_{h} \left(T_{h,i} - T_{h,o}\right) = \left(\dot{m}c_{p}\right)_{c} \left(T_{c,o} - T_{c,i}\right)$$
(4.3.71)

or from the specified exchanger effectiveness using Equation (4.3.52) and (4.3.53). For the first time, estimate the values of c_p .

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FIGURE 4.3.20 Heat transfer area as a function of shell inside diameter and effective tube length for 19.05 mm $(^{3}/_{4}$ in.) tubes on a 23.8 mm $(^{15}/_{16}$ in.) equilateral triangular tube layout, fixed tubesheet, one tube-side pass, fully tubed shell. (From Hewitt, G.F., Ed. *Heat Exchanger Design Handbook 1998*, Begell House, New York, 1998. With permission.)

Example of the Approximate Design Method

Problem Statement. Estimate the dimensions of a shell-and-tube heat exchanger to cool 100,000 lb_m/hr (12.6 kg/sec) of liquid toluene from 250 to 110°F (121.1 to 43.3°C) using cooling tower water available at 80°F (26.7°C). Use split-ring floating head construction (TEMA S) with ${}^{3}/_{4}$ in. (19.05 mm) outside diameter × 14 BWG (0.083 in. = 2.11 mm wall) low-carbon steel tubes on ${}^{15}/_{16}$ in. (23.8 mm) equilateral triangular pitch. This construction implies one shell-side pass and an even number of tube-side passes — assume two for the present. Choose cooling water exit temperature of 100°F (37.8°C). Specific heat of toluene is 0.52 Btu/lb_m°F (2177 J/kgK) and viscosity at 180°F (82.2°C) is 0.82 lb_m/ft hr (0.34 × 10⁻³ Nsec/m² or 0.34 cP).

Solution.

 $q_T = (100,000 \text{ lb}_{\text{m}}/\text{hr})(0.52 \text{ Btu/lb}_{\text{m}}^{\circ}\text{F})(250 - 110)^{\circ}\text{F}$ = 7.28×10⁶ Btu/hr = 2.14×10⁶ W

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5/8 (15.88) 5/8 (15.88) 3/4 (19.05) 3/4 (19.05)	13/16 (20.6) 13/16 (20.6) 15/16 (23.8)		0.90
5/8 (15.88) 5/8 (15.88) 3/4 (19.05) 3/4 (19.05)	13/16 (20.6) 13/16 (20.6) 15/16 (23.8)	→	0.90 1.04 1.00
5/8 (15.88) 3/4 (19.05) 3/4 (19.05)	13/16 (20.6) 15/16 (23.8)		1.04
3/4 (19.05) 3/4 (19.05)	15/16 (23.8)	→ <	1,00
3/4 (19.05)			
(10.05)	15/16 (23.8)		1.16
3/4 (19:03)	1 (25.4)		1.14
3/4 (19.05)	1 (25.4)		1.31
1 (25.4)	1 1/4 (31.8)		1.34
1 (25.4)	1 1/4 (31.8)		1.54
1 (23.4)	1 1/4 (31.8)		1.54

TABLE 4.3.11 Values of F1 for Various Tube Diameters and Layouts

This table may also be used for low-finned tubing in the follow ue estimated for h_o from Table 4.3.10 should be multiplied by between 0.75 and 1 for A^* A_o , the total a good application; 0.85 is a good estimate in Equation 1U divided by he ratio of the outside heat transfer area including fit s. Then this value of A_o finned tube heat transfe aren to the prain tube area (per unit ength). The result of this calculation r Equation 4.3.96 to find a volume Figure 4.3.20. is used as

Handbook 1998, Begell House, New York, G.F., Ed., Heat Exc iger i With permission

TABLE 4.3.12 Values of F₂ for Various Numbers of Tube Side Passes^a

	F ₂ Number of Tube-Side Passes				
Inside Shell Diameter, in. (mm)	2	4 6 0 1.40 1.80		8	
Up to 12 (305)	1.20	1.40	1.80	_	
$13^{1}/_{4}$ to $17^{1}/_{4}$ (337 to 438)	1.06	1.18	1.25	1.50	
$19^{1}/_{4}$ to $23^{1}/_{4}$ (489 to 591)	1.04	1.14	1.19	1.35	
25 to 33 (635 to 838)	1.03	1.12	1.16	1.20	
35 to 45 (889 to 1143)	1.02	1.08	1.12	1.16	
48 to 60 (1219 to 1524)	1.02	1.05	1.08	1.12	
Above 60 (above 1524)	1.01	1.03	1.04	1.06	

Since U-tube bundles must always have at least two passes, use of this table is essential for U-tube bundle estimation. Most floating head bundles also require an even number of passes.

Source: Hewitt, G.F., Ed., Heat Exchanger Design Handbook 1998, Begell House, New York, 1998. With permission.

LMTD_{countercurrent} =
$$\frac{(250 - 100) - (110 - 80)}{\ln \frac{250 - 100}{110 - 80}} = 74.6^{\circ}F = 41.4^{\circ}C$$

Since there are at least two tube-side passes, flow is not countercurrent, and $T_{h_c} > T_{c_c}$, estimate $F \approx 0.9$. Therefore, MTD = $0.9 (74.6^{\circ}F) = 67.1^{\circ}F = 37.3^{\circ}C.$

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4.4 Bioheat Transfer

Kenneth R. Diller, Jonathan W. Valvano, and John A. Pearce

Abstract

Heat transfer plays a major role in many processes in living systems. This chapter presents a brief review of bioheat transfer from the perspective of those aspects which distinguish it from processes in inanimate systems. Of particular interest are the effects of blood perfusion on temperature distribution, coupling with biorate processes, measurement of thermal transport properties in tissues, therapeutic processes, human thermoregulation, and effects on living tissues of elevated temperatures and subzero temperatures. An extensive compilation of properties for biomaterials is also presented.

Introduction

Analysis and control of heat transfer processes in living systems involve phenomena that are not encountered in systems that are not alive. For example, there is a continuous flow of blood through tissues and organs via a complex network of branching vessels. Heat exchange processes are influenced by vascular geometry, which consists of arteries and veins with countercurrent flow over large portions of the network of pairs. For some physiological conditions, arterial and venous blood temperatures are different, neither is equal to the local tissue temperature, and these temperature differences may vary as a function of many transient physiological and physical parameters are humans of regulation for the thermal state in tissue are quite nonlinear and have presented to apply challenge to understand and model. Nonetheless, thermoregulatory processes are crucial of non-maintenance of life and must be accounted for in the design of many types of system s hat interface with humans and a linears.

Although considerable processes as been made in defining and modeling thermal processes in living systems, and especially solver the past 15 years, many important problems remain. This section provides a brief discussion with present state of knowledge and understanding of some of the more active areas of bohow transfer. Genere at the state of additional areas of application of bioheat transfer may be found among the references ared.

Coupling of Temperature History to Rate Processes

Although the processes of life are observed at the system (macroscopic) and microscopic levels of resolution, the true essence of life lies in the multitudinous biochemical phenomena which occur continuously throughout all organisms. Over the past 50 years the science of biochemistry has elucidated many of the governing life processes at the molecular level. Today the major frontier of knowledge in the life sciences is associated with molecular biology. Increasingly, bioengineers are collaborating with molecular biologists to understand and manipulate the molecules and biochemical processes that constitute the basis of life. Temperature is a primary controlling parameter in the regulation of these rate processes. The study of thermal biology has identified that the rates of nearly all physiological functions are altered by 6 to 10% per degree Celsius over wide range of thermal states produced in response to environment conditions.¹ Likewise, temperature is often altered during therapeutic or diagnostic procedures to produce or measure a targeted effect, based on the fact that a change in local temperature will have a large effect on biochemical process rates. Thus, a knowledge of how temperature can be monitored and/or controlled in living tissues is of great value in both the assessment of normal physiological function and the treatment of pathological states.

In assessing the effects of temperature alterations on biochemical rate processes, two broad categories of state changes can be considered: temperatures both above and below the normal physiological thermal state. An extensive review of both of these thermal domains has been published recently,² to which the reader is referred for further details and bibliography.

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Let \overline{T} be the temperature rise above baseline, $\overline{T} = T - T_0$. Both the thermistor bead temperature rise (\overline{T}_b) and the tissue temperature rise (\overline{T}_m) are initially zero. The subscripts b and m refer, respectively, to the bead and the tissue (i.e., the surrounding medium).

$$\overline{T}_{\rm h} = \overline{T}_{\rm m} = 0 \quad \text{at } t = 0 \tag{4.4.3}$$

Assuming the venous blood temperature equilibrates with the tissue temperature and that the metabolic heat is uniform in time and space, the Pennes' bioheat transfer equation²⁸ in spherical coordinates is given by:

$$\rho_{\rm b}c_{\rm b}\frac{\partial\overline{T}_{\rm b}}{\partial t} = k_{\rm b}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\overline{T}_{\rm b}}{\partial r}\right) + \frac{A + Bt^{-1/2}}{\frac{4}{3}\pi a^3} \quad r < a$$
(4.4.4)

$$\rho_{\rm m} c_{\rm m} \frac{\partial \overline{T}_{\rm m}}{\partial t} = k_{\rm m} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \overline{T}_{\rm m}}{\partial r} \right) - w c_{\rm bl} V_{\rm m} \quad r > a$$
(4.4.5)

Perfect thermal contact is assumed between the finite-sized spherical thermistor and the infinite homogeneous perfused tissue. At the interface between the bead and the tissue, continuity of thermal fur and temperature leads to the following boundary conditions:

$$\overline{T}_{b} = \overline{T}_{m} \quad \text{at } r \equiv a \text{esale} \qquad (4.4.6)$$

$$O_{r}^{k} = k_{m} \frac{\partial I_{m}}{\partial r} \text{ at } \mathbf{r} = \mathbf{a}$$
(4.4.7)

The other relevant boundary conditions $r \to 0$ and $r \to \infty$. Since no heat is gained to last the center of the there is a

$$\overline{T}_{b} = \text{finite}\left(\text{or } \mathbf{k}_{b} \ \frac{\partial \overline{T}_{b}}{\partial r} = 0\right) \text{ as } \mathbf{r} \to 0$$
 (4.4.8)

Because the thermistor power is finite and the tissue is infinite, the tissue temperature rise at infinity goes to zero:

$$\overline{T}_{\rm m} \to 0 \quad \text{as } r \to \infty$$
 (4.4.9)

It is this last initial condition that allows the Laplace transform to be used to solve the coupled partial differential equations. The Laplace transform converts the partial differential equations into ordinary differential equations that are independent of time t. The steady-state solution allows for the determination of thermal conductivity and perfusion.²⁰

$$\overline{T}_{b}(\mathbf{r}) = \frac{A}{4\pi a k_{b}} \left\{ \frac{k_{b}}{k_{m}(1+\sqrt{z})} + \frac{1}{2} \left[1 - \left(\frac{\mathbf{r}}{a}\right)^{2} \right] \right\}$$
(4.4.10)

$$\overline{T}_{\rm m}({\bf r}) = \frac{A}{4\pi r \, {\bf k}_{\rm m}} \left(\frac{{\rm e}^{(1-r/a)\sqrt{z}}}{1+\sqrt{z}} \right)$$
(4.4.11)

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The last assumption has drawn criticism based on studies that indicate the temperature to be closer to tissue.^{58,63} Limitations of this model include the difficulty of implementation, and that the artery and vein diameters must be identical. Both of these issues have led to the development of the models described in subsequent sections. Studies using this model have been applied to the peripheral muscle tissue of a limb,⁴⁴⁻⁴⁶ and the model is accepted as valid for vasculature with diameters <300 µm and $\varepsilon < 0.3$.⁶³

Simplified Weinbaum-Jiji (W-J)⁴⁰

In response to the criticism that their previous model is too difficult and complex to apply, Weinbaum and Jiji simplified the three-equation model to a single equation:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k_{eff} \frac{\partial T}{\partial x} \right) + q_{met}$$
(4.4.44)

In their simplification, they derived an equation based on the temperature of tissue only. The imperfect countercurrent heat exchange is embodied in an effective conductivity tensor term.

$$k_{eff} = k \left\{ 1 + \frac{n \left[\left(\rho c \right)_{b} \pi r_{b}^{2} \overline{\nabla} \cos \gamma \right]^{2}}{\sigma_{\Delta} \cdot k^{2}} \right\} + q_{met}$$
(4.4745)

The k_{eff} term has similar parameters to the tissue and arter, very value heat exchange term in Equation 4.4.43, and a shape factor term (σ_{Δ}). In order to the blood temperature from their previous formulation, two major assumptions (the second conditions) we explain the second conditions) we explain the second conditions of the second

- 1. The mean tissue temperature $= (T_a + T_v)/2$.
- 2. Heat from a prince artery is mostly conduct a to be corresponding vein:

$$\mathbf{Pre^{Page}}_{q_{a}} \approx q_{v} \approx \sigma_{\Delta} k \left(T_{a} - T_{v}\right). \tag{4.4.46}$$

Both of these assumptions were based upon studies in rabbit thigh muscle from their previous formulation. However, to respond to criticism of these assumptions, they performed further mathematical analysis on ε and provided insights into the limits for applying these assumptions.⁴⁷ An obvious limitation of this model is that the local temperatures along the countercurrent artery and vein cannot be calculated. Another limitation is that the model is applicable only in situations where $\frac{L_e}{L} << 1$. In the example of analyzing the peripheral tissue in the arm, L is equal to the characteristic radius of the arm.⁴⁸ Weinbaum and Lemons⁴⁸ admit that this assumption breaks down under the following conditions: (1) if blood flow rates significantly increased in the larger vessel pairs of the peripheral tissue layer, and (2) if deeper muscle tissue, where the diameter of the countercurrent pair vessels are less than 300 µm, are included. This model has been tested in the porcine⁶² and canine kidney,⁶⁷ and continues to be verified by the Weinbaum group^{41,50} and other investigators.⁶⁸

Small Artery Model^{69,70}

The small artery model was developed by Anderson in studies of the canine kidney cortex. The model considers the energy balance in a control volume (i, j, k) which contains either an arterial (Q_a) or venous (Q_v) vessel. For a volume with an artery parallel to the "z" coordinate axis, the equation is

$$Q_{a} = N(VA)_{a}(\rho c)_{b}(1 + \lambda - 2\lambda\xi)\left(\frac{T_{z} - T_{z-\Delta z}}{\Delta z}\right)$$
(4.4.47).

which direct thermoregulation behavior. The hypothesis in support of the theory of energy content regulation based on Webb's experimental observations is

Heat (energy) regulation achieves heat (energy) balance over a wide range of heat (energy) loads. Heat flow to or from the body is sensed, and physiological responses defend the body heat (energy) content. Heat (energy) content varies over a range that is related to heat (energy) load. Changes in body heat (energy) content drive deep body temperatures.⁷⁸

The proposed mechanism of energy-driven thermoregulation balances the constantly changing metabolic energy production and the adjustment of heat losses to maintain the body as a system at steadystate. In contrast, the proposed mechanism of temperature-driven thermoregulation affects a coordination of physiological processes to maintain the body core temperature at a set point.

Thermoregulatory Processes

Conservation of energy for the human body must account for internal metabolic energy production plus multiple mechanisms of environmental heat and work exchange.

$$\Delta E = M - \left(W + Q_{conv} + Q_{cond} + Q_{rad} + Q_{evap} + Q_{resp}\right)$$
(4.4.51)
ergy storage in the body (W)
energy production (W)
vork (W)
at loss by convection (W)
at loss by conduction W
at loss by radia 0. (W)

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where $\Delta E = Rate$ of energy storage in the body (W)

- Μ = Metabolic energy production (W)
- W = External work (W)
- Q_{conv} = Surface heat loss by convection (W)
- Q_{cond} = Surface heat loss by conduction
- Q_{rad} = Surface heat loss by radi
- $Q_{evap} = Surface heat$ **F** $_st b e a oration (W$
- wy heat foss (W)

e charges heat with the environment, and loses heat by evaporation body produces ener of body fluids. Energ, is produced in the body by basal (resting), metabolism defined as the minimal metabolism measured a a temperature of thermal neutrality in a resting homeotherm with normal body temperature several hours after a meal and not immediately after hypothermia, and also at an increased rate due to muscle activity, including physical exercise and shivering, and by food intake. Therefore, the total energy production in the body is determined by the energy needed for basic body processes plus any external work. Since the body operates with less than 100% efficiency, only a fraction of the metabolic rate is applied to work, with the remainder dissipated as heat.⁷⁹ The mechanical efficiency, η , associated with metabolic energy utilization, is zero for most activities except when the person is performing external mechanical work such as in walking upstairs, lifting something to a higher level, or cycling on an ergometer.⁸¹ When work transferred from the environment is dissipated as heat in the human body, η is negative. An example of this case is walking downstairs.

Convection, radiation, conduction, and evaporation of sweat at the skin surface all facilitate heat transfer from the body. Heat transfer also occurs via the respiratory tract and lungs, although to a lesser extent in human beings as compared with many mammals and reptiles. Storage of energy takes place whenever there is an imbalance of production and dissipation mechanisms. In many instances, such as astronauts in space suits or military personnel in chemical defense garments, energy storage is forced due to the lack of appropriate heat exchange with the environment.⁸²

The human thermoregulatory system is quite complex and behaves mathematically in a highly nonlinear manner. It contains multiple sensors, multiple feedback loops, and multiple outputs.⁸³ The primary mechanisms by which the body responds to control the storage of energy (positive or negative) include the evaporation of sweat, shivering of the muscles, and vasoconstriction and vasodilation of the blood vessels.

Heat transfer internal to the body is due to the conductance which governs the flow of energy between the core, through the tissue, and to the surface. This transport process is governed significantly by can trigger local release of bradykinins that induce vascular dilation or constriction, thus affecting temperature rise.

The increase in tissue temperature is accompanied by an increase in cellular metabolism. The arterioles (microscopic arteries, typically about of 40 to 200 μ m inner diameter¹⁰¹) dilate in response to heating under smooth muscle control. The downstream capillary pressure increases above the homeostatic level of about 25 torr. Two consequences are an increased capillary flow and capillary pressure. First, at higher capillary pressures the gaps between the endothelial cells (thin epithelial cells comprising the vessel wall) tend to widen and, at higher pressures, edema may form (an increase in the fluid in the extracellular compartment).^{102,103} Second, higher capillary flow results in rapid clearing of cellular metabolites and increases convection heat transfer, delivery of tissue oxygen, cellular nutrients, antibodies, and leukocytes (white blood cells) including monocytes which are necessary in the healing response. Neutrophils are white blood cells that initially release proteolytic enzymes to digest dead tissue and, later, phagocytose (eat) the debris.¹⁰⁴ Monocytes are other leukocytes that remove damaged and dead cells by phagocytosis.

Tissue Effects: Elevated Temperatures

Exposure to temperatures above normal physiologic ranges (>42°C) can result in measurable irreversible changes in tissue structure or function. Cell death or tissue alterations may be detrimental — for example, skin burns — or beneficial, as in vessel sealing or tumor destruction. Tissues of the central nervous system are the most thermally sensitive, exhibiting irreversible changes for long-term exposures above about 42°C. The specific pathologic end point may be evaluated histologically, h stocoonically, and/or physiologically. Most assays of thermal alterations are qualitative in names prover, several end points which are inherently quantitative lend themselves well to face process descriptions of their thermal kinetics. Even for qualitative processes, thermal is a to choosels often provide useful descriptions and so provide helpful insights into the under the principles of tissue thermal damage.

Here we use "damage" into the more three three three an ulds of irreversible alterations, therapeutic or not. In the Arrienite hodels, irreversible threat tamage is exponentially dependent on temperature and in a by experiment on time of exposure. Many damage processes can be modeled as first-order rate processes for which two experimentally derived coefficients are sufficient. Second-order processes require four parameters again derived from experiment. The first-order rate process models often apply well to the prediction of damage thresholds and less well as the damage becomes complete or severe since several of the fundamental assumptions are violated. In order to be useful in evaluating thermal insult, the kinetic model must be coupled to quantitative pathological analysis. This subsection describes several quantitative markers of thermal damage and experimental methods for estimating relevant kinetic coefficients both in constant temperature and transient thermal history experiments. As expected, transient *in vivo* thermal history data yield a noisy kinetic plot; however, estimates of the appropriate rate coefficients can be made.

Theory of Rate Process Descriptions

The original work on the application of rate process models to thermal damage was reported by Moritz and Henriques in a series of seminal papers entitled "Studies of Thermal Injury" in 1947.¹⁰⁵⁻¹⁰⁸ They applied flowing water at elevated temperatures to pig skin and measured exposure times required to create first-, second-, and third-degree burns. In their work, the damage was quantified using a single parameter, Ω , which ranges on the positive real axis and is calculated from an Arrhenius integral:

$$\Omega(\tau) = \int_0^{\tau} A \, e^{\left[\frac{-E}{RT}\right]} dt \qquad (4.4.56)$$

where A is a frequency factor (s⁻¹), τ the total heating time (s), E an activation energy barrier (J/mole), R the universal gas constant (8.314 J-mole⁻¹-K⁻¹), and T the absolute temperature (K).

Generally, [C*] is neither known nor calculable; however, at sufficiently low concentrations of C* the steady-state principle asserts that for short-lived activated states the rate of formation can be considered equal to the rate of disappearance. The activated state, [C*], forms at a rate k_a [C]², relaxes back to inactivated at rate k_b [C][C*], and denatures at the rate k_c [C*]. Consequently:

$$k_a[C]^2 = k_b[C][C^*] + k_c[C^*]$$
(4.4.65)

and so:

$$[C^*] = \frac{k_a [C]^2}{k_c + k_b [C]}$$
(4.4.66)

We actually need an overall reaction velocity, k, which relates [C] to its rate of disappearance:

$$-\frac{d[C]}{dt} = k[C] \tag{4.4.67}$$

There are two limiting cases for Equation 4.4.66: first, the concentration of remaining undamaged material, [C], may be large enough that deactivation at k_b dominates the k_c pathway, so $[C^*] \cong COc /k_b$ for which the overall formation rate, $k = k_c k_a/k_b$ and a first-order process results and second, if the remaining undamaged material concentration, [C], is small, $k_c >> k_b$ [C] and the process is second order since from Equation 4.4.66 k = k_a [C]. In liquid-phase system: we have belable concentrations of native state molecules, the first condition should apple so the line of exposure at damaging temperatures such that [C] is very smale $k_c \in V$. C and a second-order process results:

$$\begin{array}{c} \textbf{preview} \quad \textbf{page} \quad 5 \underbrace{\textbf{60}}_{dt} = \mathbf{k}[C]^2 \quad (4.4.68) \end{array}$$

where for simplicity the [C] dependence has been removed from k:

First-Order Solution

Equation 4.4.67, then, is a Bernoulli differential equation with the solution:

$$C(\tau) = C(0)e^{\{\int k dt\}}$$
(4.4.69)

Equations 4.4.59 and 4.4.60 may be used to relate k to ΔH^* and ΔS^* . It should be noted at this point that the energy barrier, E, (Figure 4.4.9) is in fact $\Delta H^* + RT$; however, in practice { $\Delta H^* _ 5 _ 105$ } >> {RT $\cong 3 \times 10^3$ }, so we may assume that E $\cong \Delta H^*$.

The pre-exponential term in Equation 4.4.59 suggests that it is temperature dependent; however, the linear dependence of A on 1/T is extremely weak compared to the exponential dependence in the final term.

Second-Order Solution

The second-order process of Equation 4.4.67 can be solved by dividing both sides by C² and using the substitution that y = 1/C. With that substitution:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}C}\frac{\mathrm{d}C}{\mathrm{d}t} = -\frac{1}{C^2}\frac{\mathrm{d}C}{\mathrm{d}t} = k \tag{4.4.70}$$

and the solution is straightforward:

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TABLE 4.4.9 Henriques and Moritz Skin Burn Threshold Data ($\Omega = 1$ Means a Second-Degree Burn Was Observed)¹⁰⁸

Exposure Time (s)	Temperature (°C)
25,000	44
11,000	45
5,000	46
2,400	47
1,100	48
570	49
300	50
160	51
90	52
35	54
16	56
5	60
2	65
1	70



FIGURE 4.4.15 Plot of damage rate for the original Henriques and Moritz data¹⁰⁷ (solid circles) and the enzyme deactivation model of Xu and Qian123 vs. normalized temperature.

Enzyme Deactivation Model

Xu and Qian¹²³ presented an enzyme deactivation model for skin burn data which combines some of the first- and second-order aspects in a single calculation. The method is based on a general simple enzyme-catalyzed reaction sequence from substrate, S, to products, P:

$$E_{a} + S \xrightarrow{k^{+}} E_{a}S \xrightarrow{k} E_{a} + P$$
(4.4.76)

and

$$E_a \xrightarrow{k_d} E_i \tag{4.4.77}$$

where E_a is activated enzyme, E_i is inactivated denatured enzyme (at velocity k_d), and E_aS is the enzymesubstrate complex. The analysis is completed by assuming that the enzyme inactivation process is slow compared to the main reaction sequence. Nondimensional temperature, z, is used in the analysis, where: the molecular structure and regimentation of the tropocollagen molecules from which collagen fibrils are built. Tissue birefringence has two components, intrinsic and form birefringence. Intrinsic birefringence is secondary to the α -helical conformation of the polypeptides that form the individual contractile proteins of muscle and tropocollagen molecules and accounts for 12 to 30% of the total birefringence of these tissues. The remaining birefringence, form birefringence, is related to the crystalline-like array of the actin and myosin molecules in the sarcomere and the regimented longitudinal array of tropocollagens within the collagen fibrils.^{131,133-146}

Thermally induced changes of muscle birefringence are associated with dissociation and disruption of the molecules, as revealed by TEM. Partial loss of birefringence in skeletal and cardiac muscle has been identified with the breakdown of the relationships of actin and myosin in the sarcomere (loss of form birefringence). The source of the birefringence of striated muscle is the central portion of the sarcomere, the A or anisotropic band, in which the actin and myosin are arranged in parallel crystalline-like array. The I or isotropic band of the sarcomere includes portions of adjacent sarcomeres that contain actin molecules which are attached to the Z band, an attachment zone for the actin molecules. Total loss of birefringence is associated with increased temperature-time histories and is probably related to the reduction of the individual fibrillar contractile proteins to thermally denatured granular profiles (loss of intrinsic birefringence) as shown in TEM of heated myocardium.^{147,148}

Thermally induced birefringence changes in collagen involve decreasing intensity of the birefringent image until it is totally lost at higher temperatures. TEM of heated collagens shows a gradual unranging of the fibers with disappearance of the characteristic periodicity of collagen, reflecting the ticcorution of the regimented organization of native collagen as it is heated.^{129,146}

The onset of the birefringence changes of collagens varies depending on the experimental conditions, anatomic site, and the age of the organism. Native tiscue to agents reside in complex environments of different proteins, glycoproteins, and cells that are grand, influenced by the oral and systemic physiology of the living organism. The entire relationships of the collagent with these tissue constituents are regulated by water and the concentrations, ionic on coval mobile and hormonal and metabolic effects. Collagens extra confirm tissues are not in their solural environment, therefore *in vitro* experiments to this collagen *in vitro* experiments of birefringence will not predict the native behavior of collagen *in vitra* and vio.¹⁴²⁻¹⁴⁴ In addition, as the organism ages, the collagens, especially the Type I collagen found in most tissues, form more molecular cross-links that alter the temperature-time thresholds of fiber dissociation.¹⁴³

Polarizing microscopy can be performed on paraffin sections prepared for routine light microscopy; thus the birefringent images can be correlated directly to morphologic structure. The intensity of the birefringent images is governed by the thickness of the section, the type of dyes used, and in the case of collagens, the collagen fiber thickness.^{128,149}

Water-Dominated Effects

The role of water vaporization becomes dominant as the tissue temperatures approach 100°C. The tissue effects depend on: (1) the rate of water vapor formation, (2) the temperature of the water vapor, (3) the rate of diffusion (escape) of the vapor from the tissue, and (4) the tissue mechanical properties.¹⁵⁰⁻¹⁵⁹ All tissues exposed to air will lose water vapor to the atmosphere because of the differences between the partial pressure of water vapor across the tissue/air boundary. Below 100°C water vapor will diffuse toward the surface to escape. The water vapor is in thermodynamic equilibrium with liquid water and, in the tissues deep to the surface, the relatively low partial pressure gradients favor the liquid over the vapor phase. Meanwhile, at the surface, the tissues dry out as the water vapor escapes leaving the hard, not infrequently brittle, solid tissue components behind.

At or just above 100°C water vapor is generated volumetrically, equilibrium is pushed toward the vapor phase, and, for sufficiently high volume generation rates, q^{'''}, more vapor is produced than can escape by simple diffusion. The excess vapor is trapped within the cell and in the extracellular tissue layers forming vacuoles or dissections along planes of mechanical weakness.¹⁵⁹⁻¹⁶¹ The surface desiccates



FIGURE 4.4.19 Simplified representation of an aqueous equilibrium phase bigg are Physiological solutions have solute concentrations that are quite small (about 300 mOsm) in care the concentration the eutectic state. As freezing progresses, the liquid phase concentration may increase by note than 20 old, subjecting care to intense osmotic stress.

 L_p is the membrane permeability to value, ω , the permeability to solute (CPA for applications in cryopreservation); and coll the coupling between the two lows (reflection coefficient). The average concentration of compasse solute between the estimatellular and intracellular solutions is given by C_s , a dame to average flux and solute fluxes by J_v and J_s , respectively. Equation 4.4.82 shows a provision for volume flow occur in the care action of multiple solutes.

An alternative, more generalized formulation of this type of phenomenon was also posed by Katchalsky in terms of network thermodynamics.¹⁸² The network thermodynamic model has been applied successfully to the analysis of the cellular response to freezing processes.¹⁸³⁻¹⁸⁵ It can readily be applied for inverse solution to quantify the transport coefficients from either freezing data or CPA addition data.^{184,186}

Tissues and Organs

The preceding models hold for the analysis of individual cells. However, many important biological systems of interest for cryopreservation consist of organized tissues which have a three-dimensional geometry that exerts a significant effect on the osmotic behavior. For these systems transport occurs not only across the individual cell membranes, but the interstitial volume is also involved in both the transport and storage of water and CPA. In this case the model must be expanded to include both parallel and serial transport and storage for all mobile chemical species. Network thermodynamics provides an effective modeling format for describing these processes.

The cryopreservation of whole organs, which is still in the research stage, will demand even more complex coupled thermal and chemical transport analyses. CPAs are added and removed via perfusion through the vascular network, and this hydrodynamic flow is coupled to the osmotic, diffusional, and viscoelastic energy domains of the organ. The network thermodynamic approach to modeling is very well suited to analysis of these coupled multidomain processes and has been applied successfully to the design of CPA perfusion protocols.¹⁸⁷

Influence of Cell Membrane Permeability

It is well documented that the permeability of cell membranes to water and to CPA is a strong function of the cell species, and that for all species it is a function of temperature.¹⁸¹ For example, the water

is irradiated with a laser wavelength that is absorbed preferentially by the cell cytoplasm over the extracellular solution. For a solution of individual cells having characteristic diameters of about 6 μ m, warming rates on the order of 10¹¹°C/s can be achieved within the cells, while the extracellular matrix remains frozen. When the heating process is terminated in an approximate step-wise manner after 7 ns, heat transfer from the extracellular matrix to the cells produces an intracellular cooling rate of about 10^{6°}C/s, which is adequate to produce a glass state in an aqueous solution. The vitrified cells may then be warmed rapidly to suprafreezing temperatures (>0°C) without nucleation and with no measured injury. If this process is to realize broad clinical or commercial application, there is considerable engineering work to be accomplished in scale-up from these microscopic level processes to applications in larger tissue and organ systems.

Since the initial achievements in the cryopreservation of cells, many applications have been developed in both the biological and medical fields. There are now numerous for-profit concerns that have developed successful techniques for preserving human tissues for subsequent transplantation. Many of these corporations use proprietary processing protocols to preserve specific tissues, and the number of commercial participants in this field has been growing steadily over the past 10 years. In like manner, there are many for-profit and not-for-profit groups which are marketing the cryopreservation of a very broad spectrum of nonhuman tissues. Some of the more prominent applications involve mammalian and amphibian gametes, tissue culture collections, and plant germplasm. A potentially important new area of application will be the cryopreservation of living materials manufactured by the tissue engineering techniques which are being developed.²⁰⁴ Bioartificial systems such as skin and organs that contain living colls will require a technology for maintaining them in a viable but latent state between the im of manufacture and implementation in a clinical setting. Cryopreservation provides and funable potential for meeting ill ar c this unique need, but the processing protocols w be of signed to ensure successful storage and recovery for each type of manufactured tissue.

Cryosurgery

All of the foregoine is Sussion has been device it cryopreservation, for which the objective is to maxim 2 the survival of cells from the frozen state to ensure their living function after thawing. The initial survival of this work 1 cP survey, for which the objective is to maximize destruction of a target tissue *in situ*.²⁰⁵ Cryostrgery has long been practiced as an effective means for killing surface lesions, and recently it has been adopted more widely as a tool for treating internal tumors that are difficult to resect mechanically.²⁰⁶

Many of the recent gains in the successful application of cryosurgery are due to the development of new imaging methods that enable the surgeon to follow the growth of the solid-liquid interface, to which the zone of cellular destruction is coupled, in real time during the freezing process. The initial imaging methodology adapted for this purpose was ultrasonography.²⁰⁷ Subsequently, other imaging modalities that can be coupled to the transient temperature field are being developed, including NMR²⁰⁸ and spectroscopy.²⁰⁹ Advances in cryosurgical probes that provide more effective removal of heat from tissues via enhanced thermal transport mechanisms have also contributed to the recent growth in cryosurgery.²¹⁰ Finally, over the past decade there has been a steady advance in understanding the mechanisms by which freezing and thawing act to cause tissue death as a function of the thermal parameters of the solidification process and tissue properties. Many of these fundamental advances have resulted from the application of bioengineering analysis of the tissue freezing process,²¹¹⁻²¹³ which portends the potential for engineering to further contribute to the field of cryosurgery.

A new approach to improving the efficacy of cryosurgery is derived from techniques long applied to enhance cryopreservation processes. Namely, the tissue is modified by addition of chemical agent prior to the initiation of freezing. However, for applications in cryosurgery the desired result is an increased level of cell killing. Antifreeze proteins (AFP) are proving to be effective for this purpose.²¹⁴⁻²¹⁵ AFPs are chemical compounds synthesized by many differing types of plants and animals to provide protection against freezing injury at high subzero temperatures.²¹⁶⁻²¹⁷ It has also been demonstrated that AFPs

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Organ	Blood Flow (ml/min/g)	Species (* = Unanesthetized)	Appendix Ref.	Technique
duodenum	3.31 ± 0.64 (SD)	rat* (24 mo old)(n = 5)	33	μs
duodenum	2.00 ± 0.76 (SD)	rat $(n = 7)$	33	µs (decerebrate)
jejunum	0.642 ± 0.086 (SE)	$\log (n = 11)$	36	µs (pentobarbital)
jejunum	0.263 ± 0.034 (SE)	$\operatorname{cat}(n = 12)$	35	µs (pentobarbital)
ileum	0.498 ± 0.083 (SE)	$\log (n = 11)$	36	µs (pentobarbital)
small bowel	1.00 ± 0.45 (SD)	dog (n = 12)	39	Rb ⁸⁶ (pentobarbital)
small bowel	2.09 ± 0.17 (SE)	neonatal sheep [*] $(n = 13)$	37	μs
small bowel	0.81 ± 0.06 (SE)	sheep* $(n = 7)$	37	μs
small bowel	2.81 ± 0.46 (SE)	$rat^{*}(n = 8)$	31	μs
small bowel	1.73 ± 0.18 (SE)	rat $(n = 7)$	31	µs (pentobarbital)
small bowel	1.67 ± 0.13 (SE)	rat $(n = 7)$	31	µs (chlorurethan)
small bowel	1.52 ± 0.20 (SE)	rat $(n = 7)$	31	µs (decerebrate)
cecum	1.89 ± 0.19 (SE)	rat^{*} (n = 8)	31	μs
cecum	1.52 ± 0.08 (SE)	rat $(n = 7)$	31	μs (pentobarbital)
cecum	1.51 ± 0.15 (SE)	rat $(n = 7)$	31	µs (chlorurethan)
cecum	1.55 ± 0.22 (SE)	rat $(n = 7)$	31	us (decerebrate)
colon	1.24 ± 0.57 (SD)	$\log (n = 12)$	39	Rb ⁸⁶ (pentobarbital)
colon	0.711 ± 0.065 (SE)	$\log(n = 11)$	36	us (pentobarbital)
large bowel	0.92 ± 0.06 (SE)	neonatal sheep* $(n = 13)$	37	us 📕
large bowel	0.55 ± 0.09 (SE)	sheep* $(n = 7)$	37	us
large bowel	1.33 ± 0.29 (SE)	rat^{*} (n = 8)	31	
large bowel	0.66 ± 0.09 (SE)	rat (n = 7)	31	u (oentobarbital)
large bowel	0.59 ± 0.07 (SE)	rat (n = 7)		us (chlorurethan)
large bowel	0.70 ± 0.13 (SE)	rat (n = 7)	31	us (decerebrate)
Pancreas	0.008-0.016			P (
Pancreas	0.545 ± 0.070 (SE)	dev = 10	30	us (pentobarbital)
Pancreas	0.513 ± 0.010 (5)	dog (n = 12)	39	Rb ⁸⁶ (pentobarbital)
Liver				no (pentocarchai)
hep art. + porta	65 ± 0.530	(n=4)	25	
hep arter logial	2.53 ± 0.184	(n = 6)	25	
Perat poral	-3.17+ (2)	nig (n = 6)	25	
hep art $+$ portal	4(+ 0 - 1 SE)	$r_{r} = \frac{1}{2} (n = 8)$	31	115
hep art + portal	1.19 ± 0.07 (SE)	rat $(n = 7)$	31	us (pentobarbital)
hep art $+$ portal	1.22 ± 0.09 (SE)	rat (n = 7)	31	us (chlor-urethan)
hep art. + portal	1.25 ± 0.16 (SE)	rat $(n = 7)$	31	us (decerebrate)
via portal vein	1.25	rat	25	µs (decereorate)
via portal vein	1.19 ± 0.12 (SE)	rat^{*} (n = 8)	31	115
via portal vein	0.98 ± 0.05 (SE)	rat (n = 7)	31	us (pentobarbital)
via portal vein	0.90 ± 0.00 (SE) 0.98 ± 0.10 (SE)	rat $(n = 7)$	31	us (chlor -urethan)
via portal vein	0.93 ± 0.12 (SE)	rat $(n = 7)$	31	us (decerebrate)
via penatic art	0.95 ± 0.12 (SE) 0.24 ± 0.04	nig	3	µs (decerebrate)
via hepatic art	0.24 ± 0.04 0.22 ± 0.04	pis nis*	19	
via hepatic art	0.15 ± 0.03	rat(n - 9)	27	
via hepatic art	0.13 ± 0.05 (SE)	$rat^*(n - 8)$	31	116
via hepatic art	0.22 ± 0.05 (SE)	rat (n = 3)	31	μs μs (nentobarbital)
via hepatic art.	0.21 ± 0.03 (SE) 0.23 ± 0.02 (SE)	rat (n = 7)	31	us (chlor -urethan)
via hepatic art.	0.23 ± 0.02 (SE) 0.31 ± 0.07 (SE)	rat (n = 7)	31	us (decerebrate)
via hepatic art	0.51 ± 0.07 (SE)	fatal sheep* $(n - 12)$	31	μs (decerebrate)
via hopatic art.	0.10 ± 0.03 (SE)	neopetal sheep $(n - 12)$	32	μs
via hepatic art.	$0.10 \pm 0.03 \text{ (SE)}$ 0.14 ± 0.05 (SE)	shoop* $(n = 7)$	27	μs
via hopatic art.	0.14 ± 0.03 (SE)	sheep $(II = 7)$ rot* (12 mo old)(n = 5)	37	μs
via hepatic art.	$0.02 \pm 0.02 (SD)$	rat $(12 \text{ IIIO OId})(n = 3)$	33 22	μs us (nontobarbital)
via nepatic art.	$0.07 \pm 0.04 (SD)$	rat (11 - 10) $rat^* (24 mo c^{1})(n - 5)$	33 22	
via nepatic art.	0.03 ± 0.01 (SD)	rat $(24 \text{ mo old})(n = 5)$	33	μs us (dependente)
via nepatic art.	0.11 ± 0.07 (SD)	$\operatorname{rat}(n = /)$	33	μs (decerebrate)
via nepatic art.	$0.1/\delta \pm 0.050$ (SE)	uog (n = 11)	30	µs (pentobarbital)
via hepatic art.	0.31 ± 0.30 (SD)	rabbit $(n = 5)$	42	μs

wide range of thermal conductivity provides the means to control the temperature or heat flow of objects or areas.

Heat Transfer in Thermal Insulation

Efficient use of thermal insulation requires an understanding of the heat transfer that occurs in insulation material. The heat transfer can be considered to occur by three assumed independent mechanisms: conduction through the solid material or gas in the insulation, gas convection, and radiation. Each will be discussed separately and then combined to give an overall apparent thermal conductivity correlation. Additional discussion can be found in References 1 through 3.

Solid Conduction

Solid conduction occurs as heat is transferred through the solid portions of the insulation material, from the hot to the cold boundaries. It is characterized by the thermal conductivity of the continuous solid material of the insulation. In a fibrous insulation this is the conductivity of the material which forms the fibers, the type of contact between the fibers, and the number of fibers per unit volume, i.e., the insulation density. The solid conductivity of the insulation can be modeled by the following formula:

$$\mathbf{k}_{\mathrm{s}} = \mathbf{A} \mathbf{k}_{\mathrm{m}}$$

where $k_s =$ the solid conductivity of the insulation material

- k_m = the solid conductivity of the solid phase material
- **e**.**co.uk**^{4,5,1)} A = empirical constant which can be related to the l_{1} nsulation and the nature of the contacts or structure of the solid p a idi te n

The reader is referred to Reference in u th 6 for models that

Gas Conduction and Convection Gas connection is formally suppressed in the storactical industrial insulation materials because of the issuation density in a fit or 5 p (0.5 cell insulation, the closed-cell properties of some solid insulation, Esu ative density in a fill or s of 0 or the gas being at a w desited evacuated insulation. In some insulation systems, such as doubleor triple-pane windows, or low-density open-cell or fibrous insulation, convection can be of importance and will be included in the manufacturers' thermal conductivity data sheets. Gas conduction is of importance in most insulations and will be briefly discussed.

Gas conduction at higher-pressure ranges will follow a similar equation as solid conduction.

$$\mathbf{k}_{g} = \mathbf{B} \ \mathbf{k'}_{g} \tag{4.5.2}$$

where k_g = the gas conduction component of the insulation

- k'_{g} = the gas thermal conductivity at the mean temperature
 - B = an empirical constant

This shows the insulation conductivity can be reduced by replacing the air in the insulation with a lower-conductivity heavy gas such as CO₂, argon, or a fluorocarbon. The conductivity of the insulation can be increased if the air in the insulation is replaced by a higher-conductivity gas such as helium or hydrogen. The conductivity ratio of krypton to air is 0.21 while helium to air is 5.7 at STP (273 K and 1 atm pressure).

Trapping a fuel such as hydrogen or an oxidizer such as oxygen in the insulation can be very dangerous and precautions must be taken to purge or vent the insulation space as a safety measure. In a cryogenic system there is also the possibility of condensing liquid air in the insulation. The liquid air is enriched in oxygen and will be hazardous if the insulation is combustible.

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 $k = k_s + k_g + k_r$ (4.5.6)

or

$$k = A k_m + B k'_g + \sigma T^3/D$$
 (4.5.7)

The average apparent conductivity of the insulation between two temperatures becomes

$$k_{a} = A k_{m} + B k'_{g} + \sigma (T_{2}^{4} - T_{1}^{4}) / (4 D * (T_{2} - T_{1}))$$
(4.5.8)

Application of this equation to specific types of insulation systems is discussed in the following sections.

Most manufactures report thermal conductivity as the average between two temperatures — typically ambient and a higher or lower temperature. The relation between the point thermal conductivity and the average is

$$k_{a} = \frac{1}{T_{2} - T_{1}} \int_{T_{1}}^{T_{2}} k(t) dt$$
(4.5.9)

n and Vacuum)

COLUK

Insulation Systems (Nonvacuum and Vacuum)

Powder Systems

Powder insulation consists of particulate materials such a checke, vermiculite, beaded Styrofoam, microcell, and some types of aerogels. The advince of mis type of insulation is that the particulate material can generally be powed or blo in into the space which is to be insulated and will flow to some extent around equipment and billing. The material can also be easily removed by gravity flow, fluidized transfer, or evaluation has is an advantage in congrest process equipment that has to be accessible for coart of natited and will ensure out on we can also be placed around in the material consult on a subambient temperature system to diminate water vapor migration into the powder and condensing or freezing. If the latter occurs, the thermal conductivity of the insulation will be degraded.

The thermal performance of the insulation is limited by the conduction of the gas filling the pores of the powder. The solid conductivity component is reduced by the multiple point contacts between particles and by selecting a material with low intrinsic solid conductivity. In some powders the particles are not solid but are porous, such as perlite or vermiculite, or are agglomerates of smaller particles, such as in fumed silica. Radiation in powder insulation can be reduced through the selection of the proper particle size or the addition of opacifiers such as carbon or reflective metal flakes of aluminum or copper.^{12,13}

Some of the most common powder insulation material properties and thermal conductivies between various temperature ranges are given in Table 4.5.1. The thermal conductivity curves of perlite, vermiculite, and aerogel at various densities are shown in Figure 4.5.2. As shown in Figure 4.5.2, the conductivities of gas-filled perlite and vermiculite powders at the same temperature do not vary greatly from one another since both are dominated by the gas filling the insulation pores. The aerogel powder insulation has a pore size approaching the mean free path of the gas, resulting in greatly reduced thermal conductivity.^{14,15}

To reduce the conductivity, the gas can be removed from the insulation. This will reduce the apparent conductivity of the insulation by an order of magnitude or more. The level of vacuum which has to be achieved to reduce the gas conductivity to a negligible value is of practical importance. Therefore, the thermal conductivity of various powder insulations are presented as a function of the gas pressure. The evacuated thermal conductivity curves for perlite, aerogel, and microcel with 10% aluminum flakes are shown in Figure 4.5.3. The differences in the powder conductivity and the effect of particle size can be seen in these figures.

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TABLE 4.5.1	Powder	Insulation

Materials	Maximum Operating Temperature (K)	Density (kg/m³)	Hot Side Temp (K)	Cold Side Temp (K)	Apparent Thermal Conductivity (W/m-K)
Perlite	1255	112	327	294	0.0418
Vermiculite	1033	144	327	294	0.0677
Silica aerogel	977	80	327	294	0.0303
Silica aerogel with 15 wt% carbon	977	116	327	294	0.0135
Cork	366	208	327	294	0.049
Diatomaceous silica	1144	240	327	294	0.0577



FIGURE 4.5.2 Powder insulation thermal conductivity.



FIGURE 4.5.3 Evacuated powder thermal conductivity from 300 to 77 K.

 TABLE 4.6.2
 Energy Audit Summary for Industrial Facilities

Electric Systems
ility Analysis
 Electrical energy use profile (building signature) Electrical energy use per unit of a product Electrical energy use distribution (cooling, lighting, equipment, process, etc.) Analysis of the electrical energy input for specific processes used in the production line (such as drying) Utility rate structure (energy charges, demand charges, power factor penalty, etc.)
a-Site Survey
List of equipment that use electrical energy Perform heat balance of the electrical energy Monitor electrical energy use of all or part of the equipment Determine the by-products of electrical energy use (such pollutants)
gy Use Baseline
Review electrical drawings and production flow charts frequent by secase model (using my baselining method) whence the base-case model to sing utility data or metered data)
Energy efficient motors
Variable speed drives Air compressors Energy efficient lighting HVAC system retrofit EMCS Cogeneration (possibly with solid waste from the production line) Peak demand shaving Bower foctor improvement

The energy audit of the envelope is especially important for residential buildings. Indeed, the energy use from residential buildings is dominated by weather since heat gain and/or loss from direct conduction of heat or from air infiltration/exfiltration through building surfaces accounts for a major portion (50 to 80%) of the energy consumption. For commercial buildings, improvements to the building envelope are often not cost-effective due to the fact that modifications to the building envelope (replacing windows, adding thermal insulation in walls) typically are very expensive. However, it is recommended to systematically audit the envelope components not only to determine the potential for energy savings but also to ensure the integrity of its overall condition. For instance, thermal bridges, if present, can lead to heat transfer increase and to moisture condensation. The moisture condensation is often more damaging and costly than the increase in heat transfer since it can affect the structural integrity of the building envelope.



TABLE 4.6.5 Building Construction Materials

FIGURE 4.6.2 Monthly actual electrical energy consumption.

- It was found that the building had been retrofitted with energy-efficient lighting systems. The measurement of luminance levels throughout the working areas indicated adequate lighting. To determine an estimate of the energy use for lighting, the number and type of lighting fixtures were recorded.
- It was observed that the cooling and heating temperature set-points were 25.5°C and 24.5°C, respectively. However, indoor air temperature and relative humidity measurements during the field survey revealed that during the afternoon the thermal conditions were uncomfortable in several office spaces, with average air dry-bulb temperature of 28°C and relative humidity of 65%. A discussion with the building operator indicated that the chillers were no longer able to meet the cooling loads following the addition of several computers in the building during the last few years. As a solution, an ice storage tank was then added to reduce the peak cooling load.

- It was discovered during the survey that the building is heated and cooled simultaneously by two systems: constant air volume (CAV) and fan coil unit (FCU) systems. The CAV system is complemented by the FCU system as necessary. Two air-handling units serve the entire building, and about 58 FCUs are on each floor.
- The heating and cooling plant consists of three boilers, six chillers, three cooling towers, and one ice storage tank. The capacities of the boilers and the chillers are

Boilers: 13 MBtu/h (2 units) and 3.5 MBtu/h (1 unit)

Chillers: 215 tons (5 units) and 240 tons (1 unit)

The thermal energy storage (TES) system consists of a brine ice-on-coil tank. The hours of charging and discharging are 10 and 13, respectively. The TES system is currently controlled using simple and nonpredictive storage-priority controls.

The internal heat gain sources are shown in Table 4.6.6. Operating schedules were based on the discussion with the building operators and on observations during the field survey.

Step 3: Energy Use Baseline Model

To model the building using DOE-2, each floor was divided into two perimeter and two core zones. Figure 4.6.3 shows the zone configuration used to model the building floors. The main reason for this zoning configuration is the lack of flexibility in the

TABLE 4.6.6Internal Heat Gain Levelfor the Office Building

Internal Heat Gain	Design Load		
Occupancy	17 m ² /person		
	Latent heat gain: 45 W		
	Sensible heat gain: 70 V		
Lighting	14 W/m ²		
Equipment	16 W/m		
Ventilation	1.7.CN/person		

DOE-2 SYSTEMS (Energy Information Agence 1980) program. Although the actual building is conditioned by the combination of constant air volume (CAV) and fan call with CoUCrystems, the SYSTEM module of DOE-2.1E cannot much the different types of HeAC systems serving one zone. Therefore, a simplification has been made to simulate the actual HVAO system of the building. This simplification consists of their thomas: the perimeter acree is conditioned by the FCUs, while the core zone is conditioned by CAV. Since all materials are located at the perimeter, this simplification is consistent with the actual HVAC system operation.



FIGURE 4.6.3 Building zoning configuration for DOE-2 computer simulation.

Figure 4.6.4 shows the monthly electrical energy consumption predicted by the DOE-2 base model and the actual energy use recorded in 1993 for the building. It shows that DOE-2 predicts the actual energy use pattern of the building fairly well, except for the months of September and October. The difference between the annual metered energy use in the building and the annual predicted electricity use by the DOE-2 base-case model is about 762 MWh. DOE-2 predicts that the building consumes 6% more electricity than the actual metered annual energy use. To perform the DOE-2 base-case model, a TRY-type weather file of Seoul was created using the raw weather data collected for 1993. Using the DOE-2 base-case model, a number of ECOs can now be evaluated.

Figure 4.6.5 shows the distribution by end uses of the building energy consumption. The electrical energy consumption of the building is dominated by lighting and equipment. The electricity utilized for lighting and office equipment represents about 75% of the total building electric consumption. As

Electricity Cost (MWon)	LNG Cost (MWon)	Total Cost (MWon)	Capital Cost (MWon)	Saving (%)	Savings (MWon)	Payback Period (years)
984.4	139.1	1123.5				
940.8	49.8	990.5	465.5	11.8	133.0	3.5
979.1	139.1	1118.2	42.4	0.6	5.3	8.0
977.9	126.9	1104.8	280.5	1.7	18.7	15.0
983.7	106.4	1090.1	16.7	3.0	33.4	0.5
972.6	138.7	1111.4	60.5	1.1	12.1	5.0
911.6	144.8	1056.4	268.4	6.0	67.1	4.0
	Electricity Cost (MWon) 984.4 940.8 979.1 977.9 983.7 972.6 911.6	Electricity Cost (MWon)LNG Cost (MWon)984.4139.1940.849.8979.1139.1977.9126.9983.7106.4972.6138.7911.6144.8	Electricity Cost (MWon)LNG Cost (MWon)Total Cost (MWon)984.4139.11123.5940.849.8990.5979.1139.11118.2977.9126.91104.8983.7106.41090.1972.6138.71111.4911.6144.81056.4	Electricity Cost (MWon)LNG Cost (MWon)Total Cost (MWon)Capital Cost (MWon)984.4139.11123.5—940.849.8990.5465.5979.1139.11118.242.4977.9126.91104.8280.5983.7106.41090.116.7972.6138.71111.460.5911.6144.81056.4268.4	Electricity Cost (MWon)LNG Cost (MWon)Total Cost (MWon)Capital Cost (%)Saving (%)984.4139.11123.5——940.849.8990.5465.511.8979.1139.11118.242.40.6977.9126.91104.8280.51.7983.7106.41090.116.73.0972.6138.71111.460.51.1911.6144.81056.4268.46.0	Electricity Cost (MWon)LNG Cost (MWon)Total Cost (MWon)Capital Cost (MWon)Saving

TABLE 4.6.7 Economic Analysis of the ECOs

be implemented for the audited office building. In particular, the VAV conversion reduces the total energy consumption more than 10%, and the daylighting control saves about 6% of the total energy consumption.

Verification Methods of Energy Savings

Energy conservation retrofits are deemed cost-effective based on predictions of the amount of energy and money a retrofit will save. However, several studies have found that large discrepancies exist between actual and predicted energy savings. Due to the significant increase in the activities of energy service companies (ESCOs), the need became evident for standardized methods for measurement and verification of energy savings. This interest has led to the development of the *NorthAmmean Energy Measurement and Verification Protocol* published in 1996 and later of protocol nor revised under the *International Performance Measurement and Verification Protocol*.

In principle, the measurement of the fet of it energy savings cance obtained by simply comparing the energy use during pre- and post-remoit periods. Unfortunately, the change in energy use between the pre- and post-remoit periods is not only due to the retrofit neelf but also to other factors such as changes is weather end thous, levels of occupancy and AVAC operating procedures. It is important to account for all these changes to accupate hypercemine the retrofit energy savings.

Several methods have been proposed to measure and verify savings of implemented energy conservation measures in commercial and industrial buildings. Some of these techniques are briefly described below.

Regression Models

The early regression models used to measure savings adapted the Variable-Base Degree Day (VBDD) method. Among these early regression models, the PRInceton Scorekeeping Method (PRISM) which uses measured monthly energy consumption data and daily average temperatures to calibrate a linear regression model and determine the best values for nonweather-dependent consumption, the temperature at which the energy consumption began to increase due to heating or cooling (the change-point or base temperature), and the rate at which the energy consumption increased. Several studies have indicated that the simple linear regression model is suitable for estimating energy savings for residential buildings. However, subsequent work has shown that the PRISM model does not provide accurate estimates for energy savings for most commercial buildings (Ruch and Claridge, 1992).

Single-variable (temperature) regression models require the use of at least four-parameter segmented linear or change-point regressions to be suitable for commercial buildings. Katipamula et al. (1994) proposed multiple linear regression models to include as independent variables internal gain, solar radiation, wind, and humidity ratio, in addition to the outdoor temperature. For the buildings considered in their analysis, Katipamula et al. found that wind and solar radiation have small effects on the energy consumption. They also found that internal gains have generally modest impact on energy consumption. Katipamula et al. (1998) discuss in more detail the advantages and the limitations of multivariate regression modeling.
In general, a large, slow-running compressor is more efficient and quieter than a small, high-speed compressor. However, the initial cost is higher. As compressor design advances, the trend is to smaller, faster-running compressors, with better silencing techniques and more sophisticated controls to keep operating costs down.

Most portable air compressors are driven by diesel engines, except those at the lower end of the airflow rates (below 100 cfm $[0.047 \text{ m}^3/\text{s}]$), where gasoline engines are often used. Electric motor-driven portables are used in some areas. Stationary compressors are driven by electric motors. The majority of them operate at either 1800 rpm with a 60-Hz line frequency or at 1500 rpm where the line frequency is 50 Hz. Very large compressors in process plants are often driven by steam turbines.

Most portable compressors use a drive coupling to transmit the torque from the engine to the compression module, although small compressors are often belt driven. The drive coupling can be a troublesome component and must be carefully selected to minimize operational problems. Ideally, the compressor should be rugged enough to withstand the torque pulses from the engine, so that a torsionally stiff drive coupling can be used. This is often not possible, however, especially with the larger compressors, and a torsionally soft coupling must be used to isolate the "airend" from the engine. In these cases, the engine speed must be maintained well above the torsional critical speed to prevent drive train failures. The torque pulses from an electric motor are mild compared to those from a diesel engine. As a result, the coupling is often eliminated by directly coupling the compressor and motor. This is accomplished by mounting the compressor drive gear on the motor shaft. If a coupling is used, it can be torsional stiff.

Maintenance Requirements

One of the primary reasons for the popularity of oil-flooded screw_one is their ability to operate continuously for many years with only routine maintenance. Every amtenance is most important, since the incoming air is mixed with the oil and any projection in incoming air and p in the oil. This same oil is used to lubricate the bearings and barti les in the oil will decrease beering fife. Typically, a 25- μ m (0.64 μ m) air filter is specified, where leaning or changing required when the restriction exceeds 25 in. of water (6.2 kbar thebe system also has a 22 km (0.64 μ m) filter, as well as an oil separation element, which (a) these particles as small as room (0.25 μ m). Any contamination entering the system ends up in the separator element, these horeming its life. The use of finer air and oil filters will increase the time between separator element enanges. This is usually desirable, since changing the air and oil filters is much easier than changing the separator element.

The coolers must also be kept clean. Dirty or fouled coolers reduce the LAT, sometimes so much that the compressor shuts down because of overheating. Another undesirable consequence of dirty coolers is a shorter bearing life due to higher oil temperatures.

Regular filter maintenance is just as important on dry screw and dynamic compressors. A restricted air inlet reduces the compressor inlet pressure and raises the pressure ratio across it. This can cause overheating of a dry screw compressor and surge in a dynamic compressor.

Thermal Issues

Performance

Assuming adiabatic compression and applying the first law of thermodynamics, the rate of work of compression for a single stage compressor is given by

$$\dot{W} = \dot{m}c_{p}(T_{2} - T_{1}) \tag{4.7.2}$$

where \dot{m} is the mass flow rate of the compressed air; c_p , the specific heat; T_2 , the outlet temperature; and T_1 , the inlet temperature. Treating the air as a perfect gas, this becomes

$$\dot{W} = P_1 \dot{V}_1 \frac{k}{k-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right] \frac{1}{\eta}$$
(4.7.3)

4.8 Pumps and Fans

Robert F. Boehm

Introduction

Pumps are devices that impart a pressure increase to a liquid. Fans are used to increase the velocity of a gas, but this is also accomplished through an increase in pressure. The pressure rise found in pumps can vary tremendously, and this is a very important design parameter along with the liquid flow rate. This pressure rise can range from simply increasing the elevation of the liquid to increasing the pressure hundreds of atmospheres. Fan applications, on the other hand, generally deal with small pressure increases. In spite of this seemingly significant distinction between pumps and fans, there are many similarities in the fundamentals of certain types of these machines as well as with their application and theory of operation.

The appropriate use of pumps and fans depends upon the proper choice of device and the proper design and installation for the application. A check of sources of commercial equipment shows that many varieties of pumps and fans exist. Each of these had special characteristics that must be appreciated for achieving proper function. Preliminary design criteria for choosing between different types is given by Boehm (1987).

As is to be expected, the wise applications of pumps and fans requires knowleds of this flow fundamentals. Unless the fluid mechanics of a particular application are understood, we design could be less than desirable.

In this section, pump and fan types are briefly defined in arbitrar, typical application information is given. Also, some ideas from fluid mechanics there expectally relevant to ture and fan operation are reviewed.

Paising of water from a cleane citters is the earliest form of pumping (a very detailed history of early applications is given by Ew ank (842). Modern applications are much broader, and these find a wide variety of machines in use. Modern pumps function on one of two principles. By far the majority of pump installations are of the *velocity head* type. In these devices, the pressure rise is achieved by giving the fluid a movement. At the exit of the machine, this movement is translated into a pressure increase. The other major type of pump is called *positive displacement*. These devices are designed to increase the pressure of the liquid while essentially trying to compress the volume. A categorization of pump types has been given by Krutzsch (1986), and an adaptation of this is shown below.

I. Velocity head

Pumps

- A. Centrifugal
 - 1. Axial flow (single or multistage)
 - 2. Radial flow (single or double suction)
 - 3. Mixed flow (single or double suction)
 - 4. Peripheral (single or multistage)
- B. Special Effect
 - 1. Gas lift
 - 2. Jet
 - 3. Hydraulic ram
- 4. Electromagnetic
- II. Positive displacement
 - A. Reciprocating
 - 1. Piston, plunger
 - a. Direct acting (simplex or duplex)

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2. Diaphragm (mechanically or fluid driven, simplex or multiplex)

B. Rotary

- 1. Single rotor (vane, piston, screw, flexible member, peristaltic)
- 2. Multiple rotor (gear, lobe, screw, circumferential piston)

In the next subsection, some of the more common pumps are described.

Centrifugal and Other Velocity Head Pumps

Centrifugal pumps are used in more industrial applications than any other kind of pump. This is primarily because these pumps offer low initial and upkeep costs. Traditionally, pumps of this type have been limited to low-pressure-head applications, but modern pump designs have overcome this problem unless very high pressures are required. Some of the other good characteristics of these types of devices include smooth (nonpulsating) flow and the ability to tolerate nonflow conditions.

The most important parts of the centrifugal pump are the *impeller* and *volute*. An impeller can take on many forms, ranging from essentially a spinning disk to designs with elaborate vanes. The latter is usual. Impeller design tends to be somewhat unique to each manufacturer, as well as finding a variety of designs for a variety of applications. An example of an impeller is shown in Figure 4.8.1. This device imparts a radial velocity to the fluid that has entered the pump perpendicular to the impeller. The volute (there may be one or more) performs the function of slowing the fluid and increasing the present. A good discussion of centrifugal pumps is given by Lobanoff and Ross (1992).



FIGURE 4.8.1. A schematic of a centrifugal pump is shown. The liquid enters perpendicular to the figure, and a radial velocity is imparted by clockwise spin of the impeller.

A very important factor in the specification of a centrifugal pump is the *casing orientation* and *type*. For example, the pump can be oriented vertically or horizontally. Horizontal mounting is most common. Vertical pumps usually offer benefits related to ease of priming and reduction in required net positive suction head (see discussion below). This type also requires less floor space. Submersible and immersible pumps are always of the vertical type. Another factor in the design is the way the casing is split, and this has implications about ease of manufacture and repair. Casings that are split perpendicular to the shaft are called *radially split*, while those split parallel to the shaft axis are denoted as *axially split*. The latter can be *horizontally split* or *vertically split*. The number of *stages* in the pump greatly affects the pump-output characteristics. Several stages can be incorporated into the same casing, with an associated increase in pump output. Multistage pumps are often used for applications with total developed head over 50 atm.

Whether or not a pump is self-priming can be important. If a centrifugal pump is filled with air when it is turned on, the initiation of pumping action may not be sufficient to bring the fluid into the pump. Pumps can be specified with features that can minimize priming problems.

There are other types of velocity head pumps. *Jet pumps* increase pressure by imparting momentum from a high-velocity liquid stream to a low-velocity or stagnant body of liquid. The resulting flow then

goes through a diffuser to achieve an overall pressure increase. *Gas lifts* accomplish a pumping action by a drag on gas bubbles that rise through a liquid.

Positive-Displacement Pumps

Positive-displacement pumps demonstrate high discharge pressures and low flow rates. Usually, this is accomplished by some type of pulsating device. A piston pump is a classic example of positive-displacement machines. Rotary pumps are one type of positive-displacement device that do not impart pulsations to the existing flow (a full description of these types of pumps is given by Turton, 1994). Several techniques are available for dealing with pulsating flows, including use of double-acting pumps (usually of the reciprocating type) and installation of pulsation dampeners.

Positive-displacement pumps usually require special seals to contain the fluid. Costs are higher both initially and for maintenance compared with most pumps that operate on the velocity head basis. Positive-displacement pumps demonstrate an efficiency that is nearly independent of flow rate, in contrast to the velocity head type (see Figure 4.8.2 and the discussion related to it below).

Reciprocating pumps offer very high efficiencies, reaching 90% in larger sizes. These types of pumps are more appropriate for pumping abrasive liquids (e.g., slurries) than are centrifugal pumps.

A characteristic of positive displacement pumps which may be valuable is that the output flow is proportional to pump speed. This allows this type of pump to be used for metering applications. Also a positive aspect of these pumps is that they are self-priming, except at initial start-up.

Very high head pressures (often damaging to the pump) can be developed in positive-diplicament pumps if the downstream flow is blocked. For this reason, a pressure-relief valve by ress must always be used with positive-displacement pumps.

Pump/Flow Considerations

Performance characteristics of the pump in its be considered in system disign Shaple diagrams of pump applications are shown in Figure 4. O First, consider the laferiand figure. This represents a flow circuit, and the pressure applications to the piping, ming, values, and any other flow devices found in the circuit rules to estimated using the laws of this mechanics. Usually, these resistances (pressure drops) we have a processor of the square of the liquid flow rate. Typical characteristics are shown in Figure 4.8.3. Most sump demonstrate a flow vs. pressure rise variation that is a positive value at zero flow and decreases to zero at some larger flow. Positive-displacement pumps, as shown on the right-hand side of Figure 4.8.3, are an exception to this in that these devices usually cannot tolerate a zero flow. An important aspect to note is that a closed system can presumably be pressurized. A contrasting situation and its implications are discussed below.

The piping diagram show on the right-hand side of Figure 4.8.2 is a once-through system, another frequently encountered installation. However, the leg of piping through "pressure drop 1" shown there can have some very important implications related to *net positive suction head*, often denoted as **NPSH**. In simple terms, NPSH indicates the difference between the local pressure and the thermodynamic saturation pressure at the fluid temperature. If NPSH = 0, the liquid can vaporize, and this can result in a variety of outcomes from noisy pump operation to outright failure of components. This condition is called **cavitation**. Cavitation, if it occurs, will first take place at the lowest pressure point within the piping arrangement. Often this point is located at, or inside, the inlet to the pump. Most manufacturers specify how much NPSH is required for satisfactory operation of their pumps. Hence, the actual NPSH (denoted as **NPSHR**). If a design indicates insufficient NPSH, changes should be made in the system, possibly including alternative piping layout, including elevation and/or size, or use of a pump with smaller NPSH requirements.

The manufacturer should be consulted for a map of operational information for a given pump. A typical form is shown in Figure 4.8.4. This information will allow the designer to select a pump that satisfied the circuit operational requirements while meeting the necessary NPSH and most-efficient-operation criteria.

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FIGURE 4.8.6. Shown are characteristics of a centrifugal fan. The drawbacks to operating away from optimal conditions are obvious from the efficiency variation.



FIGURE 4.8.7. A variety of vane types that might be used on a centrifugal fan are shown.



FIGURE 4.8.8. Efficiency variation with volume flow of centrifugal fans for a variety of vane types is shown.

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Equation (4.9.12) can also be plotted on the chart to give the *packing capability line*. For a required approach, the *operating point* of the tower is the intersection of the cooling demand curve and packing capability line. Charts of cooling demand curves are available (Cooling Tower Institute, 1967; Kelly, 1976). Correlations of the form of Equation (4.9.11) do not necessarily fit experimental data well. A dependence $g_m a \alpha L^{1-n}G^n$ is implied and, in the past, experimental data were often forced to fit such a relation. If the $g_m a$ correlation does not have the form of Equation (4.9.11), the NTU cannot be plotted as a line on a cooling demand chart.

With the almost universal use of computers and the availability of suitable computer programs, one can expect less use of cooling demand charts in the future. The major sources of error in the predictions made by these programs are related to nonuniform air and water flow, and the correlations of packing mass transfer and pressure drop experimental data. The experimental data are obtained in small-scale test rigs, in which it is impossible to simulate many features of full-size towers — for example, nonuniform flow due to entrance configuration, nonuniform wetting of the packing, and, in the case of counterflow towers, the effect of spray above the packing and rain below the packing. Furthermore, since testing of packings in small-scale test rigs is itself not easy, considerable scatter is seen in such test data. Correlations of the data typically have root mean square errors of 10 to 20%.

Legionnaires' Disease

Legionnaires' disease is a form of pneumonia caused by a strain of legionnella bacteria (sero groph I). Smokers and sick people are particularly vulnerable to the disease. Major outbreaks have been easily at conventions and in hospitals, for which the source of the bacteria has been eased to cooling towers of air-conditioning systems. The bacteria require nutrients such as elgebrated bacteria in sludge, and thrive if iron oxides are present. However, properly designed one that, and maintained cooling towers have never been implicated in an outbreak of the disease. Key require neutrients to be met include the following:

- 1. Mist (drift) eliminators should be effective.
- The tower an uld be located so as to min to ze the possibility of mist entering a ventilation system.
 Torosion in the tower a to very lines should be minimized by use of glass fiber, stainless steel, and coated steel
 - 4. The design should facilitate inspection and cleaning, to allow early detection and remedy of sludge buildup.
 - 5. Water treatment and filtration procedures should meet recommended standards.

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FIGURE 4.10.9 (continued).

Details affecting the size and shape of the fusion zone are strongly dependent upon the type of wilding process and its inherent energy transfer capability. Very high-energy processes at the celectron beam welding enable relatively narrow and deep fusion zones. In such the processes, the thermal energy is highly focused. Both the fusion zone and the He Z are relatively small. However, the thermal gradients introduced can be quite large. For thes wild energy processes the accesstructure in the fusion zone represents a more rapidly cooled as sold lifed structure, with the potential or liner grain size. Openflame welding, or torch welding, employs a much lever energy source. As a result, temperature gradients are smaller but the bulk te temperatures are distance from the fusion zone are higher. It follows that the extended the HAZ and the region of a felter parent microstructure become large.

In all fusion welds there is a region around the weld zone where the parent metal has been heated, in the solid state, to temperatures sufficient to cause structural and property changes. This region is called the HAZ. Heat transfer in the HAZ of a welded component determines the temperature profile and cooling rates. Based on the knowledge, or prediction, of the temperature profiles and cooling rates, microstructural variation can be estimated. Prediction of the form of the changing microstructure can then be used to determine if metallurgical problems are to be expected, and whether welding process modifications must be made. The extent of the HAZ is dependent upon many factors, including the heat source energy, the thermal mass of parent material, the thermal conductivity of the parent, and the workpiece geometry.

The impact that the varying temperatures and cooling rates of the HAZ have upon the material microstructure is dependent upon the parent material and its pre-weld condition. For instance, two pure aluminum plates might be joined by MIG welding. The resulting microstructural variation across the weld zone and the HAZ is shown schematically in Figure 4.10.10.

As indicated in the Figure 4.10.11, the microstructural variation can be quite severe, with yield strengths dropping to levels below half that of the unaffected parent. While this microstructure is highly affected by the welding process, the performance change is relatively uniform from the center of the fusion zone outward, not showing any localized regions of severely degraded properties. However, the resulting microstructure is directly related to the type of strengthening mechanisms available in the material and, therefore, to the material itself. Thus, it is useful to discuss some specific cases in which the effects in the HAZ can become quite deleterious, with the potential for localized failure. In addition to cold working, other common strengthening mechanisms should be discussed. Many aluminum alloys





Solutions of this equation are available inite or semi-infinite riety of mode or plates. Two cases of interest at ng point source of heat and the moving line source of heat **P**t e discussed next

nt Source of Heat

The moving point sou 🔛 s gable description of welding thick sections, or for depositing a weld . bead on a plate (see Figure 4.10.15). Relative motion between the source and the plate, in the x-direction, at a velocity, v, is assumed. By applying symmetry to the steady-state solution for a constant rate of



FIGURE 4.10.15 Moving point heat source.

Heat Treatment

The term *heat treatment* is used to describe a variety of processes that are applied to modify the properties of metals with the goal of generating improvements in hardness, strength, ductility, or toughness. One thing that all heat treatments have in common is that they all rely on the transfer of heat to or from the workpiece material. Heat treatments can be subdivided into two categories, nonhardening thermal processes and hardening processes (Mangonon, 1999). Depending upon the specific heat treatment, different aspects of thermal processing must be controlled. Nonhardening processes generally rely on relatively long times at temperature and are often considered isothermal processes, while it is more common that the control of thermal gradients is critical in the application of the principles of heat transfer for hardening processes.

Nonhardening Thermal Processes

Nonhardening thermal processes include processes such as thermal stress relief, annealing, and normalizing. Residual stresses can be generated by many manufacturing processes, ranging from casting and welding to mechanical deformation. Such stresses can lead to component distortion, cracking, stress corrosion, and in some cases delayed failure. By holding the workpiece at an elevated temperature the internal stresses diminish. Thermal stress relief consists of uniformly heating the workpiece to a temperature below the recrystallization temperature, in the recovery range, holding it for a predetermined period of time related to the severity of the stress state, and then uniformly cooling it Aster all nonhardening thermal processes, temperature gradients are to be minimized and he processes are typically considered isothermal.

An annealing heat treatment is performed to soften a material treat guincant amounts of cold work have been performed. This softening may be desired to enable curther mechanical working, or to adjust the metallurgical properties for a specific polication. Annealing is performed a temperatures above the recrystallization temperature and result in a softer, more equiaxed grain structure. The process, like thermal stress relief in a twee relatively slow having and obding, with the time at optimum temperature used to control the final grain structure and size it long periods of time at high temperature lead to the fit by annealed condition, while size or times at the same temperature can be said to yield subcritical innealing. The temperature courted for such a heat treatment are often given based on equilibrium heating and cooling rates. Since real processes do require finite heating and cooling rates, it must be noted that the critical temperatures will be shifted up with increasing heating rates and down with increased cooling rates.

Normalizing is a term generally reserved for discussions regarding steels. In this nonhardening heat treatment, the cooling rate is specified as natural or air cooled. A normalized steel generally exhibits a uniform fine-grained microstructure which enhances its homogeneity and the machinability; the normalization process is often applied after a shape has been generated by casting or forging. The strength and hardness of a normalized steel may either increase or decrease, depending upon the prior state of the workpiece material.

Hardening Heat-Treating Processes

Hardening heat treatment processes are only applicable to materials which show multiple phases at low temperature, can be locked into a nonequilibrium microstructure by rapid cooling, and can be driven toward a desirable microstructural state during subsequent heating. The results of such a sequence are generally considered the principal strengthening approaches for metals. For steels, the Martensitic transformation is critical and is driven by the ability to rapidly cool the workpiece, generating the nonequilibrium structure, Martensite. Other materials, such as aluminum, are strengthened through a mechanism described as precipitation hardening, and still others grow stronger by solid solution strengthening. However, the ability to lock in the nonequilibrium structure and then subsequently adjust the material microstructure via thermally induced diffusion is shared among these hardening heat-treatment processes.

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article in *Chemical Engineering*² gives more details about the experience of various companies in using pinch technology and the benefits obtained.

Fundamental Principles and Basic Concepts

Pinch technology is based on thermodynamic principles. Hence, in this section we will review the important thermodynamic principles and some basic concepts.

Temperature-Enthalpy Diagram

Whenever there is a temperature change occurring in a system, the enthalpy of the system will change. If a stream is heated or cooled, then the amount of heat absorbed or liberated can be measured by the amount of the enthalpy change. Thus, $Q = \Delta H$. For sensible heating or cooling at constant pressure where $CP = mc_p$,

$$\Delta H = CP \ \Delta T \tag{4.11.1}$$

For latent heating or cooling, $\Delta H = m\lambda$ (specific latent heat of condensation or evaporation). If we assume that the temperature change for latent heating or cooling is 1°C, then $CP = m\lambda$.

Equation (4.11.1) enables us to represent a heating or cooling process on a temperature-e-thalpy diagram. The abscissa is the enthalpy and the ordinate is the temperature. The slope of the line is (\sqrt{SP}). Figure 4.11.1a shows a cold stream being heated from a temperature of 20° (To 80°C with CP = 2.0 kW/°C.



FIGURE 4.11.1 The temperature enthalpy diagram.

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Multistream Design Problem

Consider a four-stream problem lists the starting and targe eratures of all the streams s the CP value. W fit pressure drop constraints on the individual involved in the flowsheet. lt a the heat transfer coefficience we shall not take into account this constraint. streams deter se (unrealistically) t ensure that transfer coefficient value of 1 kW/m² K for all streams. provide the second seco Our objective is to device 22 of outlined in Table 4.11. Further, let Δt_{min} denote the minimum temperature difference between any hot process stream and any cold process stream in any exchanger in the network. We shall use this problem to illustrate the concepts of pinch technology.

TIDEE WITT STORM DUM FOR U DOSIGN FROM				
Stream Number	Stream Name	T _s (°C)	T _t (° C)	CP (kW/°C)
1	C1	20	160	40
2	C2	120	260	60
3	H1	180	20	45
4	H2	280	60	30

TABLE 4.11.4 Stream Data for a Design Problem

Targets for Optimization Parameters

Energy Targets

Composite Curves

Let us plot the heat curves for all the hot streams on the T-H diagram (see Figure 4.11.10). We can divide the diagram into a number of temperature intervals, defined by the starting and target temperatures for all the streams. Between two adjacent temperatures we can calculate the total heat content of all the streams that are present in this temperature interval. For example, between 180°C and 60°C the sum total of the heat available is calculated as:





The interval temperatures now h on guarantees that for ot and cold stream will always be a given T_{int} , the actual the 1 m het greater that of

t and cold streams are sorted in descending order and l temperatures f r botł the ved (see Table 4.11.5). duplicate in

For each interva, an y balance is made. The enthalpy balance for interval *i* is calculated using the following equation:

$$\Delta H_i = \left(T_{\text{int},i} - T_{\text{int},i+1}\right) \left(\sum_{j=1}^{N_{\text{streams}}} CP_{c_j} - \sum_{j=1}^{N_{\text{streams}}} CP_{h_j}\right)$$

where ΔH_i = net heat surplus or deficit in interval *i*

 CP_c = mass specific heat of a cold stream

 CP_h = mass specific heat of a hot stream

For the example problem, this calculation is shown in Table 4.11.6. Within each interval the enthalpy balance will indicate that there is either a heat deficit or surplus or the interval is in heat balance. A heat surplus is negative and a heat deficit is positive.

The second law of thermodynamics only allows us to cascade heat from a higher temperature to a lower temperature. Thus, a heat deficit from any interval can be satisfied in two possible ways — by using an external utility or by cascading the surplus heat from a higher temperature interval. If no external utility is used, the heat cascade column of Table 4.11.6 can be constructed. All intervals in this heat cascade have negative heat flows. This is thermodynamically impossible in any interval (it would mean heat is flowing from a lower to a higher temperature). To correct this situation, we take the largest negative flow in the cascade and supply that amount of external hot utility at the highest temperature interval. This modification will make the heat cascade feasible, i.e., none of the intervals will have



FIGURE 4.11.14b Shell targeting for hot streams on the balanced composite curves. Hot streams will require nine shells.

Step 2: Estimate the total number of shells required by the hot process streams and the hot utility streams (see Figure 4.11.14b).

• Starting from the hot stream initial temperature, drop a vertical line on the balanced composite curve until it intercepts the cold composite curve. From this point, construct the horizontal and

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FIGURE 4.11.18b Algorithm for source subnetwork design at the pinch point.



FIGURE 4.11.19 Identifying matches and stream splitting using the CP table.

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For example, for the process GRCC shown in Figure 4.11.23, we want to find out how much low-pressure superheated steam can be generated. The saturation temperature of the low-pressure steam is 170°C and boiler feed water is available at 130°C. The superheat is 10°C. Using a simple trial and error procedure, we can find out how much steam will be generated.

- Assume the amount of steam that is generated.
- Develop a heat curve for the low-pressure steam generation on the shifted temperature scale.
- As generation of low-pressure steam will be a cold stream, the temperature of the stream will be increased by $\Delta t_{min}/2$.
- Keep on increasing the amount of steam generated till the steam generation heat curve touches the process GRCC at any point.

Once the utility levels are decided, introduce them into the stream data and obtain the balanced composite curves. The number of pinch points will increase. In addition to the original process pinch point, each utility level will introduce at least one pinch point. A balanced grid diagram that includes all the utility streams and all the pinch points identified on the balanced composite curve can now be used along with the network design algorithms to develop a network that achieves the target set.

Data Extraction

Process integration studies start from a base case flowsheet. This flowsheet may be the developed from the designer's experience. To conduct pinch analysis properties it is comportant to extract the flowrate, temperature, and heat duty data correctly.

The stream target and starting temperatures should be closed so that we do not generate the original flowsheet.¹² To illustrate this, consider the nows be asnown in Figure 41.00.17 we extract the data as two streams then we might endupy with the original flowsheet. If the drum temperature is not important then we can consider it to be one stream and we stiml a charce for finding new matches. The drum and the pump can be one stream and we stiml a charce for finding new matches.



FIGURE 4.11.24 Flowsheet for data extraction example.

While extracting data extra care should be taken when streams are mixing nonisothermally.¹² Consider the system shown in Figure 4.11.25. Stream A is being cooled to the mixed temperature and stream B is being heated to the mixed temperature. This happens due to mixing the streams. The mixed stream is then heated to a higher temperature. If we extract the data as shown in Figure 4.11.25a and if the

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The Stirling evce was originally developed by Robert Stirling in 1816 for use in a hot-air engine (Walker, 1981). Tay, Stirling encouplers a crused in many cryogenic thermal applications, including cooling spacecraft instrument extends, inducting thermal imaging systems, and superconducting elements.

A schematic of the Stirling cryocooler is shown in Figure 4.12.5. The unit consists of a cylinder enclosing the power piston and the displacer piston. The two chambers at either end of the cryocooler are connected through a regenerator, which is one of the critical components of the system.

The sequence of processes for the Stirling cycle is as follows. First, the working fluid (typically, helium gas) is compressed by the power piston while energy is rejected to cooling coils to achieve a near-isothermal compression. Next, the displacer piston moves the high-pressure gas through the regenerator (which was cooled during a prior process), and the gas is cooled to a lower operating temperature.



FIGURE 4.12.5 Stirling cycle refrigerator.

 Q_h is the energy added to the gas in the hot chamber from a source at T_h , and Q_c is the heat added to the gas in the cold chamber from a source at T_c .

An advantage of the VM cryocooler is that the thermal input may be provided by a wide variety of sources, including solar energy and isotope thermal energy. This feature makes the VM cryocooler attractive for cryogenic cooling in long-term space exploration and in applications where mechanical vibrations of a drive engine must be avoided (Walker, 1983a). Because the VM refrigerator operates at a relatively low speed (approximately 600 rpm), the reliability of the system is particularly good. For example, one VM cryocooler was operated continuously and unattended for 20,000 hours (833 days) (Pitcher, 1973).

Gifford-McMahon (GM) Cryocooler

The GM cryocooler was developed by McMahon and Gifford (1959) for use as a miniature cryogenic refrigerator. A schematic of the GM cryocooler is shown in Figure 4.12.7. The sequence of processes for the GM cryocooler is as follows.



FIGURE 4.12.7 Gifford-McMahon (GM) cryocooler.

Pressure build-up process — With the displacer piston at the bottom of the cylinder and the outlet valve closed, the inlet valve is opened to allow the working fluid (typically helium gas) to enter the upper expansion space.

Intake stroke — The displacer piston is moved from the bottom of the cylinder to the top, while the inlet valve remains open and the outlet valve remains closed. The warm gas originally in the upper space is moved through the regenerator to the lower space. The gas is cooled as it flows through the regenerator, and additional gas is drawn in through the inlet valve to maintain the gas pressure constant within the system.

Expansion process — With the displacer at the top of the cylinder, the inlet valve is closed and the outlet valve is slowly opened. The gas within the lower space expands adiabatically, and the gas temperature decreases.

fin configurations have been used, including straight fins, wavy fins, herringbone fins, and serrated fins. Various stacking arrangements may be used to produce flow configurations such as counterflow, cross-flow, and multipass heat exchangers. The counterflow configuration is most commonly used in cryogenic systems because of the better thermal performance of a counterflow exchanger for a given heat transfer surface area.

A schematic of the perforated plate heat exchanger is shown in Figure 4.12.10. The perforated plate exchanger consists of a series of parallel perforated plates separated by low-thermal-conductivity spacers or gaskets (Fleming, 1969). The plates are constructed of a high-thermal-conductivity metal, such as aluminum or copper. The spacers are bonded (sealed) to the plates to prevent leakage between the two streams. Older models of the perforated plate exchangers used a plastic material for the spacers to reduce the problem of longitudinal conduction. More recent models have used stainless steel spacers to achieve better mechanical reliability (Hendricks, 1996). The configuration for the perforated plate exchanger is usually counterflow.



Small perforations (diameters ranging from 1.5 mm or 1/16 in. to less than 0.4 mm or 1/64 in.) are made into the plates so that a large heat transfer coefficient is achieved. The ratio of the plate thickness (length of the hole in the plate) to the diameter of the hole is on the order of 0.75, so the thermal and hydrodynamic boundary layers do not become fully developed within the perforations, which results in high heat transfer coefficients and correspondingly high friction factors.

Wire screens have been used to replace the perforated plates in some compact heat exchangers used in small cryocoolers (Lins and Elkan, 1975; Vonk, 1968). The fluid streams flow through the wire layers, and heat is exchanged between the two streams through conduction along the wires. The wire screens are separated by plastic or resin-impregnated paper spacers, or stainless steel metallic spacers that are diffusion-bonded or brazed to the perforated plates.

Cryogenic Heat Exchanger Design Problems

Three problems may become significant in the design of heat exchangers for cryogenic service:

- 1. The effect of variation of fluid properties with temperature variation of the fluid specific heat is particularly important (Chowdhury and Sarangi, 1984; Soyars, 1991).
- 2. The effect of longitudinal conduction along the separating surface between the two streams this effect is especially critical for short heat exchangers that must have a high efficiency (Kroeger, 1967).
- The effect of flow maldistribution the performance deterioration due to flow distribution is most pronounced for high-efficiency heat exchangers (Fleming, 1967).

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The barometric pressure and temperature of atmospheric air changes with increasing altitude above sea level. Standard atmospheric conditions are defined at sea level where the pressure is 29.921 in. of mercury (in.Hg) and the temperature is 59°F. The U.S. Standard Atmosphere defines the variation of pressure and temperature with altitude and is generally used to estimate these properties at different altitudes. The troposphere (lower atmosphere) is taken to consist of dry air behaving as an ideal gas. The acceleration due to gravity is also taken to be a constant, 32.1740 ft/s². The pressure and temperature are given by

$$T = 59 - 0.00356616Z \tag{4.13.3}$$

$$p = 29.92 \left(1 - 6.8753 \times 10^{-6} Z\right)^{5.2559}$$
(4.13.4)

where Z is the altitude (ft), p is the barometric pressure (in.Hg), and T is the temperature (°F). Equations (4.13.3) and (4.13.4) are accurate from sea level to an altitude of 36,000 ft. More extensive information including higher altitudes, other physical data, and SI units may be obtained from NASA (1976).

Thermodynamic Properties of Saturated Water

The thermodynamic saturated state is discussed in Chapter 3 of this handbook. Relationships given the thermodynamic properties of saturated water have been developed by Hyland and Weder (198 a) over the temperature range from -80 to 300°F. The determination of these properties is used on an ideal temperature scale almost identical to the generally used practical temperature scale used for measurements. As an example, the standard boiling temperature of cate at 14.696 psia or 29.921 in.Hg) is 211.95°F rather than the usual value of 212°F on the product at a 14.696 psia or 29.921 in.Hg) is estimated liquid water have value the indust zero at the triple point, 32.01°T. Water at a temperature between its triple point and citizal point may coexist in troostates, liquid and vapor. When these phases are in equilibrium chapter physical features of the system remain constant with time and the vapor and figure posturated.

Over inquid water, the are also pressure for the temperature range from 32 to 392°F is

$$\ln(p_{ws}) = \frac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_6 \ln T$$
(4.13.5)

where $C_1 = -1.044039 \times 10^4$, $C_2 = -1.1294650 \times 10^1$, $C_3 = -2.7022355 \times 10^{-2}$, $C_4 = 1.2890360 \times 10^{-5}$, $C_5 = -2.4780681 \times 10^{-9}$, and $C_6 = 6.5459673$, and where T is the absolute temperature (°R) and the subscript "ws" indicates saturated water. The saturation pressure over ice in terms of temperature for a range from -148 to 32° F is

$$\ln(p_{ws}) = \frac{C_7}{T} + C_8 + C_9 T + C_{10} T^2 + C_{11} T^3 + C_{12} T^4 + C_{13} \ln T$$
(4.13.6)

where $C_7 = -1.0214165 \times 10^4$, $C_8 = -4.8932428$, $C_9 = -5.3765794 \times 10^{-3}$, $C_{10} = 1.9202377 \times 10^{-7}$, $C_{11} = 3.5575832 \times 10^{-10}$, $C_{12} = -9.0344688 \times 10^{-14}$, and $C_{13} = 4.1635019$. The ASHRAE *Handbook of Fundamentals* (1997) gives the thermodynamic properties of water at saturation.

Thermodynamic Properties of Moist Air

Hyland and Wexler (1983a; 1983b) developed formulas for the thermodynamic properties of moist air based on the "thermodynamic temperature scale," an ideal temperature scale almost identical to the generally used practical scale used for physical measurements. The ASHRAE *Handbook of Fundamentals* (1997) gives the thermodynamic properties of moist air based on the Hyland and Wexler equations. These thermodynamic properties are

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Dry Bulb Temperature

FIGURE 4.13.2 Air conditioning process directions on a psychrometric chart.

particularly changes in dry-bulb temperature, humidity ratio re thatpy. Observe that the horizontal ry bub temperature or a sensible change. On the movement across the chart involves only a chan other hand, a vertical movement the chart represents a pure latent dange where the dry-bulb temperature remains constant but the numidity ratio charge a Figure 4.13.2 shows a schematic view of the psychrometric charge the process of retions common to air conditioning are shown. several examples where the use of the psychrometric chart is demonstrated. Cooling and Dehumi

Moist air, when cooled to a temperature below its dew point, will condense a portion of the water vapor initially in the mixture. Equipment that could be used to carry out the cooling and dehumidifying process is shown in Figure 4.13.3, where a refrigerant enters and leaves an evaporator coil. The air inlet and exit conditions are defined as well as the condensed water drained from the pan below the coil. If the system operates on a steady-state flow basis (all variables remain constant with time) then the process may be described by the following water mass and energy equations:

$$\dot{m}_a W_1 = \dot{m}_a W_2 + m_w \tag{4.13.20}$$

$$\dot{m}_a h_1 = \dot{m}_a h_2 + \dot{m}_w h_w + q \tag{4.13.21}$$

where q is the heat transfer from the moist air to the coil. These two equations may be rearranged to give

$$\dot{m}_{w} = \dot{m}_{a} (W_{1} - W_{2}) \tag{4.13.22}$$

$$q = \dot{m}_a \Big[(h_1 - h_2) - (W_1 - W_2) h_w \Big]$$
(4.13.23)

where the equation for q indicates that the cooling and dehumidifying process includes sensible and latent heat transfer. The sensible and latent heat transfer quantities are

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an hourly basis. The sum of all the hourly heat losses over the heating season is the annual energy consumption needed to heat the space.

Many thermostats incorporate a temperature setback feature to reduce energy consumption of the heating system. The temperature is lowered several degrees at night and raised during the day when the space is occupied. It turns out that the early morning hours is the time when the heat load is the largest. Additionally, the morning time is typically when the thermostat calls for increasing the temperature demand, during which time the occupants may experience discomfort. The time interval needed to warm the space after thermostat setback may be estimated using the heat balance method (Pederson et al., 1998).

The determination of the heat load assumes that no solar energy enters the space, internal heat gains are negligible, and that the outdoor and indoor temperatures are constant. The calculation of the design heat load from a space is as follows:

- 1. Choose outdoor design conditions: temperature, humidity, wind direction, and wind speed.
- 2. Choose indoor design conditions.
- 3. Determine temperatures in any adjacent unheated spaces, if any.
- 4. Choose the heat transmission coefficients and calculate the heat loss rates for the walls, floors, ceilings, glass, doors, and all other surfaces where heat is transferred to the outside.
- 5. Calculate the heat load resulting from the infiltration of air and all other outdoor air entering the space.
- 6. Sum all the contributions to the heat load from transmission and infiltration O

Outdoor Design Conditions

Heating design-day dry and wet bulb temperatures above who wind speed for numerous geographical locations are compiled from recent data by ASE CAE (Handbook of Liu damentals, 1997). Winter climatic data are based on hourly we are robservations over at less a 12 year period. Winter month conditions are reported in the basis of 99.6 and 9.0 annual percentiles. These annual percentiles represent, respectively, 39 and 88 h from alge and 9760 h. For example, Asheville, NC has a heating a your remperature equal to 11°F the 9.6 percentile, which means that for 35 h out of the year the temperature is likely to be a ten 11°F. Other climatic data, if available for a particular condition, suggesting lower dry-balb temperatures and/or higher wind velocities should be considered in selecting the design conditions.

Indoor Design Conditions

Generally, the design dry-bulb temperature is chosen from the lower portion of the acceptable range, therefore preventing oversizing of the heating system. A heating system operating at conditions less than the design load does so at less efficiency. For this reason, a heating design dry-bulb temperature equal to 70°F is usually chosen. The relative humidity level is typically set at maximum value of 30% to provide, on the one hand, a healthy environment, while on the other hand assure moisture integrity of the building's envelope.

Heat Transmission Loss

Heat transfer through all the building envelope materials comprising the walls, ceiling, roof, floors, windows, and doors is sensible and is calculated using

$$q = UA(T_i - T_o) \tag{4.13.42}$$

where U is the overall heat transfer coefficient or U-factor (Btu/h ft²°F), A is the normal surface area through which the heat transfer passes (ft²), T_i is the inside design temperature (°F), and T_o is the outside design temperature (°F). The U and A values are generally different for walls, ceilings, roofs, and windows

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equations. Thus, the method of Lagrange multipliers not only yields the optimum but also the sensitivity coefficients with respect to the various constraints in the problem. The Lagrange multiplier λ may also be related to the change in the objective function U, at the optimum, with the constraint parameter E as $\lambda = -\partial U/\partial E$. Therefore, the values of the Lagrange multipliers obtained in a given problem may be used to guide slight variations in the constraints to choose appropriate sizes, dimensions, etc., that are easily available or more convenient to fabricate.

Example

As a practical example, consider an electronic circuitry in which the power source may be considered as a square with side dimension L in meters. It is desired to minimize the heat transfer from the power supply. The heat transfer coefficient h in W/m^2K is given by the expression

$$h = (2 + 10 L^{1/2}) \Delta T^{1/4} L^{-1}$$

where ΔT is the temperature difference in K from the ambient. A constraint arises due to the strength of the bond that attaches the power supply to the electronic circuit board as L $\Delta T = 5.6$. We need to calculate the side dimension L of the square that would minimize the total heat loss, solving the problem as a constrained one.

The rate of heat loss Q from the power supply is the objective function which is to be minimized and is given by the expression

$$\dot{Q} = h A \Delta T = (2 L + 10 L^{3/2}) \Delta T^{5/4} Q$$
since area A = L². The constraint is L $\Delta T = 5.6$
Therefore, the optimum is given by the equation
$$\Delta T^{5/4} (2 \pm 10 L^2) + \lambda (2 \pm 0)$$

$$\Delta T^{5/4} (2 \pm 10 L^{3/2}) \Delta T^{1/4} + \lambda L = 0$$

$$L \Delta T = 5.6$$

These equations can be solved to yield the optimum as

$$L^* = 0.04 \text{ m}; \Delta T^* = 140; \dot{Q}^* = 77.05 \text{ W}; \lambda = -17.2$$

It can be shown that if the constraint parameter is increased from 5.6 to 5.7, the heat transfer rate \hat{Q} becomes 78.77, i.e., an increase of 1.72. This is expected since the sensitivity coefficient $S_c = -\lambda = 17.2$. Therefore, the effect of relaxing the constraint on the optimum value can be determined. This problem can also be solved as an unconstrained one by substituting ΔT in terms of L from the constraint into the expression for \hat{Q} . The solution becomes simpler, but λ is not obtained.

The range of application of calculus methods to the optimization of thermal systems is somewhat limited because of complexities that commonly arise in these systems. Numerical solutions are often needed to characterize the behavior of the system and implicit, nonlinear equations that involve variable material properties are frequently encountered. However, curve fitting may be employed in some cases to yield algebraic expressions that closely approximate the system and material characteristics. If these expressions are continuous and easily differentiable, calculus methods may be conveniently applied to yield the optimum. However, only equality constraints can be considered. Therefore, inequalities must be converted to equations, as discussed earlier, to apply this approach. The method of Lagrange multipliers

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be employed. However, this is only for illustration purposes and in actual practice each test run or simulation would generally involve considerable time and effort.

There are several approaches that may be employed in search methods, depending on whether a constrained or an unconstrained problem is being considered and on whether the problem involves a single variable or multiple variables. The classification of these methods and the major techniques are discussed here.

Elimination Methods

In these methods, the domain in which the optimum lies is gradually reduced by eliminating regions that are determined not to contain the optimum. We start with the design domain defined by the acceptable ranges of the variables. This region is known as the initial *interval of uncertainty*. Therefore, the region of uncertainty in which the optimum lies is reduced until a desired interval is achieved. Appropriate values of the design variables are chosen from this interval to obtain the optimal design. For single-variable problems, the main search methods based on elimination are

- · Exhaustive search
- · Dichotomous search
- Fibonacci search
- · Golden section search

All these approaches have their own characteristics, advantages, and applicability. These net of scan also be used for multivariable problems by applying the approach to one variable at the.

Uniform exhaustive search — As the name suggests, this methas an any uniformly distributed locations over the entire domain to determine the objective mean fine number of runs n is chosen and the initial range L_o of variable x is subdivided by pacing n points enformly over the domain. Therefore, n+1 subdivisions, each of with D ((n+1)), are obtained. A each of the on points, the objective function U (x) is evaluated undig comulation of the system. The interval containing the optimum is obtained by eliminate regions where inspect on indicate that it does not lie. Thus, if a maximum in the objective function is desired, the region between the location where the smaller value of U (x) is obtained in two runs eacher active boundary is eliminated, as shown in Figure 4.14.7 in terms of the results from three runs in eigen exists to the region between A and C. Similarly, in Figure 4.14.7b, the region between the lower domain boundary and point B is eliminated.



FIGURE 4.14.7 Elimination of regions in the search for a maximum.

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FIGURE 4.14.16 The hemstitching method with return to the constraint obtained by keeping (a) x_1 fixed, and (b) x_2 fixed.

difference is greater than one, the move may be made in a direction that yields the greatest change in the objective function.

Figure 4.14.16 shows the hemstitching method for a two-variable, single-constraint problem. The first step involves reaching the constraint by keeping one of the two variables, x_1 or x_2 , fixed, and varying the other till the constraint is satisfied. For example, if the constraint is

$$\mathbf{x}_1^2 \, \mathbf{x}_2^3 = 35 \tag{4.14.26}$$

we can keep either x_1 or x_2 fixed to obtain the value of the other variable at the constraint as

$$x_1 = \left(\frac{35}{x_2^3}\right)^{1/2}$$
 or $x_2 = \left(\frac{35}{x_1^2}\right)^{1/3}$ (4.14.27)

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Kreith F., Timmerhaus K., Lior N., Shaw H., Shah R.K., Bell K. J., etal.."Applications." *The CRC Handbook of Thermal Engineering.* Ed. Frank Kreith Boca Raton: CRC Press LLC, 2000

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FIGURE 4.15.4 Examples of commercial structured boiling surfaces. (From Pate, M.B. and i) *Compact Heat Exchangers*, Hemisphere Publishing, New York, 1990. With permission.)

These structured boiling surfaces, developed for refriger non-and process opplications, have been used as "heat sinks" for immersion-cooled-microsceneration chips.

The behavior of tube bundles is a read forent with struct red-surface takes. The enhanced nucleate boiling dominates, and the convective boiling enhancement, found in plain tube bundles, does not occur. Active enhancement convective boiling enhancement, found in plain tube bundles, does not occur. Active enhancement convective boiling enhancement, found in plain tube bundles, does not occur. Active enhancement convective boiling enhancement, found in plain tube bundles, does not occur. Active enhancement convective boiling enhancement, found in plain tube bundles, does not occur. Active enhancement is also and the original structure enhancement of the surface of the heated surface. Although active techniques are effective in reducing the also also and/or increasing the critical heat flux, the practical applications are very limited, largely because of the difficulty of reliably providing the mechanical or electrical effect.

Compound enhancement, which involves two or more techniques applied simultaneously, has also been studied. Electrohydrodynamic enhancement was applied to a finned tube bundle, resulting in nearly a 200% increase in the average boiling heat transfer coefficient of the bundle, with a small power consumption for the field.

Convective Boiling/Evaporation

The structured surfaces described in the previous section are generally not used for in-tube vaporization, because of the difficulty of manufacture. One notable exception is the high-flux surface in a vertical thermosiphon reboiler. The considerable increase in the low-quality, nucleate boiling coefficient is desirable, but it is also important that more vapor is generated to promote circulation.

Helical repeated ribs and helically coiled wire inserts have been used to increase vaporization coefficients and the dry-out heat flux in once-through boilers.

Numerous tubes with internal fins, either integral or attached, are available for refrigerant evaporators. Original configurations were tightly packed, copper, offset strip fin inserts soldered to the copper tube or aluminum, star-shaped inserts secured by drawing the tube over the insert. Examples are shown in Figure 4.15.5. Average heat transfer coefficients (based on surface area of smooth tube of the same diameter) for typical evaporator conditions are increased by as much as 200%. A cross-sectional view of a typical "microfin" tube is included in Figure 4.15.5. The average evaporation boiling coefficient is increased 30 to 80%. The pressure drop penalties are less; that is, lower percentage increases in pressure drop are frequently observed.

Convective Condensation

This final section on enhancement of the various modes of heat transfer focuses on in-tube condensation. The applications include horizontal kettle-type reboilers, moisture separator reheaters for nuclear power plants, and air-conditioner condensers.

Internally grooved or knurled tubes, deep spirally fluted tubes, random roughness, conventional innerfin tubes have been shown to be effective for condensation of steam and other fluids.

The microfin tubes mentioned earlier have also been applied successfully to in-tube condensing. As in the case of evaporation, the substantial heat transfer improvement is achieved at the expense of a lesser percentage increase in pressure drop. By testing a wide variety of tubes, it has been possible to suggest some guidelines for the geometry, e.g., more fins, longer fins, and sharper tips; however, general correlations are not yet available. Fortunately for heat-pump operation, the tube that performs best for evaporation also performs best for condensation.

Twisted-tape inserts result in rather modest increases in heat transfer coefficient for complete condensation of either steam or refrigerant. The pressure drop increases are large because of the large wetted surface. Coiled tubular condensers provide a modest improvement in average heat transfer coefficient.

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Further Information

This section gives some indication as to why heat transfer enhancement is one of the fastest growing areas of heat transfer. Many techniques are available for improvement of the various modes of heat transfer. Fundamental understanding of the transport mechanism is growing, but, more importantly,



FIGURE 4.16.3 Thermal resistance network in a heat pipe.

Design Example

Design a water heat pipe to transport 80 W of waste heat from an electronics package to cooling water. The heat pipe specifications are

- 1. Axial orientation complete gravity-assisted operation (condenser above the evaporator; $\psi = 180^{\circ}$).
- 2. Maximum heat transfer rate 80 W.
- 3. Nominal operating temperature -40° C.
- 4. Inner pipe diameter 3 cm.
- 5. Pipe length 25 cm evaporator length, 50 cm adiabatic section, and 25 cm condenser length.

The simplest type of wick structure to use is the single-layer wire mesh screen wick shown in Table 4.16.2. The geometric and thermophysical properties of the wick have been selected as (this takes some forethought)



FIGURE 4.17.2 Cumulative spray drop number and volume distributions.

(or mass) distribution function, $F(D^3)$; this gives more weight to the large drops in the distribution. In this case, a volume median diameter (VMD) or a mass median diameter (MMD) can also be defined, as indicated in Figure 4.17.2.

Various other mean diameters are also in common use. These are ariz d 1 the standard notation of Mugele and Evans² as

(4.17.1)

$$\mathbf{O}(\mathbf{A}_{jk})^{j} = \frac{D^{T}f(D) dD}{\int_{D_{min}}^{D_{max}} D^{k}(Q) dD}$$
(4.17.1)
where $f(D) = dF(D)/d\mathbf{E}^{j}$ the hard ize probability density function (usually normalized such that $\int_{D_{min}}^{D_{max}} f(D) dD = 1$). Commonly used mean diameters are D_{10} (i.e., $j = 1$, $k = 0$, sometimes called the length mean diameter³ and D_{32} (i.e., $j = 3$, $k = 2$, called the Sauter mean diameter or SMD). The Sauter mean diameter has a useful physical interpretation in combustion applications since drop vaporization rates are proportional to the surface area of the drop. It represents the size of that drop that has the same volume-to-surface area ratio as that of the entire spray.

Several distribution functions have been found to fit experimental data reasonably well. Among these are the Nukiyama–Tanasawa and the Rosin–Rammler distributions which have the general form³ f(D) $= aD^{p} \exp(\{-bD\}^{q})$, where the constants a, p, b, and q characterize the size distribution. The higher the parameter, q, the more uniform the distribution, and typically 1.5 < q < 4. Other distributions have been proposed which consist of logarithmic transformations of the normal distribution, such as f(D) = a $\exp(-y^2/2)$, where $y = \delta \ln(\eta D/(D_{\text{max}} - D))$, and a, δ , and η are constants. In this case, the smaller δ , the more uniform the size distribution. It should be noted that there is no theoretical justification for any of these size distributions. Spray drop size distributions can be measured nonintrusively by using optical laser diffraction and phase/Doppler instruments. A discussion of these techniques and their accuracy is reviewed by Chigier.4

Atomizer Design Considerations

Atomization is generally achieved by forcing a liquid or a liquid–gas mixture through a small hole or slit under pressure to create thin liquid sheets or jets moving at a high relative velocity with respect to the surrounding ambient gas. Desirable characteristics of atomizers include the ability to atomize the

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liquid over a wide range of flow rates, low power requirements, and low susceptibility to blockage or fouling. In addition, atomizers should produce consistent sprays with uniform flow patterns in operation.

Atomizers can be broadly characterized as those producing hollow cone or solid cone sprays, as depicted in Figure 4.17.3. In solid cone (or full cone) sprays the spray liquid is concentrated along the spray axis, Figure 4.17.3a. These sprays are useful in applications requiring high spray penetration, such as in diesel engines. In hollow cone sprays the axis region is relatively free of drops, giving wide spray dispersal, Figure 4.17.3b. These sprays are often used in furnaces, gas turbines, and spray-coating applications.



Many different atomizer de igns are found in appreciations. Common atomizer types include pressure, rotary, twin-fluid corressist, air-blast, ef e ve conte flashing, electrostatic, vibratory, and ultrasonic atomizer as discussed next.

Atomizer Types

In pressure atomizers atomization is achieved by means of a pressure difference, $\Delta P = P_{\text{res}} - P_{\text{gas}}$, between the liquid in the supply reservoir pressure, P_{res} , and the ambient medium pressure, P_{gas} , across a nozzle. The simplest design is the plain orifice nozzle with exit hole diameter, d, depicted in Figure 4.17.3a. The liquid emerges at the theoretical velocity $U = \sqrt{2\Delta P}/\rho_{\text{liquid}}$, the (Bernoulli) velocity along the streamline A–B in Figure 4.17.3a, where ρ_{liquid} is the density of the liquid. The actual injection velocity is less than the ideal velocity by a factor called the discharge coefficient, C_D , which is between 0.6 and 0.9 for plain hole nozzles. C_D accounts for flow losses in the nozzle.

Four main jet breakup regimes have been identified, corresponding to different combinations of liquid inertia, surface tension, and aerodynamic forces acting on the jet, as shown in Figure 4.17.4. At low injection pressures the low-velocity liquid jet breaks up due to the unstable growth of long-wavelength waves driven by surface tension forces (Rayleigh regime). As the jet velocity is increased, the growth of disturbances on the liquid surface is enhanced because of the interaction between the liquid and the ambient gas (the first and second wind-induced breakup regimes). At high injection pressures the high-velocity jet disintegrates into drops immediately after leaving the nozzle exit (atomization regime). Criteria for the boundaries between the regimes are available.⁵ Aerodynamic effects are found to become very important relative to inertial effects when the jet Weber number, We_j > 40, where We_j = $\rho_{gas}U^2d/\sigma$, ρ_{gas} is the gas density, and σ is the liquid surface tension.

Experiments show that the unstable growth of surface waves is aided by high relative velocities between the liquid and the gas, and also by high turbulence and other disturbances in the liquid and gas flows, and by the use of spray liquids with low viscosity and low surface tension.

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Further Information

sprays can be obtained through participation tems (JLACS-1-metricus, -Europe, -Japan, -Information about recent work in the field of atomization in the Institutes for Liquid Atomization on Spraying Systems (LASS-Korea). These regional ILASS rections hold annual meeting. An international congress (ICLASS) is also held biomielty utore information is a glabe or the ILASS-Americas homepage at http://ucicherg.ac-du/lass.Affiliated with an DAss organizations is the Institute's Journal publication http://ueiei.eug.uc.cdu/ilass. Affiliated with $a \in \hat{U}$ Asso organizations in a on iz and Sprave nublished by Clegen House, Inc., New York.

the doors open after personnel or products have passed through them. The proper management of cooland cold-storage rooms principally involves avoiding wide temperature fluctuations, keeping the cooling coils properly defrosted, and preventing heat from entering the room.

In the cool and cold storage of perishable foods, the following optimum conditions are of considerable significance: optimum temperature, optimum relative humidity, condensation of water vapor on the product, optimum air movement, optimum stacking, and sanitation.

Controlled Atmosphere (CA) Storage

The controlled atmosphere is derived from the fact that the composition of the atmospheric gases in contact with the products is controlled at precise levels during storage or transportation in order to prolong and extend the storage and market life of fresh fruits and vegetables. The major and obvious benefit of controlled atmosphere systems, as a supplement to refrigeration, is the extended storage life of food products. The immediate implication of this is that food products can be shipped longer distances and sea transportation becomes a viable alternative to air freight.

In CA storage, the oxygen (O_2) level is reduced and carbon dioxide (CO_2) increased. Lower respiration occurs, thus extending the life of some products in storage and providing additional distribution and marketing possibilities for fresh products. It is important to maintain the proper relationship of O_2 and CO₂ depending on the product and temperatures of storage. In most instances, O₂ and CO₂ are maintained between 2 and 5% volume, with nitrogen (N_2) comprising the remainder of the mixture. The print gas composition varies for each commodity and cultivar. The composition of an ordinary calculationage room atmosphere varies within the range 19 to 21% O2 and 0 to 2% CO2 depending of le temperature, kind, and quantity of food products stored, and the air-tightness of the part of (Dewey, 1983). Possibly 10% of all commercial apples grown for fresh market pr o 😪 🕶 stored in a controlled atmosphere, \mathbf{V}_{2} 7% \mathbf{O}_{2} and from 5 4.5°C. Recently, CA and optimum operating conditions are 0 to 8% -Τa alle have received considerable attention. applications to different fruits and

Refrigerated Transport

Large another of Guits and vegetables are test every year, not only because of the lack of cooling and non-rented transport facilities, but to because of the incorrect choice of temperature and humidity, and sometimes due to me making of different commodities in the same container.

The world's annual production of perishable food products is estimated by the United Nations Food and Agriculture Organization to be over 450 million tons. Of this, only about 10% is traded internationally; the rest is consumed locally or wasted. About 61% (28.8 million tons) of the total international traded amount (47.3 million tons) is transported by sea. Of the remainder, about 34% (16.3 million tons) is transported by sea. Of the remainder, about 34% (16.3 million tons) is transported by land and about 5% (2.2 million tons) by air. It is estimated that of 28.8 million tons of perishable cargo, about 20.1 million tons (i.e., 70%) is shipped in reefer vessels and 8.7 million tons (i.e., 30%) in containers (Dincer, 1997). Therefore, for perishable products the transport from producer to consumer is one of the main steps in the cold chain and is essential for the quality of products. The packing is designed to protect the products during transport and distribution. Refrigerated facilities play a significant role when transporting perishable food products to marketplaces; the products may be onboard for up to a month. Therefore, good temperature control and air distribution is vital for product quality. So, extensive research is required to make refrigerated transportation capable of performing at its maximum potential level.

The requirements for the product environment during transportation and distribution are generally the same as for cold storage. But the time of exposure is normally much shorter and technical difficulties in maintaining low temperatures are greater. Also, there is a need for better temperature control in transportation. In transport by sea, some problems (e.g., mainly temperature rise) in maintaining satisfactory conditions are relatively slight and the situation has been improved considerably over the past few years. The types of insulation and the refrigeration unit used are much the same as in any stationary cold store. Suitable ships are now available to satisfy every requirement for any type of refrigerated

Cooling Process Parameters

In food processing, there are many situations in which the temperature at any point in the product is a function of time and its coordinate, leading to the transient heat transfer. The most notable food processing examples of transient heat transfer are heating, cooling, freezing, drying, blanching, etc. For practical food cooling applications, the factors that influence the temperature change and cooling rate are temperature and flow rate of cooling medium (coolant), thermal properties of food product, and physical dimensions and shape of food product.

The primary objective of a cooling study is to produce usable data and technical information that will help improve the existing cooling systems and provide optimum operation conditions. People who work in the food cooling industry need the following: (1) a procedure for analyzing cooling process parameters, (2) a procedure for using cooling data to design cooling systems for efficient food cooling applications, and (3) a basic data documentation protocol.

Regardless of the type of cooling technique, knowledge and determination of the cooling process parameters are essential to provide efficient and effective food cooling at the micro- and macroscales. Some major design process factors for a food cooling process are

- Cooling process conditions in terms of temperature, flow rate, and relative humidity
- Arrangement of the individual products and/or product batches
- Depth of the product load in the cooling medium
- Initial and final product temperatures

o.uk are time, and seven-eighths The parameters in terms of the cooling coefficient, lag factor, had cooling time (Figure 4.18.5) are the most important and many many according for a food cooling process, and these are used to evaluate and present the cooling in the data and cooling inchavior of the food products. A number of experimental and modeling is follies to determine these aratheters for various food products, ee Dincer, 1995a; Wade, 1984). particularly for fruits and vent 61 have been undertak



FIGURE 4.18.5 Representation of cooling times on a cooling profile. (From Dincer, I., Heat Transfer in Food Cooling Applications, Taylor & Francis, Washington, D.C., 1997. With permission.)

Cooling coefficient — This is an indication of the cooling capability of a food product subject to cooling and denotes the change in the product temperature per unit time of cooling for each degree temperature difference between the product and its surroundings.

Lag factor — It is a function of the size and shape and the thermal properties of the product, such as the effective heat transfer coefficient, thermal conductivity, and thermal diffusivity, and quantifies the resistance to heat transfer from the product to its surroundings in relation to the Biot number.

Half-cooling time — The half-cooling time is the time required to reduce the product temperature by one-half of the difference in temperature between the product and the cooling medium. The food cooling rate data are formed in the cooling times.

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Further Information

Other sources on thermal processing in food preservation technologies which may be consulted for more detailed information are Mohsenin (1980), Cleland (1990), and Dincer (1997).


FIGURE 4.19.2 Transmission electron microscopy (TEM) image of a metal-silicon c mac, thick has failed during a brief electrical pulse. The heat generated in the tungsten plug raises the temperature and induces severe atomic diffusion. (From Banerjee, K. et al., *Proc. Int. Reliab. Plays for the 10*).

during a brief pulse of electrical our ert. Such current pulses are caused by electrostatic-discharge (ESD), which can occur during thip in a unacture and packaging () gare 4.19.3 shows a cross-sectional micrograph of a high v lengt transistor for a smith power or cuit. High-voltage and high-power transistors are in the *Comput-power circuits* for versiles and are particularly susceptible to thermal failure because of the large rates of heat the era or and the electrical noise they must withstand. In this particular device, heat diffuses laterally it a single-crystal silicon layer of thickness as low as 200 nm. Figure 4.19.4 shows an electron micrograph of a silicon cantilever, in which the heat is conducted by a suspended, singlecrystal silicon layer. This device is used for high-density thermomechanical data storage, during which Joule heating in the cantilever softens the surface of an organic substrate and atomic-scale forces form a data bit of diameter below 50 nm. The bit dimensions and writing rate are governed by conduction along the cantilever and the tip.



FIGURE 4.19.3 Cross-sectional SEM image of a high-voltage silicon-on-insulator transistor. Electrons flow from the source to the drain and generate heat within the silicon layer, which can be as thin as 200 nm. This device, which is called a lateral diffusion metal-oxide-semiconductor (LDMOS) transistor, is used to block hundreds of volts for smart-power electronic circuits. (From Leung, Y.K. et al., *IEEE Electron Device Lett.*, 18, 414-416, 1997.)

time less than the time constant of device or interconnect cooling to the substrate, then heat travels only micrometers or less from the surface of the chip during the heating pulse. The energy, therefore, must be absorbed by a very small volume, which dramatically increases the temperature rise. Such brief heating phenomena are often the result of electrical overstress (EOS), which is a major reliability concern for both high-power circuits and compact low-power circuits in computers. Electrostatic discharge (ESD) can induce a current pulse into the circuit terminals of a duration between about 1 and 150 ns and of a magnitude as high as 5 A (e.g., Amerasekera et al., 1992).

This section summarizes the theory and experiments that are used to study heat conduction in electronic microstructures. The following subsection describes simulation approaches for the temperature fields in microstructures, including the basics of solid-state theory. After that, theory and measurement approaches for the effective thermal conductivities of thin electronic films are presented. Finally, this section provides an overview of the available metrology for determining thermal conductivities of thin films and temperature distributions in microstructures.

Simulation Hierarchy for Solid-Phase Heat Conduction

Heat conduction governs the temperature distributions in many micromachined electronic structures. For many macroscopic heat conduction problems, it is possible to obtain accurate predictions without knowing the mechanism responsible for heat transport. For isotropic materials, the heat flux is related to the temperature gradient by Fourier's law,

$$\mathbf{q}'' = -k\nabla T \qquad (4.19.1)$$

where k is the thermal conductivity. Equation 415 a car be used to acculately aredict the temperature distribution in macroscopic structure, viscout knowledge of what is carrying the heat, as long as the appropriate values of the thermal conductivity are a ken from experimental data.

The value of (b) conductivity can be a turnet d by considering the microscopic mechanism of a mpo this childepends on the medium. No ecular motion carries heat in gases, for example, for which the kinetic theory yields e.e. Vir emi and Kruger, 1986; Rohsenow and Choi, 1961)

$$k = \frac{1}{3}Cv\Lambda \tag{4.19.2}$$

The conductivity is proportional to the heat capacity per unit volume C [J m⁻³ K⁻¹], the mean molecular speed v, and the mean free path Λ of molecules between consecutive collisions. In solid metals and many metallic alloys, heat conduction is dominated by the motion of electrons. In dielectric and semiconducting materials, the coupled vibrations of atoms are responsible for heat conduction. Quantum theory accounts for atomic vibrational energy in discrete units called phonons, which can be visualized as packets of energy traveling at speeds near that of sound in the material. Conduction in solids can be simulated using the kinetic theory of gases with expressions analogous to Equation 4.19.2 for electrons and phonons.

Accurate simulations of microstructures must consider the heat conduction mechanism. Figure 4.19.6 summarizes the hierarchy of simulation approaches for the semiconducting and dielectric regions in electronic microstructures. The regimes of applicability are distinguished using the mean free path Λ and the wavelength λ of phonons, which are the dominant heat carriers in these materials. In semiconducting crystals such as silicon, the phonon mean free path and wavelength near room temperature are approximately 100 and 1 nm, respectively. Many simulations can effectively use the continuum heat diffusion theory based on Fourier's law, Equation 4.19.1, if care is taken to use accurate values of the thermal conductivities and interface resistances of the constituent films. Phonon transport theory becomes important when the dimensions of a structure are comparable with the phonon mean free path. For this

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Figure 4.19.8 Dispersion relationship and group velocity for the plane and of atoms in Figure 4.19.7a.

In a three-dimensional crystal, the group propagation of sound. long waveleng hs For wavelengths comparable quations 4.19.5 and 4.19.6 show that the n he ir atomic spacing group velocity approact t A orighboring atoms oppose each other and are zero because the vior 01 an 2a, it can be shown that the atomic displacements out of pha **2**. For wavelengths sh for with wavelength longer than 2a (e.g., Kittel, 1986). The with wavenumbers k_w the pore-bound the useful dispersion relationship, which lies within the first $=\pm\pi$ Brillouin zone.

The next level of complexity for atomistic calculations connects atoms with springs of two different constants, K_1 and K_2 . The atoms lie along a linear chain as in Figure 4.19.7b. This is a more realistic model of the situation within a three-dimensional crystal, in which the basic repeating unit, called the *unit cell*, often includes two or more atoms. In this case, the first Brillouin zone is bounded by $k_w = \pm \pi/2a$ and contains two solutions. The two frequencies and group velocities for the long wavelength limit ($k_w a \ll 1$) are

$$\omega = \sqrt{\frac{2(K_1 + K_2)}{M}}; \quad v = 0 \qquad optical \ branch \qquad (4.19.8)$$

$$\omega = \sqrt{\frac{\left(K_1 K_2\right)}{2M\left(K_1 + K_2\right)}} k_w a; \quad \nu = \sqrt{\frac{\left(K_1 K_2\right)}{2M\left(K_1 + K_2\right)}} a \quad acoustic \ branch \tag{4.19.9}$$

The vibrations with finite frequency in the long-wavelength limit have zero group velocity. This branch of solutions accounts for the motion of two atoms within a given unit cell with respect to each other, approximately 180° out of phase. Because these vibrations can be caused by oscillating electric fields they belong to the *optical branch* of solutions. Equation 4.19.9 presents a limit not dissimilar to that in Equation 4.19.5 and yields a finite group velocity. Because these vibrational waves carry sound in solids, they belong to the *acoustic* branch.

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where N is the density per unit volume of unit cells in the atomic lattice. In the Einstein model, Equation 4.19.13 collapses to

$$C = 3Nk_{B} \left(\frac{\theta_{E}}{T}\right)^{2} \frac{\exp(\theta_{E}/T)}{\left[\exp(\theta_{E}/T) - 1\right]^{2}}$$
(4.19.15)

where the Einstein temperature is $\theta_E = \hbar \omega_E / k_B$. The high-temperature limit is

$$C = 3Nk_B; \ T >> \theta_E \tag{4.19.16}$$

which is consistent with experimental data for the volumetric heat capacity and the law of Dulong and Petit (e.g., Kittel, 1986). However, at low temperatures Equation 4.19.15 yields a specific heat that varies too rapidly with decreasing temperature and strongly underpredicts the experimental data. The problem is that many vibrational states exist with frequencies below ω_E and are more easily accessed and occupied at low temperatures.

The *Debye model* uses a more complicated expression for the density of states, which solves this problem. The model assumes an isotropic linear dispersion relationship, $\omega = v k_w$, where v is equal to both the group and phase velocities of the vibrational waves. The density of states with a fiven polarization is

$$D(\omega) = \frac{\omega^2}{2\pi^4 v^3}; \ \omega \le \omega_5 \qquad (4.19.17)$$

$$D(\omega) = 0; \ \omega > \omega_5 \qquad (4.19.18)$$
The maximum returney ω_D is dictated by Ω_5 and Ω_5 models and ω_5 (4.19.18)
The maximum returney ω_D is dictated by Ω_5 and Ω_5 models and ω_5 (4.19.19)
 $\omega_D = v \sqrt[3]{6\pi^2 N}$

$$(4.19.19)$$

sets the total number of states equal to the number of unit cells. Because there can be multiple atoms per unit cell, the cutoff frequency is often calculated using the total number of atoms per unit volume, N_a rather than N. This approach yields the correct total number of states in the crystal and is reasonably effective at predicting the measured heat capacity. There is a characteristic *Debye temperature* associated with this cutoff frequency,

$$\theta_D = \frac{\hbar\omega_D}{k_B} = \frac{\hbar\nu}{k_B} \sqrt[3]{6\pi^2 N_a}$$
(4.19.20)

Table 4.19.1 provides Debye temperatures, group velocities, unit-cell number densities, and atomic number densities for several solids. The heat capacity is calculated by combining Equations 4.19.13 and 4.19.17 to 4.19.20, which yield

$$C = 9 N_a k_B \left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 \exp(x)}{\left[\exp(x) - 1\right]^2} dx$$
(4.19.21)

where x serves as an integration variable here. Equation 4.19.21 is compared with experimental data for diamond in Figure 4.19.10. The high and low temperature limits of the heat capacity are

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FIGURE 4.19.11 Contributions of the phonon branches to the volumetric key of silicon. (Adapted from M. Asheghi, Stanford University.)

Another approach is to calculate the de explicitly from n relationships, such m 🌢 that $D(\omega)$ varies continuously ∇ evector and is different for each branch. This approach yields in he v av alculating the heat e 4.19.11 plots the result of this approach the greatest precision i ii γU relative contribution \mathbf{f} for silicon oranches to the specific heat. h pasic Properties of all c Metals

Electron transport in solids is modeled by integrating the Schroedinger wave equation subjected to a periodic potential function representing the atomic lattice (e.g., Ashcroft and Mermin, 1976). The solutions yield the electronic band structure in the material, which describes the variation of the electron energy with the electron wavevector. The wavevector description of electrons recognizes that they behave as waves while traveling through the atomic lattice. The electronic band structure is analogous to the dispersion relationship for phonons, because it relates the energy and group velocity of electron waves to the direction of propagation and wavelength. A detailed discussion of band structure and electron properties is beyond the scope of this treatment and can be found in basic texts on the solid state.

Here, we are most concerned with electron conduction in metals, in which electrons behave much as free charged particles in a vacuum. In contrast to phonons, which can multiply occupy a given state, the Pauli exclusion principle restricts occupation of electron states to two, with opposite spin directions. At zero temperature the free electrons fill the states with the lowest energy in the system up to the *Fermi energy*

$$E_F = \frac{\hbar^2}{2m} \left(3\pi^2 N_e\right)^{2/3} \tag{4.19.24}$$

where N_e is the number density of free electrons in the metal and $m = 9.10956 \times 10^{-31}$ kg is the free electron mass. Because of the Pauli exclusion principle, only those electron states with energy close to the Fermi level can contribute significantly to heat conduction. Electrons occupying states with lower energies travel within the lattice but yield no net charge or heat current because the states with opposite direction are occupied as well. This makes the velocity of electrons near the Fermi level very important,

The gradient operators yield derivatives in physical space, described using the subscript **r**, and wavevector space, described using the subscript \mathbf{k}_w . The group velocity **v** of particles usually depends on the wavevector \mathbf{k}_w and has magnitude v. The time rate of change of the distribution function (first term on left) is caused by spatial gradients in the distribution function (second term on left), external forces that change the wavevector (third term on left), and by scattering (first term on right). The third term on the left is only relevant for electrons, for which an electric field vector **E** induces changes in the electron state according to

$$\frac{\partial \mathbf{k}_{\mathbf{w}}}{\partial t} = \frac{e}{\hbar} \mathbf{E} \tag{4.19.32}$$

where $e = 1.60219 \times 10^{-19}$ C is the proton charge. The scattering term on the right of Equation 4.19.31 can be expressed as an integral over initial and final states for the electrons or phonons involving the probabilities for the transitions and the distribution functions (e.g., Zimon, 1960). This integral is often modeled using the equilibrium distribution function and the relaxation time approximation,

$$\left(\frac{\partial f}{\partial t}\right)_{S_{cat}} = \frac{f_{EQ} - f}{\tau} \equiv \frac{-f_{DE}}{\tau}$$
(4.19.33)

The equilibrium distribution function f_{EQ} at the local temperature is calculated using either Equation 4.19.11 for phonons or Equation 4.19.28 for electrons. It may be need that for electrons, the value of f must lie between zero and unity. For phonons, the orthogona number can be arbitrarily large and f is positive and unbounded. The distribution function $f_{DE} = f - f_{F0}$ is the distribution of the particle system from equilibrium. The relaxation line τ describes the rate at which is number of particles in the state would return to equilibrium on the absence of spatial gradients. Because there are a variety of scattering mechanisms helpful to break the canoring rate τ^{-1} into components using *Matthiessen's rule*. $\tau^{-1} = \sum_{i} \tau_{j}^{-1}$ (4.19.34)

The sum in Equation 4.19.34 is over all of the significant scattering mechanisms. For phonons, the summation accounts for phonon-phonon scattering, phonon scattering on imperfections or impurities, and phonon-electron scattering (Ziman, 1960). For electrons, the summation is dominated by scattering on phonons and imperfections. Equation 4.19.34 can be augmented to account for scattering on thin-film and grain boundaries using $\tau \sim d/v$, where *d* is the characteristic film or grain size and v is the velocity magnitude. Subsequent sections of this review will show that more detailed models are available for handling scattering due to film and superlattice interfaces and grain boundaries. For superconducting materials, Equation 4.19.34 can be augmented to account for phonon scattering with the fraction of the electron gas remaining in the normal state at a given temperature (Uher, 1990).

The Boltzmann equation can be integrated most simply for the steady state and when spatial derivatives of the departure from equilibrium, f_{DE} , are negligible. The second condition is satisfied when the boundaries do not significantly perturb the distribution function, which essentially requires that the medium is large compared to the particle free path. Under these circumstances, we may write

$$\mathbf{v} \cdot \nabla_{\mathbf{r}} f \cong \mathbf{v} \cdot \nabla_{\mathbf{r}} T \frac{\partial f_{EQ}}{\partial T}$$
(4.19.35)

and

a relatively small influence on conduction. An average of phonon free paths over all directions in a columnar grain is well approximated by

$$\tau_{G}^{-1} = \frac{2\nu}{\pi d_{G}} \left[1 - \exp\left(-\frac{\pi^{2}}{4} \eta_{G}\right) \right]$$
(4.19.69)

which yields a scattering rate with an upper bound dictated by the grain size.

Figure 4.19.22 compares the data for the vertical thermal resistance of thin layers with calculations using Equations 4.19.46 and 4.19.69. The scattering strength is governed by point and extended defects with cross sections taken from research on thicker diamond films (Graebner, 1993). The calculations are reasonably successful for the data included in the plot, which were measured for films deposited at high temperatures. However, agreement for films deposited at lower temperatures requires the use of an internal scattering term, which is independent of the grain size (Touzelbaev and Goodson, 1998). There is only one study, to our knowledge, reporting both the in-plane and out-of-plane thermal conductivities of a given set of diamond films. Verhoeven et al. (1997) observed a particularly large degree of anisotropy for films with predominantly heteroepitaxial grains, with the in-plane conductivity smaller by about one order of magnitude than the out-of-plane conductivity.



FIGURE 4.19.22 Predictions and data for the out-of-plane thermal conductivity of polycrystalline diamond films. (From Touzelbaev, M.N. and Goodson, K.E., *Diamond Relat. Mater.*, 7, 1, 1998, and Verhoeven, H. et al., *Diamond Relat. Mater.*, 6, 298, 1997.)

Amorphous Oxide and Organic Films

Process-dependent material structure and stoichiometry influence the thermal conductivities and volumetric heat capacities of highly disordered films, including those of amorphous glasses and organic materials. The best representative data available at present are for silicon-dioxide and polyimide films. Figure 4.19.16 shows that polyimide films exhibit anisotropic conductivities due to the partial alignment of molecular strands in the film plane, which is sensitive to the spin-coating parameters. The conduction of atomic vibrational energy is more effective by means of the electronic bonds coupling atoms along a molecule than by the forces acting between neighboring molecules. The volumetric heat capacity in amorphous regions. The nature of disorder in this material renders most of the modeling concepts discussed in this review inappropriate at room temperature. The relevant length-scales for observing size effects, such as the anisotropy shown for polyimide films in Figure 4.19.16, are not known. While the molecular weight and the molecular radius of gyration might be assumed to play a role, there has been no systematic study of thermal transport in organic films as a function of these parameters. Past studies of the conductivity focused primarily on demonstrating anisotropy and were not accompanied by detailed spectroscopy. However some progress can be made using the extensive literature on heat conduction in bulk organic materials, which has been reviewed (e.g., Ward, 1975).

The series model can be used to interpret thermal conductivity anisotropy data for organic films. The molecular strands are modeled locally within the material as elongated segments with anisotropic intrinsic conductivities. The longitudinal conductivity $k_{a,f}$ describes the transport of atomic vibrational energy along a strand, and the lower out-of-plane conductivity $k_{a,f}$ is used to model the weaker coupling between neighboring strands. While these two conductivities would be extremely difficult to observe experimentally for a single strand, they can be considered equal to the strand-parallel and strand-normal conductivities in a polymer consisting of perfectly oriented molecules. The anisotropy in the material is then given by (Henning, 1967)



The angle φ is between the run normal and a given strict direction. The operator $\langle \rangle$ yields the average cosine accordeding all strands and φ all dated from an assumed orientation distribution function φ the cosine accorded ing all strands and φ one that for perfectly random orientation. Figure 4.19.24 shows



FIGURE 4.19.24 Predicted impact of the molecular orientation on the thermal conductivity anisotropy in organic films. (Adapted from Kurabayashi, K. and Goodson, K.E., *Proc. 5th ASME/JSME Thermal Eng. Joint Conf.*, San Diego, CA, March 1999.)



The transmission in this model is assumed to be negligible for frequencies above the minimum Debye frequency of the two materials.

The diffuse mismatch model yields reasonably good agreement with experimental data for a wide variety of interfaces below 77 K. At higher temperatures, two complicating effects become important. First, many phonons experience inelastic collisions at the interface, which can augment energy transmission (e.g., Stoner and Maris, 1993). Another complication is the growing importance of nearinterfacial disorder with increasing temperature. Thin films rarely have homogeneous microstructural quality and purity and, in general, the poorer material is concentrated near interfaces. While a detailed simulation might attempt to resolve the varying thermal resistance in volume elements near an interface, a practical compromise is to model the near-interfacial disorder as an interface resistance. Note that this approach assumes that the interface resistance can be modeled independently from the resistance of the volume of that material. If transport normal to a film is important, and the mean free path of programs in the film is comparable to the film thickness, it is no longer possible to isolate resistance contait is the of the interface and the volume. For this situation, it is recommended that calculation, exploy effective film conductivities for a given thickness and direction of transport

Measurement Techniques The small dimensions of electron conclustructures make experimental data particularly important, because the governing actuations and many of the paperoprint elassumptions for simulations are still the subject of rescalar Small dimensions of a make experiments more challenging by requiring new regime of spatial and temporarity oution. This section provides an overview of the experimental experimental conductivities and interference in the subject of the experimental methods. echniques that determine the first ve thermal conductivities and interface resistances of thin electronic films as well as the temperature distributions in electronic microstructures. A comprehensive overview of the film thermal conductivity measurement techniques can also be obtained by reading the articles of Graebner (1993), Goodson and Flik (1994), Cahill (1997), and Goodson and Ju (1999). More information about microscale thermometry techniques can be obtained from the reviews of Majumdar et al. (1996) and Goodson et al. (1998).

Thin-Film Thermal Properties

When measuring film thermal conductivities, care must be taken to obtain data that are appropriate for simulating practical devices. Many techniques use a film structure that does not resemble the functional device and, in many cases, requires processing that alters the purity or structural quality of the film. For example, several techniques measure the in-plane thermal conductivity using free-standing films, which experience a different stress history and are exposed to more contaminants than those within a device. We focus here on techniques that are deemed to be most appropriate for use in conjunction with electronic device design.

The techniques use either lasers or electrical currents for heating, which offer contrasting advantages. Laser heating avoids contact with the sample film, which helps for small samples and when adhesion is a problem. It is nearly impossible to consistently know the precise magnitude of incident laser power absorbed by the film, which renders the absolute magnitude of the temperature response of little use. However, the precise temporal control of intensity, which is made possible through picosecond and nanosecond pulsed lasers and optical modulators, allows thermal properties to be extracted from the in the temperature. This approach was refined using a thermocouple junction at the tip of an atomic force microscope probe by Majumdar et al. (1993; 1996). This technique maintains the tip-surface separation using the force-feedback mechanism of the atomic force microscope, while using the thermocouple signal for measurements of temperature. The data obtained using this approach for semiconductor laser and transistor structures promise unrivaled spatial resolution, well below 100 nm, and have been documented extensively in review articles (e.g., Majumdar et al., 1996). These methods are ideally suited for obtaining images with high spatial resolution. However, thermometry methods based on scanning probe microscopy have proven very difficult to calibrate and, therefore, are most useful for imaging qualitative distributions of temperature.

Optical methods use radiation interaction with either the surface of a device or interconnect, in some cases probed through transparent passivation, or with a specially deposited thin layer on the surface. Far-field laser reflectance thermometry calibrates the temperature dependence of the surface reflectivity of an interconnect or device structure. Because of their small radiation penetration depths in metals are especially well suited for laser-reflectance imaging. The metal yields a precisely defined location for the measurement and minimizes the interaction of the probe beam with the electrical currents flowing in the circuit. This motivated Ju et al. (1997) to include an aluminum layer within the overlying passivation of a SOI power transistor. Figure 4.19.29 shows the temperature distribution across the corner of an aluminum interconnect on an organic passivation layer. The temperature image was captured 100 ns after the initiation of an electrical current pulse in the interconnect, and is compared with a pled analysis of the temperature and potential distributions. Additional optical methods include up weight near-field laser-reflectance imaging (Goodson and Asheghi, 1997), thermin any sturg me Raman effect n Wickramsinghe, 1987), and (Ostermeier et al., 1992), photothermal displacement imaging (Marting) whose differing vantages are discussed fluorescence thermal imaging (Barton and Tangy in the review of Goodson et al. (1998)



FIGURE 4.19.29 Temperature distribution across the corner of a patterned aluminum alloy interconnect subjected to a brief electrical current pulse. (Adapted from Ju, Y.S. and Goodson, K.E., *IEEE Electron Device Lett.*, 18, 512, 1997b.)

Summary

The decreasing dimensions of micromachined electronic structures motivate research on new regimes of simulation and measurement tools for heat conduction. This section has provided an overview of the simulation methods together with key elements of the solid-state physics needed to understand them. The goal has been to emphasize the most important equations and governing relationships which are used most frequently in practice. This chapter section has also summarized the experimental techniques that are most effective at measuring temperature distributions, thermal conductivities, and interface resistances in electronic microstructures. A particularly large challenge for the near future includes the further integration of many of these techniques into the research and development infrastructure of the semiconductor industry. It is hoped that this overview will facilitate this process.



FIGURE 4.20.2 X-section of high-density interconnect PCBs with and without filled vias. (Courtesy of Motorola.)



where f_i is the fractional toward e of copper for later i. (1) the 4.20.4 shows the in-plane and through conductivity for 5-2-10 thick 2-layer, 4-4 ger 4-layer, 8-rayer, and 10-layer PCB with $\frac{1}{2}$ oz/ft² copper or the french layers (assuming the one layers consist only of copper and FR-4, i.e., no vias), where me thermal conductivity for the ray is 390 W/mK and thermal conductivity for FR4 is 0.25 W/mK. Figure 4.20.4 indicates that the $K_{in-plane}$ is two orders of magnitude higher than $K_{through}$. Heat flow through the copper layers dominates the heat transfer because of the large differential in the thermal conductivities



FIGURE 4.20.4 Conductivity vs. number of layers for a 55-mil-thick FR-4 PCB with $\frac{1}{2}$ oz/ft² copper on the internal layers.



FIGURE 4.20.10 Power dissipation in typical actively and passively cooled portable computers. The computers compared in the figure have different equipment. Actively cooled portable computers can dissipate more for any temperature specification of the computer casing temperature. (Based on data from Inc., 1990).



FIGURE 4.20.11 Computer surface temperature vs. power dissipation for typical actively cooled and passively cooled portable computer for an ambient temperature of 25°C. The computers compared in the figure have different equipment. (Based on data from Intel, 1998b.)

computers for an ambient temperature of 25°C. The bottom surface of the computer is insulated. The top (keyboard) and side walls are used as a radiator. The notebook casing is made of ABS/PC plastic or a painted metal chassis (emissivity = 0.95) and has an isothermal casing temperature. The data presented in Figures 4.20.10 and 4.20.11 are for an actively cooled portable computer $10 \times 12.3 \times 1.0$ in. base unit (excluding lid thickness) and a passively cooled portable computer with a base size of $8.2 \times 10.2 \times 0.65$ in.

In general, actively cooled portable computers can dissipate more power than passively cooled portable computers and still meet the temperature specification for the computer casing temperature (Figure 4.20.10). While the CPU is not the only component dissipating heat, it is certainly one of the major contributors to an increase in computer surface temperature. Other major contributors are the graphics card, memory, hard drives, and power supplies. For operation at 15°C above ambient temperature (25° C), about 15 to 17 W of total platform power (entire notebook power minus the display power) can be managed passively in a typical notebook. The allowable power dissipation for an actively cooled portable computer is in the neighborhood of 24 to 25 W (Figure 4.20.11).

Material Properties	Magnesium Alloy	
	AM60A-F	AZ91A-F
Tensile strength — yield (ksi)	16.68	21.76
Poisson's ratio	0.35	0.35
Modulus of elasticity (psi)	6.52e+06	6.52e+06
CTE (ppm/°C)	25.6	26
Thermal conductivity (W/m-K)	62	72
Specific heat (J/g-°C)	1	1.05

TABLE 4.20.12Properties for Typical MagnesiumAlloys Used for Computer Housings

Typical Heat Transfer Schemes

Figure 4.20.12 shows some of the typical active and passive heat transfer schemes used for portable computing applications. Figure 4.20.12 (left) shows a passive heat transfer scheme using a heat pipe along with a heat spreader to conduct the heat from the microprocessor. Figure 4.20.12 (right) shows an active heat transfer scheme using a finned heat sink and fan.



FIGURE 4.20.12 Some typical heat transfer schemes for portable computers. (Adapted from Sterner, J., *Proc. IMAPS Int. Sys. Packaging Symp.*, 107-112, 1997.)



FIGURE 4.20.13 Some common fan configurations for AT and ATX form-factor desktop computer cases (Steinbrecher, 1998; AMD, 1998).

In desktop computers, fans are used to evacuate the hot air from the case. Figure 4.20.13 shows some of the common fan configurations in AT and ATX form-factor cases. Case fans come in many flavors, but usual sizes are 80×80 mm and 92×92 mm. The size and location of the fan depends on the



Package Type

FIGURE 4.20.19 Comparison of the thermal fatigue reliability for the partial-array BGA (0.5 mm pitch, 48 I/O), Elastomer-on-Flex package (0.65 mm pitch, 48 I/O), Flex-substrate BGA (0.8 mm pitch, 48 I/O), and Glob-top BGA (1 mm pitch, 196 I/O) vs. OMPACTM (1.5 mm pitch, 68 I/O).

Acceleration Transforms

Environmental usage profile has been developed based on the phone usage profile, typical environmental extremes, and the measured thermal behavior of the radio. The profiles (Figure 4.20.20) have been developed for two distinct environmental conditions — extreme (heat or cold). Nonlinear finite entities simulations have been run to evaluate the field life and correlate the LLTS cycle to correst of field life.



Time (Minutes)

FIGURE 4.20.20 Schematic of temperature profile for extreme hot and extreme cold environments (Data from Lall et al., 1998).

Acceleration transforms have been derived based on nonlinear finite element model predictions. An acceleration transform quantifies the acceleration factor for an accelerated test environment compared to actual field use environments. The acceleration transform is in turn determined by several variables including the component and board geometry, materials, architecture, and most importantly the field profile. Ideally, one would like to have one acceleration transform which can be used to evaluate the field life once the life in accelerated test has been evaluated.

Unfortunately there is no universal acceleration transform — further, there can be no such universal relationship. The reason being that the variables which influence this acceleration transform vary from component to component and, thus, can be derived only for a class of components, e.g., plastic BGAs,

Selected U.S. Electronic Thermal Management Patents

Housing with recirculation control for use with banded axial-flow fans. Yapp, M.G., Needham, MA; Houten, R.V., Winchester, MA; Hickey, R.I., Concord, NH; Patent 5,489,186, Feb. 6, 1996.

Solid-state imaging device having temperature sensor; Miyaguchi, Kazuhisa; Hamamatsu, Jpn, Muraki, Tetsuhiko, Hamamatsu, Jpn; Patent 5,508,740, April 16, 1996.

Active vaned passage casing treatment, Hobbs, D.E., South Windsor, CT; Patent 5,431,533, July 11, 1995.

Appliance for attaching heat sink to pin grid array and socket, Perugini, M.N., Monroe, CT, Romatzick, Milford, D.H. Jr., CT; Patent 5,396,402, March 7, 1995.

Cast casing treatment for compressor blades, Privett, J.D., North Palm Beach, FL, Byrne, W.P., Jupiter, FL, and Nolcheff, N.A., Palm Beach Gardens, FL, Patent 5,474,417, Dec. 12, 1995.

Centrifugal compressor with a flow-stabilizing casting, Heinrich, K., Nussbaumen, Switzerland, Patent 5,466,118, Nov. 14, 1995.

Clip for clamping heat sink module to electronic module, Harmon, R.A., Hudson, MA; and Urrata, G., Wakesfield, MA; Patent 5,381,305, Jan. 10, 1995.

Colored liquid crystal display having cooling, Hyatt, G.P., La Palma, CA; Patent 5,398,041, Mar. 14, 1995. Epoxy molding composition for surface mount applications, Gallagher, M.K., Lansdale, PA; Petti, M.A., North Wale, PA; Patent 5,434,199, July 18, 1995.

Flexible thermal transfer apparatus for cooling electronic components, Tousignant, J.A., Shar view, MN; Patent 5,411,077, May 2, 1995.

Heat sink, Voorhes, D.W., Winchester, MA; Goldman, R.D. Soog to MA; and Lopez, R.R., Boxford, MA; Patent 5,390,734, Feb. 21, 1995.

Heat sink attachment assembly, Jordan, W.D., et al., X, and Smithers at C. Lewisville, TX; Patent 5,396,338, Jan. 31, 1995.

Heat sink assembly for solid state devices, Tate P., Johnton, D., and Rife, W.B., Greenville, RI; Patent 5,397,919 (Mark, 1995).

High externance centrifugal pure having an open-faced impeller, Lehe, J.-R., Vernon, France; Dearran, A., Chamble, France B.s. a, R., Vernon, France; Tiret, E., La Chapelle, Reanville, France; Patent 5,427,498, June 27, 1995.

Liquid crystal display having conductive cooling, Hyatt, G.P., La Palma, CA; Patent 5,432,526, July 11, 1995.

Organic coatings with ion reactive pigments especially for active metals, Mosser, M.F., Sellersville, PA; Harvey, W.A., III, MontClare, PA; Patent 5,409,970, April 25, 1995.

Powder epoxy resin coating composition, Matsuzaki, K., Yono, Japan; Ono, K., Tokyo, Japan; Iwamoto, S., Soka, Japan; Osa, M., Saitama, Japan; and Watanabe, T., Soka, Japan; Patent 5,418,265, May 23, 1995.

Ultrathin film thermocouples and method of manufacture, Rimai, D.S., Webster, NY, Anne, R.E., Caledonia, NY; and Bowen, R.C., Rochester, NY; Patent 5,411,600, May 2, 1995.

Custom conformal heat sinking device for electronic circuit cards and methods of making the same, Santilli, M.A., Lake Shastina, CA; and Morgan, W.Z., Dallas, TX; Patent 5,403,973, April 4, 1995.

Active thermal isolation for temperature responsive sensors, Martinson, S.D, Hampton, V., Gray, D.L., Newport News, VA; Carraway, D.L., Virginia Beach, VA; and Reda, D.C., San Jose, CA; Patent 5,311,772, May 17, 1994.

Air distribution system and manifold for cooling electronic components, Bartilson, B.W., Chippewa Falls, WI and Jirak, J.J., Jim Falls, WI; Patent 5,321,581, June 14, 1994.

Apparatus for an air impingement heat sink using secondary flow generators, Lindquist, S.E., Boylston, MA and Bailey, D.A., Concord, MA; Patent 5,304,845, April 19, 1994.

Circuit with built-in heat sink, Mehta, A.M., Plantation, FL and Desai, V.D., Plantation, FL; Patent 5,307,519, April 26, 1994.



FIGURE 4.21.2 Relationship between nondimensional gas velocity and average Nusselt number, $U_{\infty} = gas$ velocity, $U_0 =$ liquid velocity at inlet (From Jacobs, H.R. and Dolan, S.M., *J. Energy, Heat Mass Transferrent* 77, U.94. With permission.)



FIGURE 4.21.3 Effect of U_{∞} changes on frictional drag.

Nu =
$$\sqrt{\frac{6}{\pi}} \operatorname{Re}^{12} \operatorname{Pr}^{1/2}$$
 (4.21.1)

where $\text{Re} = \frac{uD}{v}$ is the film thickness and u is the mean film velocity. For a wavy film the heat transfer coefficient will be increased by 15% (Levich, 1962) based on an assumed similarity to gas absorption. Similarly for turbulent films, mass transfer analogs should offer some assistance although there exist few controlled experiments (Levich, 1962).

4.22 Temperature and Heat Transfer Measurements

Robert J. Moffat

There are two different kinds of material to consider with respect to experimental methods: the unit operations of measurement (transducers and their environmental errors) and the strategy of experimentation. This section deals only with the unit operations: transducers, their calibrations, and corrections for environmental errors.

Temperature Measurement

An International Practical Temperature Scale (IPTS) has been defined in terms of a set of fixed points (melting points of pure substances) along with a method for interpolating between the fixed points. The IPTS agrees with the thermodynamic temperature scale within a few degrees Kelvin over most of its range. The IPTS is the basis for all commerce and science, and all calibrations are made with respect to the IPTS temperature. The scale is revised periodically.

Accurate calibrations are not enough to ensure accurate data, however. If a sensor has been installed to measure a gas temperature or a surface temperature, any difference between the sensor temperature and the measurement objective due to heat transfer with the environment of the sensor is an "error" In most temperature-measuring applications, the environmental errors are far larger than the calibration tolerance on the sensor and must be dealt with just as carefully as the calibration

Thermocouples

Any pair of thermoelectrically dissimilar materials can reduce as a thermocoupte. The pair need only be joined together at one end and connected to a charge-measuring inclusion to the other to form a usable system. A thermocoupte de (e) pairs signal in response to the temperature difference from one end of the pair to the other. The temperature at one mit, k own as the *reference junction* end, must be known accurate to be the temperature at one mit, k own as the *reference junction* end, must be known accurate to be the temperature at one mit, k own as the *reference junction* end, must be known accurate to be the temperature at one mit, k own as the *reference junction* end, must be known accurate the most of monly used electrical output sensors for temperature measurement. With different material for the temperature ranges, thermocouples have been used from cryogenic temperatures (a few Kelvin) to over 3000 K. In the moderate temperature range, ambient to 1200°C, manufacturer's quoted calibration accuracy can be as good as $\pm 3/8\%$ of reading (referred to 0°C) for precision-grade base metal thermocouples. Broader tolerances apply at very high temperature and very low temperatures.

Thermocouple signals are DC voltages in the range from a few microvolts to a few tens of microvolts per degree C. Because of their low signal levels, thermocouple circuits must be protected from ground loops, galvanic effects, and from pickup due to electrostatic or electromagnetic interactions with their surroundings. Thermocouples are low-impedance devices. Multiple channels of thermocouples can be fed to a single voltage reader using low-noise-level scanners or preamplifiers and electronic multiplexers.

The alloys most frequently used for temperature measurement are listed in Table 4.22.1. These alloys have been developed, over the years, for the linearity, stability, and reproducibility of their EMF vs. temperature characteristics and for their high-temperature capability.

Calibration data for thermocouples are periodically reviewed by the National Institutes of Science and Technology based on the then-current IPTS. Values in Table 4.22.1 illustrate the approximate levels which can be expected, and are from the National Bureau of Standards Monograph 125. Maximum temperatures listed in this table are estimates consistent with a reasonable service lifetime. Allowable atmosphere refers to the composition in contact with the thermoelements themselves. Accuracy estimates are provided for two levels of precision: standard grade and precision grade where these data are available.

Noble metal and refractory metal thermocouples are often used with substitute lead wires, as a costsaving measure. These lead wires, described in Table 4.22.2 are cheaper and easier to handle than the high temperature thermocouples. They have the same temperature–EMF characteristics as their primary thermoelements, but only over the range of temperatures the lead wires will usually encounter (up to a few hundred degrees C). Except for the substitute alloys, thermocouple extension wires have the same

- R = radius of the disk, m k = thermal conductivity of the disk, W/m·C
- t = thickness of the disk, m
- q'' = heat flux absorbed by the disk, W/m² (must be uniform)

The output signal is thus directly proportional to the heat flux on the disk. Cooling passages are frequently built into the support structure to maintain the edge of the disk (the heat sink for the foil disk) at constant temperature.

Calibration

Calibration of the Gardon-type heat flux meters is most easily done by comparison, using a radiation calibrator.

Planar gauges can be calibrated either by conduction or radiation, but the results will depend on the calibration method for some guages.

Sensor Environmental Errors

Temperature sensors generate signals in response to their own temperatures, but are usually installed to measure the temperature of some fluid or solid. There is heat transfer between the sensor and all of its surroundings, with the result that the sensor usually equilibrates at some temperature different first the fluid or solid it is installed in. This difference is considered an error in the measurement

Similarly, heat flux gauges are generally installed so one can infer the text flux which would have been there had the gauge not altered the system behavior. But heat the tauges do disturb the system, and the heat flux at the gauge location, when the gauge is the entry be significantly different from that which would have been there without the gauge. The system disturbance of the most also be considered an error.

Steady-State Errors in Gas Temperature Vetas frime.

All impressed-type temperature sensors (then accouples, resistance detectors, and thermistors) are subject to the same environmentation of which are frequently larger than the calibration errors of the sensors. Large probes are usually affected more than small ones; hence, RTDs and thermistors (selected by investigators who wish to claim high accuracy for their data) are more vulnerable to environmental errors (due to their larger size and their self-heating errors). This aspect of accuracy is sometimes overlooked.

Sensor installations for gas-temperature measurements should be checked for all three of the usual steady-state environmental errors: velocity error, radiation error, and conduction error. The same equations apply to all sensors, with appropriate dimensions and constants.

$$E_{v} = (1 - \alpha) \frac{V^{2}}{2g_{c} J c_{p}}$$
(4.22.11)

radiation error:

$$E_r = \frac{\sigma \varepsilon}{h} \left(T_{\text{sens}}^4 - T_{\text{surr}}^4 \right)$$
(4.22.12)

conduction error:

$$E_{c} = \frac{T_{\text{gas}} - T_{\text{mount}}}{\cosh\left[L_{\sqrt{\frac{hA_{c}}{kA_{k}}}}\right]}$$
(4.22.13)

where $E_v =$ velocity error, °

 α = recovery factor, —

V = velocity, ft/sec

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esale.co.uk FIGURE 4.22.14 System disturbance errors caused by an attached thermocouple (worst case).

- h = heat transfer coefficient, TC to fluid, W/m, °C
- D = outside diameter of TC, m
- k_w = effective thermal conductivity of
- $k_{\rm s}$ = thermal conductivity of s

cident readiation and the heat transfer coefficient The effective gas tempe ire as

m

$$Pre Page T_{\infty}^* = T_{\infty} + \frac{\alpha GA_R}{hA_C}T_{\infty}^*$$
(4.22.14)

where T_{∞} = actual gas temperature, °C

- α = absorptivity of the TC for thermal radiation
- G = incident thermal radiation flux, W/m²
- A_{R}/A_{c} = ratio of irradiated surface to convective surface
 - h = heat transfer coefficient between the TC and the gas, W/m°C

Steady-State Errors in Heat Flux Gauges for Convective Heat Transfer

If the gauge is not flush with the surface, it may disturb the flow, and if it is not at the same temperature as the surface, it will disturb the heat transfer. Thus, the gauge may properly report the heat flux which is present when the gauge is present, but that may be significantly different from the heat flux which would have been there if the gauge had not been there.

For planar gauges, both effects are usually small. The thermal resistance of such a gauge is generally small, and they are thin enough to avoid disturbing most flows. Circular foil gauges pose a more serious problem, since they are often cooled significantly below the temperature of the surrounding surface. Dropping the wall temperature at the gauge location can significantly increase the local heat load in two ways: one due to the fact that, for a given value of h, a cold spot receives a higher heat load from the gas stream. The second effect arises because the value of the heat transfer coefficient itself depends on the local wall temperature distribution: a local cold spot under a hot gas flow will experience a higher heat transfer coefficient than would have existed had the surface been of uniform temperature.

den Ouden, C. and Hoogendoorn, C.J., Local convective heat transfer coefficients for jets impinging on a plate: experiments using a liquid crystal technique, in *Proc. of the 5th Int. Heat Transfer Conf.*, Vol. 5, AIChE, New York, 1974, 293–297.

Personal Communication from Erwin Meinders, March 1996. Work in progress at the Technical University of Delft under Prof. Hanjalic.

Akino, N. and Kunugi, T., ASME HTD, Vol. 112, 1989.



4.23 Flow Measurement

Jungho Kim, Sherif A. Sherif, and Alan T. McDonald

Introduction

Flow measurements are perhaps the most common type of measurements made. Many types of flow measuring devices are available. The choice of meter type depends on the required accuracy, range, cost, ease of reading or data reduction, and service life. The simplest and cheapest device that gives the required accuracy should be chosen.

Flow measurement can be classified into four broad types: (1) direct methods, (2) flow restriction methods, (3) linear flow meters, and (4) traversing methods. Each of these are discussed in the subsections below.

Direct Methods

Tanks can be used to determine the flow rate for steady liquid flows by measuring the volume or mass of liquid collected during a known time interval. If the time interval is long enough, flow rates may be determined precisely by using tanks. Compressibility must be considered in gas volume measurements. It is not practical to measure the mass of gas, but a volume sample can be collected by one of an inverted "bell" over water and holding the pressure constant by counterweights. No call ration is required when volume measurements are set up carefully; this is a great advance of a great advance of the state.

Restriction Flow Meters for Flow in Such

Most restriction flow meters toriat in 1 flow (except the lamina flow dement) are based on acceleration of a fluid stream thread is some form of nozzle, new mechanically in Figure 4.23.1. Flow separating from the shart efficient are nozzle throat form a construction zone shown by the dashed lines downstream in a the rozzle. The main flow small continues to accelerate from the nozzle throat to form a vena contracta at Section 2 no accelerates again to fill the duct. At the vena contracta, the flow area is a minimum, the flow streamlines are essentially straight, and the pressure is uniform across the channel section. The theoretical flow rate is

$$\dot{m}_{\text{theoretical}} = \frac{A_2}{\sqrt{1 - (A_2/A_2)^2}} \sqrt{2\rho(p_1 - p_2)}$$
(4.23.1)



FIGURE 4.23.1 Internal flow through a generalized nozzle, showing control volume used for analysis.

Equation 4.23.1 shows the general relationship for a restriction flow meter: Mass flow rate is proportional to the square root of the pressure differential across the meter taps. This relationship limits the flow rates that can be measured accurately to approximately a 4:1 range.



FIGURE 4.23.6 Permanent head loss produced by various flow metering elements. (From Miller, R. V. Now Measurement Engineering Handbook, 3rd ed., McGraw-Hill, New York, 1996. With perm store

Linear Flow Meters

The output of flow meters is typically calibrated to reach a volume flow are who procified liquid or gas under certain conditions. For example, he volume flow rate of a liquid could be expressed in terms of liters per minute (LPM) of wate, while that of a gas could be expressed in terms of standard cubic feet per minute (SCFFC) of all. When using flow meter is determine flow rate at conditions different from us tall bottom conditions, corrections to the output must be made to obtain the true flow rate. **Rotameters**

A typical rotameter consists of a float contained within a diverging tube (Figure 4.23.7). The float is raised by a combination of drag and buoyancy. The height to which the float rises in the tube can be made to vary by varying the shape of the diverging tube — tubes with linear or logarithmic variations can easily be fabricated.



FIGURE 4.23.7 Schematic of rotameter operation.

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viscosity. A field calibration should be performed when the working fluid has a viscosity significantly higher than that for water.

Paddlewheel flow meters are an inexpensive alternative to turbine flow meters when extreme accuracies are not required. The rate of rotation of a paddlewheel partially immersed in the flow is used as a measure of the flow rate. Unlike turbine flow meters, paddlewheels can be used in dirty flows.

Positive Displacement Meters

Positive displacement meters measure flow rate by separating the flow into discrete increments that are moved from one side of the flow meter to the other. An example of a positive displacement meter is shown in Figure 4.23.9. Positive displacement meters can be used to measure the flow rate of high-viscosity fluids.



FIGURE 4.23.9 A cutaway of a positive displacement meter. (Courtesy of Smith Meter, Erie, PA.)

Vortex Shedding Meters

When a bluff body is placed in a flow, vortices can be shed downstream of the body (Figure 4.23.10). Vortex shedding meters measure the frequency of the asymmetric pressure variations that occur on the body as the vortices are shed and relate this to the flow rate. The vortex shedding frequency is governed by the Strouhal number (St)



FIGURE 4.23.10 Vortices shedding off a body placed in a moving stream.

$$St = \frac{f_s d}{V} \tag{4.23.17}$$

- where f_s = vortex shedding frequency
 - d = characteristic dimension of the body
 - V = velocity of the flow

Experiments have shown that St is constant within 1% for Reynolds numbers between 10^4 and 10^6 . The frequency is directly proportional to the velocity (and thus the flow rate) around the body in this range. These flow meters are insensitive to dirty flows and can be used under a wide variety of conditions because they have no moving parts.

Thermal Mass Flow Meters

These determine flow rate of clean gases by passing the fluid through a precision tube that is heated in one section, and the temperature of the tube wall upstream and downstream of the heated section is measured (Figure 4.23.11). The difference in temperature between the two sensors is a measure of the mass flow rate. Calibration is usually performed using nitrogen and a correction factor is applied for other gases.



sensor such that its temperature is kept a fixed amount above the upstream sensor. The feedback circuit needed to do this is similar to that used in hot-wire anemometry. The amount of power supplied to the downstream sensor is a measure of the gas flow rate. The advantages of these meters are that they give a direct indication of the flow rate without requiring corrections for temperature or pressure variations, they have no moving parts, and are noninvasive.

Ultrasonic Flow Meters

These meters determine flow rate by measuring the Doppler shift of an ultrasonic signal due the movement of particles in the flow (Figure 4.23.12). Particles or bubbles in the fluid are usually required. Recently, meters have been developed that can measure the flow of clean fluids by measuring the velocity of



FIGURE 4.23.12 Schematic of ultrasonic flow meter.

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Principle of Operation

The basic setup for a popular type of LDV system is shown on Figure 4.23.22. A single, polarized, continuous-wave laser is split into two parallel beams using a beam splitter. The beams pass through a focusing lens that causes the beams to intersect in the focal plane of the lens, simultaneously reducing the beam diameter. The light beams approximate cylinders in the intersection region. The two beams constructively and destructively interfere with each other in the intersection regions, setting up alternating bright and dark fringes that are parallel to the axis of the laser beams. A particle moving through the measuring volume will scatter light, with the intensity of the scattered light depending on whether or



FIGURE 4.23.22 Basic setup for two-component LDV: (a) schematic of a dual beam system, (b) details of the beam crossing. (Courtesy of TSI Inc., St. Paul, MN.)

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Photodetectors

Two types of photodetectors are generally used to convert the scattered light collected by the collecting lens to an electronic signal. In the photomultiplier, photons strike a coating of photoemissive material causing electrons to be emitted. These electrons are accelerated by a high-voltage field and impact onto a dynode that emits even more electrons. This amplification process is repeated, resulting in final gains of 10³ to 10⁷. Frequency responses up to 200 MHz can be obtained using photomultipliers.

Photodiodes are light-sensitive semiconductor junctions. An amplification process similar to that in PMTs occurs in photodiodes, but gains are limited to about 10^3 . The frequency response tends to be lower as well. Photodiodes are much less expensive than photomultipliers, however.

Signal Processing

A typical output signal from a photodetector is shown on Figure 4.23.24. It is seen to consist of a pedestal along with the Doppler signal. Most commercial LDV systems currently on the market use FFT or correlation methods to determine the frequency.



It is important that the perfecte be able to follow the flow. Consider a spherical particle placed in a uniform flow. The drag force on the particle is due to the difference in speed between the particle and the carrier fluid, i.e.,

Force =
$$ma = m \frac{dU_{\infty}}{dt} = \rho_P \frac{\pi D_P^3}{6} \frac{dU_{\infty}}{dt} = Drag$$
 (4.23.44)

The drag on a spherical particle is given by

$$Drag = C_d \frac{1}{2} \rho_{\infty} \frac{\pi D_p^2}{4} (U_{\infty} - U_p)^2$$
(4.23.45)

where

$$C_{d} = \frac{24}{\text{Re}_{p}} = \frac{24\nu}{D_{p}(U_{\infty} - U_{p})}$$
(4.23.46)

for $0 < Re_P < 100$. Combining Equations 4.23.44, 4.23.45, and 4.23.46 and integrating yields

$$\frac{\left(U_{\infty} - U_{P}\right)}{U_{\infty}} = \exp\left(-t/K\right)$$
(4.23.47)

The result of the imaging system is to produce two sets of markers that are displaced in time. The two sets can be on a single sheet of film, or can be recorded separately on different frames of a video camera. An example of two sets of markers is shown on Figure 4.23.26a.



(a)



FIGURE 4.23.26 Example of (a) double-exposed image of particle shifted from one another (b) the corresponding spatial autocorrelation. (Courtesy of Ken Kiger, University of Maryland courts e Park, MD.)

Interrogation System

ded up into small sub-areas over To obtain the velocity vector, the posed image is first div Il be compree The in f the sub-areas should be as small as which the average velocity possible to obtain hghest number 🖊 el tors, but they should not be so small that an Jty under of particle pain results. good rule of thumb is to choose a sub-area size that contains about 10 problems in position of each of the particles is then determined, and a spatial autocorrelation performed (Figure 4.23.26b). The location of the peaks in the spatial correlation plane yields the average particle displacement over that sub-area. The largest peak results from a self-correlation and is not of interest. Two other peaks, one in the positive direction and the other in the negative direction, indicate the possible average particle displacement over that sub-area.

Flow Direction

It is not possible to determine the *direction* of the velocity vector from a doubly exposed photograph alone. This is not a problem if the main flow direction is known *a priori*, but it does present problems in highly turbulent flows or if flow reversals occur. Image shifting of the second set of particles relative to the first can be used to resolve this directional ambiguity. Suppose the second set of images is shifted by a positive amount. A stationary marker will be displaced by this amount. A moving marker will have an additional displacement as a result of its movement. A shift greater than that expected at the largest negative flow velocity then eliminates ambiguity regarding flow direction, since the second image is always on the positive side of the first image. This technique is analogous to using frequency shifting in LDV measurements to determine the direction of the particle movement.

Rotating mirrors and crystals have been used to produce this shift in images. More recently, cameras have been developed specifically for PIV that electronically shift the first set of images by a certain number of pixels before acquiring the second set, eliminating any additional mechanical complexity.

Viscosity Measurements

Viscometry is the technique of measuring the viscosity of a fluid. Viscometers are classified as rotational, capillary, or miscellaneous, depending on the technique employed. Rotational viscometers use the

many areas of science and engineering outside the field of computer science, especially for complex systems where more traditional methods have failed to be useful. The techniques of artificial neural networks (ANNs) and genetic algorithms (GAs) have been shown to be particularly useful in the thermal sciences. This section is written as an overview of the state-of-the-art in this area. It is fundamentally tutorial in nature so that the reader unfamiliar with these techniques can use it as a point of departure. We will describe the methodology and scope of these methods, and then concentrate on specific applications that illustrate their use. An extensive bibliography is also provided for readers who are interested in pursuing the subject further.

Thermal systems are natural or engineered systems involving temperature differences. The engineering discipline of thermal sciences, particularly heat transfer, specifically deals with the analysis, design, and control of such systems, and has a long history of development in response to the needs in a great variety of applications. Much of the advancement in the field has been the result of first-principle analyses for the simpler phenomena, supplemented by experimental correlations for the more complex ones for which the first principles are either not known or the problem is too complex for their use. Unfortunately, the existing methods of analyses are no longer adequate to deal with many current applications, even though analyses are still critically needed for a variety of reasons, such as, for instance, energy efficiency, environmental concern, optimum design and control, cost, and competitiveness in the market place. While the thermal sciences as a basic engineering discipline must and should continue to develop and advance our quantitative understanding of the fundamental phenomena involved, new methods of analysis must be found in the thermal which can be used as a surrogate for such understanding. This is particularly true in the analysis the control of dynamic thermal systems. Obvious examples in this area are dynamic parformente of complex heat exchangers and storage devices, network dynamics and hydronic contractive mention, thermal rackaging of microelectronic devices, and thermal processing of convention and advanced materols, but to mention a few.

AI and Its Scope

difficult to concept. O ord artificial is used to signify something man-pore ellipsic concept. In fact, one can debate whether machines can accould be to say that it is "the study of how to make computers do AI, though easy took ize, is difficult de, **m** is elligence itself is a actually mink. Working do n tie things which, at the non-off, pople do better" (Rich and Knight, 1991); the "study that encompasses computational techniques for tasks that apparently require intelligence when performed by humans" (Tanimoto, 1990); "the discipline that aims to understand the nature of human intelligence through the construction of computer programs that imitate intelligent behavior" (Bonnet, 1985); or "the study of mental faculties through the use of computational models" (Charniak and McDermott, 1985). Though every definition has its drawbacks, it appears that some elements of a knowledge base, search techniques, and logical reasoning are usually involved in AI and that the techniques usually attempt a simulation of a specific aspect of human or animal behavior. There are also related and overlapping disciplines that go by other names. Soft computing, for example, is a collection of techniques including fuzzy logic, neural networks, genetic algorithms, and probabilistic reasoning that are based on the idea that "intelligent systems should exploit, whenever possible, the tolerance for imprecision and uncertainty" (Bouchon-Meunier et al., 1995; Jang et al., 1997). Knowledge engineering and intelligent information systems are terms that are used when the emphasis is on the data base. Virtual reality, artificial life, machine intelligence and data mining are also related ideas that are being widely used. There is obviously some overlap between all these terminologies that we will not attempt to sort out here. The point is that these techniques are able to deal with complex problems that would be difficult to handle otherwise.

AI has a wide variety of applications in the natural and engineering sciences, among which are pattern recognition, decision making, system control, information processing, natural languages, symbolic mathematics, computer-aided instruction, speech recognition, vision, and robotics. These applications have spawned a large number of usable algorithms, such as optimization and search techniques, that can be used in other fields. Out of the broad set of techniques available, we will, for reasons of space, be concerned with two specific methods that have been shown to be useful in the thermal sciences: artificial

neural networks and genetic algorithms. We will exclude any discussion of topics such as knowledgebased techniques, fuzzy logic, and other AI methodologies.

Why AI?

At the outset one must ask why AI techniques have any role in the thermal sciences. The answer to this lies at the core of the subject, i.e., in heat transfer. Heat transfer, like its sister discipline fluid mechanics, is based on mechanics and thermodynamics and has long been considered as one of the basic engineering sciences. At a fundamental level it is deterministic and well understood; the equations for conduction, convection, and radiation can be easily written down, and the material properties involved can also be measured. So it is proper to discuss why AI and allied techniques are needed at all.

A major application of heat transfer is in devices such as heat exchangers which are one of the basic components of thermal systems. A wide variety of heat exchangers exists in the market and for each it is necessary, for design and selection purposes, to know the heat transfer rate under prescribed operating conditions. For a given device exchanging heat between two fluids, the heat transfer rate depends on the flow rates and the inlet temperatures of each fluid. From the heat transfer rate, secondary quantities such as the fluid outlet temperatures can be determined. Though the governing equations can be written down, it is extremely difficult to solve them numerically under actual operating condition. Among the reasons for this are: complicated heat and fluid flow geometries, turbulence in the flow, existence of hydrodynamic and thermal entrance regions, nonuniform local heat transfer rates and fluid temperatures, secondary flows in the tube bends, vortices in the neighborhood of the tube-fin junctions, all-s the now development in fin passages, heat conduction along tube walls, natural convertion within the tubes and between fins, and temperature dependence of fluid properties.

The computation must take into account the large ratio the ware the largest and the smallest length scales at which computations must be carried out we have the numerical solutions thus, cannot be obtained with reasonable effort. As a consequence of this we are usually forced to account the first-principles approach in favor of empirical information. It must be empirised that each one of the complicating factors can be have been and can even be stierto be understood, but their occurring together is what takes on public difficult.

of a large number of simple subsystems that interact with each other. The whole is more than the sum of the parts so that, even given complete understanding of the subsystems and the laws governing their interactions, it is not a trivial matter to calculate the properties or behavior of the whole. On the other hand, if the interactions between the subsystems are weak or nonexistent, the system is *decomposable* or nearly decomposable, and is much easier to handle.

Empirical information in the form of correlations are commonly used to help in the analysis of thermal systems that are complex. These correlations usually have little physical basis and are proposed to have the simplest form that will give the best accuracy. The vagueness in this procedure gives rise to a variety of correlations for even the most straightforward calculation like, for example, that of turbulent heat transfer in a duct. Furthermore, as our ability to carry out complicated calculations for the purpose of design increases, the definition of the "simplest" form changes. At this point in time, it may be said that simplicity in the correlation may no longer be of concern, but accuracy in the predictions is. In this regard, artificial intelligence has much to offer. It is typically used in instances where the ultimate purpose is to detect and make use of patterns, rather than to uncover new physics.

Heat Exchangers

Since much of the following discussion will center on compact heat exchangers, we will explain this problem a little further. Empirical information about a given heat exchanger is experimentally determined by the manufacturer and is presented to the user, i.e., the thermal system designer, as the heat transfer rate as a function of the four control variables: the two flow rates and the two inlet temperatures. Being a four-variable function, it is difficult to represent completely, though partial information can be provided in the form of charts or graphs. In principle, this function depends on the geometry of the heat exchanger,

was allowed to vary was the fin spacing, as it is the most important physical variable in such heat exchangers. On the other hand, the experimental data covered a wide range of operating conditions on the air side. McQuiston (1978b) correlated his data in terms of three air-side heat transfer coefficients, nondimensionalized as Colburn *j*-factors. These are the sensible heat j_s , the latent heat j_m , and the total heat j_t defined by

$$j_s = \frac{h_a}{G_c c_{p,a}} P r_a^{2/3}$$
(4.24.16)

$$j_m = \frac{h_{m,a}}{G_c} S c_a^{2/3}$$
(4.24.17)

$$j_t = \frac{h_{d,a}}{G_c} S c_a^{2/3}$$
(4.24.18)

where h_a is the heat transfer coefficient, and $h_{m,a}$ and $h_{d,a}$ are the mass transfer coefficients associated with the *j*-factors, G_c is the air-flow mass flux, $c_{p,a}$ is the specific heat of air, and Sc_a is the Schmidt number of air defined by $Sc_a = v_a/D_a$, where v_a is the kinematic viscosity of air and D_a is the mass diffusivity of water vapor in air. The *j*-factors were correlated as functions of Reyn 1d numbers Re_D and Re_δ and the geometrical factors $(4/\pi)(x_a/D_h)(x_b/D)\sigma_f$ and $\delta/(\delta - t)$, we can

$$\begin{array}{c} \text{Preview from } \mu_{a} \\ \text{Preview from } \mu_{a} \\ \text{Page } \mu_{a} \\ \mu_{a} \\ \mu_{a} \end{array} \tag{4.24.19}$$

D is the tube outside diameter, δ is the fin spacing, and μ_a is the dynamic viscosity of air. Also x_a and x_b are the two tube-center distances shown in Figure 4.24.8, D_h is the hydraulic diameter, *t* is the fin thickness, and σ_f is the ratio of the free-flow area of air to the frontal area.

The McQuiston (1978b) correlations are

$$j_s = 0.0014 + 0.2618 Re_D^{-0.4} \left(\frac{4}{\pi} \frac{x_a}{D_h} \frac{x_b}{D} \sigma_f\right)^{-0.15} f_s(\delta)$$
(4.24.21)

$$j_{t} = 0.0014 + 0.2618 Re_{D}^{-0.4} \left(\frac{4}{\pi} \frac{x_{a}}{D_{h}} \frac{x_{b}}{D} \sigma_{f}\right)^{-0.15} f_{t}(\delta)$$
(4.24.22)

where

$$f_{s}(\delta) = \begin{cases} \left(0.9 + 4.3 \times 10^{-5} Re_{\delta}^{1.25}\right) \left(\frac{\delta}{\delta - t}\right)^{-1} & \text{dropwise condensation} \\ 1.0 & \text{dry surface} \\ 0.84 + 4.0 \times 10^{-5} Re_{\delta}^{1.25} & \text{film condensation} \end{cases}$$
(4.24.23)



FIGURE 4.24.15 Closed-loop feedback control system and its response to a disturbance for K3 = 0.01.

constant K3 so that this disturbance is damped out and the system stabilizes at the reference air inlet temperature. As a first approximation, the steady-state energy balance between the heat given up by the water and that absorbed by the air gives the constants K1 and K2. Figure 4.24.15 also shows the response of the system to a disturbance of 5.56° C of the air inlet temperature at t = 2 and with K3 = 0.01. Here the time scale is arbitrary because of the way K1 and K2 are assigned. However, when K3 is increased, the system responds much more slowly. On further increasing K3, the system eventually becomes oscillatory and, thus, uncontrollable.

Additional Applications in Thermal Engineering

In the past applications of ANN to engineering problems have been attempted in structural engineering and engineering mechanics (Zeng, 1998). Tentative studies of applying ANN to problems in thermal systems have been carried out quite recently, with a relatively short history. With the exception of neural network system and control systems applied to HVAC systems, the studies have been somewhat sporadic and in only some distinct areas of application. For heat transfer data analysis and correlation, an ANN-based methodology was proposed by Thibault and Grandjean (1991) and a similar methodology was introduced by Jambunathan et al. (1996) to predict coefficients of heat transfer in convective-flow systems using liquid crystal thermography. Both steady and unsteady heat conduction problems were treated by

General Discussion

It must be emphasized that the implementation of ANN predictions in an industrial setting is really very simple and straightforward. For instance, once the network is trained for a given device by the manufacturer, it can simply be transmitted to the intended customer by giving him or her the sets of weights and biases and the corresponding software for the appropriate network configuration. The customer can then independently carry out the neural computing. An example of the weights and biases for heat exchanger 1 for the 4-2-1 ANN configuration is given in Table 4.24.5. It should be mentioned that the training process of an ANN is somewhat computing intensive, even though not excessive in terms of the computing resources available today, and all computations can be readily handled by personal workstations. However, once the training is completed, the predictions using the finalized sets of synaptic weights and biases can be carried out on any personal computer.



There are several inherent and excellent attributes of the ANN approach which the traditional analysis, including the dimensionless correlations of the experimental data, simply does not have. The neural network, by its ability for pattern recognition, is capable of capturing all the relevant physics, but some of this physics is often not accounted for in the common data correlations by dimensionless parameters. It should be realized, too, in this regard, that since the effects of properties, which represent part of the physics, are expected to show up in the input-output relations in the training data for the ANN, there is no need to explicitly use such properties in the ANN analysis, while such use is necessary in the traditional analysis. Unfortunately, in many applications such properties, which may be either intrinsic or apparent, may not be known or not known accurately, and thus may often lead to difficulties. Furthermore, the complexity in the physics is not an issue in the ANN approach, while such complexity is often the critical barrier in the traditional analysis. Also, it is a relatively simple matter, by continuing training with new data, to adapt to changing conditions, a situation which often exists in real-world applications. The traditional analysis is not capable of doing so, since the physical mechanisms responsible for such changing conditions are not known. In addition, dynamic thermal-system models, which are needed for developing optimal control strategies are difficult to develop by the traditional analysis even for simple systems, and are readily accessible by the ANN approach. On the other hand, there are several shortcomings relative to ANN implementations, but these can be remedied by adequate numerical experimentation, past experience in similar applications, or continuing research in the field of ANN.

The attractiveness of the ANN approach has been demonstrated in the preceding subsections by considering the static and dynamic performances of typical compact heat exchangers. Such heat exchangers are already in common use today, and the traditional analysis has not yet produced satisfactory theories to predict their behavior. Even though the applications of the ANNs have been demonstrated here by considering them as examples, the potential applications in other thermal systems are essentially unlimited, since thermal systems still have not been analyzed by the ANN approach to any large extent. In order to promote its application to thermal science and engineering problems, it would be desirable to look at application areas in which the ANN approach would make significant impacts. In this regard, the most significant areas lie where the lack of physical models, system complexities, and unavailability of properties or constitutive relations represent critical barriers to the advancement of the field in terms of product development, process development and control, manufacturing, sensitivity and risk analyses, and other similar areas which involve simulation, design, and control. This is particularly imperative in those areas where there are plentiful experimental data that can be used to train the networks. To give some indication to this vast potential, several example of critical technology areas can be mentioned for the applications of the ANN approach.

Thermal Networks

The focus so far has been on components, especially on heat exchangers. However, even if the behavior of a single heat exchanger could be accurately predicted, it would be difficult to do so for a large network of these devices as are commonly used in practice. Thermal networks, as used in heating and c oling systems in a single building or in a collection of buildings, may consist of chiled value for cooling purposes, steam for heating, air for ventilation or, more commonly commution of these. Current design is heavily based on overspecification so that it is to must be find simultaneous heating and cooling to obtain the desired comfort level, especially during periods of charge in weather. A hydraulic network may consist of a large number of later connected pipes with values theat exchangers, pumps, and other components online three simplest network, problem consists in determining the flow in each branch of the network Methods based on me solution of the network with the based on me solution of the network of t network a v id is have been developed to set this problem. The control of the flow in a given branch P charge of pump or value parameters is a more difficult problem. Though one can in principle resolve the network for me the provide the second be time-consuming for a large network and, thus, difficult to use in reartime. In addition the exact, current state of the network and its physical properties may not be known for computational purposes. Thus, for example, the heat transfer coefficients at a heat exchanger will be affected by fouling, which will in turn change its ability to transfer heat. It is possible that the response of the network be simulated by an ANN. This ANN can be trained by using experimental data so that the response of changing any parameter can be found. One of the advantages is that this can be done online so that changes in system performance are reflected in the simulation.

Thermal Processing of Materials

In almost all materials processing operations involving thermal effects, the most critical information is the direct relationship between the desired characteristics of the end product, such as, for instance, material microstructure defect elimination and the processing parameters. Physical models that provide such relationship are either not available or, if available, involve either apparent properties or constitutive relations which are not available. In the latter case, the lack of such information essentially nullifies the usage of such models. The ANN analysis simply bypasses all these uncertainties and leads immediately to the direct quantitative relationship between the desired end-product characteristics and the processing parameters. This advantage becomes particularly significant in dealing with new engineering materials, foods, and pharmaceutical and biological materials.

Environmental and Safety Studies

There are many complex thermal systems dealing with environmental and safety control in building, factories, mines, and the like. HVAC and indoor pollution systems and fire protection systems are good

The conventional of way of correlating data is to determine correlations for inner and outer heat transfer coefficients. For example, power laws of the following form

$$\varepsilon Nu_a = a \ Re_a^m \ Pr_a^{1/3}$$
 (4.24.29)

$$Nu_{w} = b \ Re_{w}^{n} \ Pr_{w}^{0.3} \tag{4.24.30}$$

are common. The two Nusselt numbers provide the heat transfer coefficients on each side and the overall heat transfer coefficient, U, is related to h_a and h_w by

$$\frac{1}{UA_a} = \frac{1}{h_w A_w} + \frac{1}{\varepsilon h_a A_a}$$
(4.24.31)

To find the constants a, b, m, n, the mean square error

$$S_{U} = \frac{1}{N} \sum \left(\frac{1}{U^{p}} - \frac{1}{U^{e}} \right)^{2}$$
(4.24.32)

must be minimized, where N is the number of experimental data sets, U^p is the prediction matches, the power-law correlation, and U^e is the experimental value for that run. The sum is recall N bars.

This procedure was carried out for the data collected. It was found that (b) S, and local minima for many different sets of the constants, the following two being wind as



From 4.24.19 shows a sector for the S_U surface that passes though the two minima, A and B. The coordinate z is a linear continuous of the constants a, b, m, and n such that it is zero and unity at the two minima. Though the values of S_U for the two correlations are very similar and the heat rate predictions for the two correlations are also almost equally accurate, the predictions on the thermal resistances on either side are different. Figure 4.24.20 shows the ratio of the Nusselt number of the air side predicted by Correlation A divided by that predicted by Correlation B. R_w is the same value for the water side. The predictions, particularly the one on the water side, are very different.

There are several reasons for this multiplicity of minima of S_U . Experimentally, it is very difficult to measure the temperature at the wall separating the two fluids, or even to specify where it should be measured, and mathematically, it is due to the nonlinearity of the function to be minimized. This raises the question as to which of the local minima is the "correct" one. A possible conclusion is that the one which gives the smallest value of the function should be used. This leads to the search for the global minimum, which can be done using the GA.

For these data, Pacheco-Vega et al. (1998) conducted a global search among a proposed set of heat transfer correlations using the GA. The experimentally determined heat rate of the heat exchanger was correlated with the flow rates and input temperatures, with all values being normalized. To reduce the number of possibilities, the total thermal resistance was correlated with the mass flow rates in the form

$$\frac{T_w^{in} - T_a^{in}}{\dot{Q}} = f\left(\dot{m}_a, \dot{m}_w\right)$$
(4.24.33)



FIGURE 4.24.20 Ratio of the predicted air- and water-side Nusselt numbers.

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Below, these matrices are formed using the MATLAB system. Once again, assuming that *T* is known at time level *j*, solve $T = T_{j+1}$ from the following:

$$CT_{i+1} = DT_i + 2\Delta t f_i.$$
(5.8)

The C-N formula in Equation 5.8 results from averaging the forward difference formula and the *backward difference* analog, based on the approximation

$$g'(t_j) \cong \frac{g(t_j) - g(t_j - \Delta t)}{\Delta t}$$

This scheme in Equation 5.8 is *implicit* (as opposed to *explicit*) in the sense that one must solve the algebraic system involving matrix C. However, since both C and D consist of only three diagonal "bands", the computation is only of order n for each time step in contrast to order n^3 if matrix C were full.

The following table shows the results using the Crank-Nicolson algorithm on the Problem in Equation 5.7. T_i denotes the exact solution and v_i the approximate solution. Columns 3 and 4 are the results with a larger time step, $\Delta t = 0.1$, and with h = 0.1, as before. Note these results are similar to the explicit scheme shown in Table 5.1 (with Δt and h = 0.1) since the C-N scheme has an error of $O(h^2 + \Delta t^2)$, as opposed to order $O(h + \Delta t^2)$ for the forward difference method. In the fifth column of Table 5.2 are the results when one decreases both steps, $h = \Delta t = 0.01$; note the improved activate due to the quadratic error in both h and Δt .



MATLAB[®] (The MathWorks, Inc., Natick, MA, 01760) is a very powerful and widely used software package for doing matrix algebra and other related computations. Below shows how the matrices for the C-N algorithm are set up and 50 time steps are taken. Assume *n* (the number of *x* values), *d* (the *n*-vector of the given *T* values at t = 0) and the "forcing" vector *f* have all been defined. The MATLAB commands are as follows, where anything following the % symbol is a comment and the semicolon both ends a statement and suppresses output.

```
dt = .01; h = 1/(n+1); lambda = dt/(h*pi)^2;
                                                           % dt is the time step
w = ones(1, n-1);
                                                           % a vector of n-1 ones
B = 2 eve(n) - diag(w, 1) - diag(w, -1)
                                                           % creates the B matrix; eye is 'identity'
C = 2*I + lambda*B; D = 2*I - lambda*B;
\mathbf{v} = \mathbf{d};
for j = 1:1:50
                                                           % doing the 50 time steps
  u = D*v + 2*dt*f;
                                                           % this solves Cv = u for v
  \mathbf{v} = \mathbf{C} \mathbf{u};
end
                                                           % sends final v to the screen
v
```

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FIGURE 5.2 Points on and near boundary.

For convenience, denote the five points in question with a local labeling 0,...,4 with point number 1 being the central point. The goal is to approximate the two derivatives in Laplace's equation at the point labeled by 1, which represents the point (x_i, y_j) . Note that the two points labeled by 0 and 3 are on the boundary, hence *T* is known there; whereas at the other three (interior) points *T* is part of the unknown vector in Equation 5.11. The problem, as illustrated by Figure 5.2, is that boundary points 0 and 3 are not full steps (*h* and *k*) from central point 1; so new approximations for T_{xx} and T_{yy} are needed. For $T_{xx}(x_i, y_j)$, expand in Taylor series about the point 1 to obtain:

$$T_{2} = T(x_{i} + h, y_{j}) = T_{1} + h T_{x}(x_{i}, y_{j}) + \frac{1}{2}h^{2} T_{x}(x_{i}, y_{j}) + \frac{1}{2}\alpha^{2}h^{2} T_{x}(x_{i}, y_{j}) + \frac{1}{2}\alpha^{2}h^{2} T_{xx}(x_{i}, y_{i}) + \frac{1}{2}\alpha^{2}h^{2} T_{$$

Note that when $\alpha = 1$ this reduces to the familiar approximation to T_{xx} . Similarly, expanding $T_3 = T(x_i, y_j + \beta k)$, as above, one gets the approximation to T_{yy} ; then adding the two gives

$$T_{xx}(x_{i}, y_{j}) + T_{yy}(x_{i}, y_{j}) \cong$$

$$2\frac{T_{0} = (1+\alpha)T_{1} + \alpha T_{2}}{\alpha(1+\alpha)h^{2}} + 2\frac{T_{3} - (1+\beta)T_{1} + \beta T_{4}}{\beta(1+\beta)k^{2}} = 0.$$
(5.16)

This second line then replaces the corresponding terms in Equations 5.10 and 5.11. In particular, the diagonal element 2*R* in Equation 5.11 is replaced by the new coefficient of $T_1 = T(x_i, y_i)$; i.e.,

$$1 + \frac{\alpha h^2}{\beta k^2}$$

Hence, for each interior element near the boundary, as the point 1 in Figure 5.2, the corresponding diagonal element of A in Equation 5.11 must be modified; and for the adjacent interior points, like the points 2 and 4, those corresponding elements in A must be altered via Equation 5.16. Moreover, for

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FIGURE 5.5 The domain and 13 nodes.

$$a + bx_1 + cy_1 = a + 0.2b + 0.2c = 1$$

$$a + bx_2 + cy_2 = a + 0.1b + 0.2c = 0$$

$$a + bx_3 + cy_3 = a + 0.1b + 0.1c = 0$$

Solving this system results in $\phi_3 = -1 + 10x$. Similarly tolking c_3 , ϕ_3 on triangle (3,2,13) gives $\phi_3 = -1 + 10y$, and on triangle (3,4,1), $\phi_3 = 1 + 10x - 10y$. Again $\phi_3 = 0$ on the rest of the domain. In a similar manner the basic functions are defined for each node.

The next step is to set up being at bis for the coefficient γ_1 in the representation $T \sim \sum \gamma_j \phi_j(x, y)$. In the FEM the γ_1 are decrements of that all boundary conditions are satisfied at those nodes. In the current case, the F voluments immediately give the values for $\gamma_4, ..., \gamma_{13}$ since the corresponding ϕ_j values are **1** but these nodes. Using the Hachelet conditions given above, one can evaluate $\gamma_4, ..., \gamma_{13}$ to get, respectively, 2.76, 277, 77, 202, 9, 3.0, 3.0, 2.93, 2.92. So in this example one eads only to solve for the coefficients γ_1 , γ_2 corresponding to the two interior nodes E_1 , E_2 ; and for γ_3 corresponding to the node E_3 on the Γ_2 portion of the boundary. Hence, one needs to solve a 3 by 3 system of linear equations. To that end, express Equation 5.24 in the current setting in which $p_1 = p_2 = 1$ and Q = 0:

$$\iint_{D} \sum_{j=1}^{3} \left(\partial_{x} \phi_{i} \partial_{x} \phi_{j} + \partial_{y} \phi_{i} \partial_{y} \phi_{j} \right) \gamma_{j} dx dy$$

$$= -\iint_{j=4}^{13} \sum_{j=4}^{13} \left(\partial_{x} \phi_{i} \partial_{x} \phi_{j} + \partial_{y} \phi_{i} \partial_{y} \phi_{j} \right) \gamma_{j} dx dy + \int_{\Gamma_{2}} \phi_{i} y dy; \quad i = 1, ..., 3$$
(5.27)

Note that the double integral sum has been split into two pieces — the sum from 1 to 3 involve unknown γ_j , whereas the second piece and the Γ_2 integral involve only known quantities and are the right side of Equation 5.27. So the left side of Equation 5.27 provides the *stiffness* matrix **K** discussed above. Also notice that, as discussed earlier, $\phi_1 = \phi_2 = \phi_3 = 0$ on Γ_1 , thus justifying the line integral in Equation 5.27 being only over Γ_2 (and here ds = dy). Turn now to the computation of the matrix **K**,

$$K_{i,j} = \iint_{D} \left(\partial_x \phi_i \partial_x \phi_j + \partial_y \phi_i \partial_y \phi_j \right) dx \, dy \quad i = 1, \dots, 3; \quad j = 1, \dots, 3.$$
(5.28)

the domain is three-dimensional, it is efficient to use more elaborate elements. Some of the more popular elements used in modern software will be discussed next.

Some Popular Finite Elements and Their Integration

The example above focused on the historically important triangular element with three nodes (the vertices of the triangle). The key feature of such elements is that they lead to piece-wise linear basis functions, the ϕ_j . While such approximating functions have been useful for decades in a number of areas of application, they suffer from the fact that linear functions do not do a very good job of approximating, e.g., quadratic, cubic, or other higher-order surfaces. More appropriate elements for dealing with problems demanding higher accuracy and/or with curved boundaries is the next subject.

Quadratic Triangular Element

Considerable detail will be given for this important case and provided are all the essential ingredients for solving a BVP using this element. Then several other cases will be discussed in much less detail. The idea here is simply to add to the three nodes at the vertices three additional nodes at the midpoints of each side of the triangle. These six nodes are exactly what is needed to determine the six constants in a quadratic in two variables:



Note that for elements in "standard" position the coordinate variables ξ and η will be used, leaving *x* and *y* for the coordinates of the original boundary value problem. Shortly it will be shown how to map a (typically small) element in the *xy*-plane over to the standard element in the $\xi\eta$ -plane. Returning to Equation 5.30, as before, one defines the basis functions as having value 1.0 at one node and 0.0 at the other five nodes. This defines the constants c_1, \dots, c_6 in Equation 5.30 for each particular node. This process leads to the following basis functions (shown in vector form):

$$\phi(\xi,\eta) = \begin{bmatrix} (1-\xi-\eta)(1-2\xi-2\eta) \\ 4\xi(1-\xi-\eta) \\ \xi(2\xi-1) \\ 4\xi\eta \\ \eta(2\eta-1) \\ 4\eta(1-\xi-\eta) \end{bmatrix}.$$
(5.31)

As the reader can easily verify, ϕ_1 has values 1.0 at (0, 0) and 0.0 at the other five nodes, and similarly for the other five basis functions. The functions in Equation 5.31 are clearly quadratic, moreover they

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In addition to the better handling of curved boundaries, the six-node triangular elements have better *interpolating* accuracy compared to the three-node elements, specifically $O(h^3)$ vs. $O(h^2)$ local error. In a modern software package the domain of interest is divided up into a fairly large number of elements. If the boundary of the domain is polygonal (consisting only of a series of straight lines) then regular, straight-sided, triangular elements will work well. But if the boundary is curved, one gets a much better approximation to it by using elements with three nodes per side since this allows for a quadratic approximation to each small piece of boundary. Figure 5.8 illustrates a portion of a domain with a curved boundary showing regular triangles in the interior and elements with one curved side along the boundary.



Recall from the earlier comments and the above example (starting with Equation 5.26) for a particular $K_{i,j}$, one only has to integrate over a few elements rather than the entire domain *D*. So focus on single element T^* in the *xy* mesh and see how the integration is performed over the standard triangle; call it *T*, in the $\xi\eta$ -plane. Changing variables and integrating over one element in the above integral:

$$\iint_{T^*} \Big[p_1 \partial_x \psi_i \partial_x \psi_j + p_2 \partial_y \psi_i \partial_y \psi_j \Big] dx \, dy = \iint_T H(\xi, \eta) |\det(J)| d\xi \, d\eta, \tag{5.33}$$

where

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

Here *H* replaces the integrand in the integral on the left and *J* is the *Jacobian* matrix of the transformation, with det(*J*) the determinate of the matrix. Now *J* can be computed using Equations 5.32 and 5.31; also observe that each of the four components of *J* is a linear function of ξ and η .

Next, assemble the integrand *H* in Equation 5.33 by replacing the integrand in *x*, *y* by its equivalent in ξ , η . Denoting $\phi_i(\xi, \eta) = \psi_i(x, y)$ and applying the chain rule:

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$$\int_{-1}^{1} g(\xi) d\xi \simeq \sum_{i=1}^{n} w_i g(\xi_i) \simeq \sum_{i=1}^{n} w_i \left[\sum_{j=1}^{n} w_j f(\xi_i, \eta_j) \right] = \sum_{i,j=1}^{n} w_i w_j f(\xi_i, \eta_j)$$

For example, if n = 3 one would need to evaluate f at the nine points shown in the figure below.



Compare this with Equation 5.39. Because of the form, this element is called *biquadratic*. The basis functions are not given here (see Reddy and Garling⁹). They are considerably more complex than the basis functions for the Serendipity element (fourth-order polynomials as opposed to cubic), so in most problems the slight improvement in accuracy does not justify the additional expense of evaluation.

Three-Dimensional Elements

Here, a small sampling of three-dimensional elements are shown; first, a linear *brick* element, then a quadratic triangular-sided (tetrahedral) element.

Possibly the most widely used three-dimensional element is the linear *brick* element based on the cube shown in Figure 5.12, also shown is a typical image in *xyz*-space.



FIGURE 5.12 The standard brick and image.

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the other nodes. Note that they are *optimally* efficient in that there are only 10 of them (the minimum number) and each one is quadratic (the minimum order).

$$\phi(\xi,\eta,\zeta) = \begin{bmatrix} (1-\xi-\eta-\zeta)(1-2\xi-2\eta-2\zeta) \\ \xi(2\xi-1) \\ \eta(2\eta-1) \\ \zeta(2\zeta-1) \\ 4\xi(1-\xi-\eta-\zeta) \\ 4\xi\eta \\ 4\eta(1-\xi-\eta-\zeta) \\ 4\zeta(1-\xi-\eta-\zeta) \\ 4\zeta\zeta \\ 4\eta\zeta \end{bmatrix}$$
(5.45)

Moreover, linear combinations of these functions will represent any quadratic polynomial: co.uk

+
$$c_2 \xi$$
 + $c_3 \eta$ + $c_4 \zeta$ + $c_5 \xi^2$ + $c_6 \eta^2$ + $c_7 \zeta^2$ + $c_8 \xi \eta$ + $c_9 \xi \zeta$ +

the sum is over 10 basis Again, the mapping into the xyz system is provided by E functions.

Another element that is particulary f connection the interior linear brick elements ne quadratic p **i m** whose standard from s lown in Figure 5.1 lement near the its utility, if a lineal brick P day places two of its n du of the boundary, one can, e.g., JU fill in above and bes the diagonal terms with the quadratic prism elements as shown in Figure 5.15. The same type of fill-in can be accomplished using the quadratic tetrahedral element discussed just above. However, it would take three tetrahedral elements to do the job of one prism, as illustrated in Figure 5.15.

Integration in Three Dimensions

 C_1

Analogous to the integration of Equation 5.41 in two dimensions and assuming the same type of quadrature, the integration over the brick becomes:

$$\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} H(\xi, \eta, \zeta) |\det(J)| d\xi \, d\eta \, d\zeta$$
$$= \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f(\xi, \eta, \zeta) d\xi \, d\eta \, d\zeta \simeq \sum_{i,j,k=1}^{n} w_i w_j w_k f(\xi_i, \eta_j, \zeta_k).$$

The weights w_i are the same as discussed above. Of course, the calculations are more involved since the matrix J is now a 3 by 3 matrix, so its inversion is more difficult. But logically the steps are the same.



FIGURE 5.14 The quadratic prism.



FIGURE 5.15 A block and two prisms.

domain to zero, or to a single point, thus providing the lowering of the dimension of the problem. These concepts now will be illustrated.

BEM and a Poisson Problem

where $\delta(x -$

In this subsection Ω denotes the domain of interest in two or three dimensions, and Γ denotes its boundary. The Poisson problem to be discussed is

$$\nabla^2 T(\mathbf{x}) = b(\mathbf{x}) \quad \text{on } \Omega,$$

$$T = f \quad \text{on } \Gamma^1,$$

$$\partial_n T = g \quad \text{on } \Gamma^2,$$

(5.63)

 $(\mathbf{x}) - T(\mathbf{x}) \partial_{\mathbf{x}} G(\mathbf{x}, \xi) d\Gamma$

where $\Gamma = \Gamma^1 + \Gamma^2$. A particularly useful choice of weighting function for this problem is the *fundamental* solution $G(\mathbf{x}, \boldsymbol{\xi})$ to Laplaces's equation. This function depends on the dimension of the problem and will be defined below. In any dimension, such a G enjoys the property:

$$\nabla^2 G(\mathbf{x}, \xi) = \delta(\mathbf{x} - \xi)$$
 (5.64)
where $\delta(\mathbf{x} - \xi)$ is the *delta* function with the property that when multiplied by any commons function $h(\mathbf{x})$
and integrated over a region including fixed point ξ , it returns the value $h(\xi)$. The returns is exploited shortly.

The first step in this presentation of the BEM is to app Condity as follows, where ξ is an interior point (i.e., not on the boundary):



$$T(\xi) + \int T(\mathbf{x}) \partial_n G(\mathbf{x}, \xi) d\Gamma = \int G(\mathbf{x}, \xi) \partial_n T(\mathbf{x}) d\Gamma + \int G(\mathbf{x}, \xi) b(\mathbf{x}) d\Omega.$$
(5.65)

Equation 5.65 is the fundamental BEM relationship involving known and unknown values of T and flux $\partial_n T$. There are two distinct uses of Equation 5.65; first, variable ξ is taken to the boundary and the result is discretized, leading to a system of linear equations for any missing T and flux values on the boundary. Secondly, when all such boundary values are known in the integrals in Equation 5.65, it is applied to compute T at desired *interior* points. In the next section these steps are taken for a two-dimensional problem, and later three-dimensional issues are discussed.

BEM in Two Dimensions

The problem to be discussed here is a case of Equation 5.63 in two dimensions. In this case the fundamental solution for Laplace's equation, $\nabla^2 G = \delta(\mathbf{x} - \xi)$, is

$$G(\mathbf{x},\xi) = \frac{-1}{2\pi} \log r(\mathbf{x},\xi),$$

$$r(\mathbf{x},\xi) = \|\mathbf{x} - \xi\| = \sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2}.$$
(5.66)

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$$(F+I)T = Gq$$

Here *I* is the *N* by *N* identity matrix and *T* is the *N*-vector of temperatures on the boundary. Finally, the i = j correction in Equation 5.70 subtracts $\frac{1}{2}$ from the diagonal of matrix *F* and results in the new matrix *H* and the system:

$$(F + \frac{1}{2}I)T = HT = Gq.$$
(5.71)

Two simple cases of the current boundary value problems, Equation 5.63 with b = 0, can now be easily addressed from Equation 5.71. First, the Dirichlet problem, in which the temperature is specific on the entire boundary. Here vector T is known, so the product $\mathbf{v} = HT$ is computed; then $Gq = \mathbf{v}$ is solved giving the flux values on the boundary (i.e., at the midpoints of each segment).

The second problem that follows easily from Equation 5.71 is the *Neumann* problem where the flux is specified on the entire boundary. In this case, the right side of Equation 5.71 is computed, say w = Gq, and then the system HT = w is solved for the temperatures on the boundary.

In either of these two cases, once both T and q are known at all midpoints of the boundary, Equation 5.69 can be used to compute $T(\xi_i)$ for any desired *interior* point.

Mixed Boundary Value Problem

The more general BVP has temperature specified on part of the boundary and flax on the cest, as in Equation 5.63. This can perhaps be best illustrated by a simple example Casicor the BVP $\nabla^2 T = 0$, with temperature specified on the vertical sides of the rectangle in Figure 2.20 and flux specified on the horizontal sides. Note that the boundary is divided into V = C segments and that the midpoints, x_j , are labeled in an unusual way which will simplify the cresentation to follow



FIGURE 5.20 Domain with eight BEM segments.

The problem with proceeding to solve the system HT = Gq is that neither T nor q is known at all eight midpoint values; in fact, each is known at four points. One must set up a new 8 by 8 system, say Au = v, to solve for the missing temperature and flux valves. First define the unknown vector u by $u = [T_1, T_2, T_3, T_4, q_5, q_6, q_7, q_8]$, where the subscripts on T and q correspond to the labeling in Figure 5.20. To construct the matrix A, equation HT = Gq is spilt so that the unknown values are on the left side and known values on the right. For i = 1, ..., 8, one thus obtains:

$$\sum_{j=1}^{4} H_{i,j}T_j - \sum_{j=5}^{8} G_{i,j}q_j = \sum_{j=1}^{4} G_{i,j}q_j - \sum_{j=5}^{8} H_{i,j}T_j = v_i$$
(5.72)

у	$T(\pi/2,y)$	$T - \mathbf{v}_{m=7, n=5}$	$T - \mathbf{v}_{m=15, n=11}$	$T - v_{m=31, n=23}$
5/3	2.55282	0.0114	0.002510	0.000567
4/3	1.76504	0.0086	0.001738	0.000392
1	1.17520	0.0057	0.001150	0.000260
2/3	0.71716	0.0033	0.000685	0.000157
1/3	0.33954	0.0013	0.000294	0.000071

TABLE 5.6Results of Solving Boundary-Value Problemin Equation 5.12 by the BEM

the exact T is known around the entire boundary. In contrast, the errors in Table 5.6 tend to be near the top boundary. The reason for this is that the BEM must compute approximations to the flux along the boundary, and with these rather crude meshes this introduces substantial error in the flux, which in turn translates into errors near the boundary.

This example shows the typical advantage of the BEM over alternative methods in that the resulting linear system is much smaller. For example, in the m = 31, n = 23 case the finite difference approach led to a linear system with 713 unknowns (and the same would be true for the FEM). Whereas the BEM led to 108 unknowns. On the other hand, in both the finite difference and FEM approaches the resulting matrices are relatively *sparse* (lots of zeros); so if the software being used takes advantage of this, the computational times become more competitive. Further comparisons between the BEM and the true TEM are made shortly.

A systematic error analysis for the BEM is not presented here. As any good by the above example, the errors in the BEM method as presented here (i.e., taking a modified as constant on each section Γ_j) will tend to converge much as the theory for the mice difference approach. When one uses more accurate descriptions of T and flux the approximation theory gets more contribution like that discussed for the FEM earlier. One way if (which the approximations contributions contributions)

More Accurate Late plations

If the top of accuracy shown is the above example is not adequate for one's purpose, or if a large twodimension or three-lineary incroblem is involved, there are basically two ways to increase accuracy. One is to take a smaller mesh, hence larger N, generally expecting the doubling of N to reduce the error by a factor of four. The other alternative is to use a more accurate integration scheme in the integrals in Equations 5.68 and 5.73. For example, consider integrals:

$$\int_{\Gamma_j} T(\mathbf{x}) F(\mathbf{x}, \xi_i) d\Gamma.$$
(5.74)

Earlier, for simplicity, *T* was taken to be constant over this segment Γ_j so that it can be factored out of the integral. Then in computing the matrix *F* one only had to integrate the function *F*. A simple way to improve the approximation of *T* (and *q*) without increasing the size of the resulting system of equations is to take a *linear* approximation to *T* over Γ_j in Equation 5.74 and use the values of *T* at the two endpoints of Γ_j (rather than the midpoint). This results in slightly more difficult integrals for matrices *F* and *G*, but this typically does not matter if these are done numerically. This linear approximation to *T* does *not* increase the size of the linear system. For details of this idea see Chapter 3 in Brebbia et al.¹²

Should one require yet more accurate approximations to *T* and *q* (for example, when they change rapidly on the boundary), some sort of quadrature involving both *T* and *F* in Equation 5.74 will be called for. To accommodate better integration accuracy and to allow for approximation to curved boundaries, the follow approach parallels that done earlier for the FEM. First, consider a section of boundary Γ_j with three points, one on each endpoint and a midpoint, labeled locally by $P_1 = (x_1, y_1), P_2 = (x_2, y_2),$ and $P_3 = (x_3, y_3)$. See Figure 5.21.

Appendices

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- A. Properties of Gases and Vapors
- B. Properties of Liquids
- C. Properties of Solids
- D. SI Units and Conversion Factors



Common name(s)	Fluorocarbons							
Chemical formula	CCl ₃ F	CCl_2F_2	CClF ₃	CBrF ₃ 13B1				
Refrigerant number	11	12	13					
CHEMICAL AND PHYSICAL								
PROPERTIES								
Molecular weight	137 37	120.91	104 46	148 91				
Specific gravity air $= 1$	4 74	4 17	3.61	5 14				
Specific volume ft^3/lb	2 74	3.12	3 58	2.50				
Specific volume, m ³ /kg	0.171	0.195	0 224	0.975				
Density of liquid (at atm hp) lb/ft ³	92.1	93.0	95.0	124.4				
Density of liquid (at atm bp), kg/m^3	1 475	1 490	1 522	1 993				
Vapor pressure at 25 deg C psia	14/5.	94.51	516	234.8				
Vapor pressure at 25 deg C MN/m^2		0.652	3 56	1 619				
Viscosity (abs) 1bm/ft-sec	7 39 × 10-6	8.74×10^{-6}	5.50	1.017				
Viscosity (abs), centinoises ⁴	0.011	0.013						
Sound velocity in gas m/sec	0.011	0.015						
Sound velocity in gas, m/sec								
THERMAL AND THERMO								
DVNAMIC PROPERTIES								
Specific heat - Div/lb dee E								
specific fleat, c_p , Btu/fb'deg F	0.14	0.146	0.154					
	0.14	0.140	0.134					
Specific heat, c_p , J/Kg·K	380.	011.						
Specific near ratio, c_p/c_v	1.14	1,14						
Gas constant R, It-Ib/Ib deg F		+03						
Gas constant R, J/kg deg C	0.005							
Thermal conductivity, Btu/hr it deg F	0.005	0.000	AUL					
Thermal conductivity, W/m·deg C	80,08			70				
Boiling point (sat 14./ psia), deg F		- 21.8	- 4.0	- 12.				
Boiling point (sat 760 mm), drg C	23.8	29.5	-81.4	-57.8				
Latent heat of evapore to the lu/lb		/1.1	63.0	51.1				
Latent heat of et al. (a) bp,, J/kg	180 00	10-000.	147 000.	119 000.				
ree in (noting) point, deg F (1 atm)	- 168.	-252.	- 294.	-270.				
Freezers (melting) point deg C (1 a m)	1 11. ⁻	-157.8	-181.1	- 167.8				
Latent heat of fusion, tub								
Latent heat of fusion, J cg								
Critical temperature, deg F	388.4	233.	83.9	152.				
Critical temperature, deg C	198.	111.7	28.8	66.7				
Critical pressure, psia	635.	582.	559.	573.				
Critical pressure, MN/m ²	4.38	4.01	3.85	3.95				
Critical volume, ft ³ /lb	0.028 9	0.287	0.027 7	0.021				
Critical volume, m ³ /kg	0.001 80	0.018	0.001 73	0.001				
Flammable (yes or no)	No	No	No	No				
Heat of combustion, Btu/ft ³	-	—		-				
Heat of combustion, Btu/lb	-	-	-					
Heat of combustion, kJ/kg		-						

TABLE A.7 (continued)Chemical, Physical, and Thermal Properties of Gases: Gases and Vapors, IncludingFuels and Refrigerants, English and Metric Units

"For N·sec/m² divide by 1 000.

Common name(s)		Helium			
Chemical formula	$C_2 ClF_5$	$C_2H_3ClF_2$	$C_2H_4F_2$	He	
Refrigerant number	115	142b	152a	704	
CHEMICAL AND PHYSICAL PROPERTIES					
Molecular weight	154.47	100.50	66.05	4.002 6	
Specific gravity, air $= 1$	5.33	3.47	2.28	0.138	
Specific volume, ft ³ /lb	2.44	3.7	5.9	97.86	
Specific volume, m ³ /kg	0.152	0.231	0.368	6.11	
Density of liquid (at atm bp), lb/ft ³	96.5	74.6	62.8	7.80	
Density of liquid (at atm bp), kg/m^3	1 546.	1 195.	1 006.	125.	
Vapor pressure at 25 deg C. psia	132.1	49.1	86.8		
Vapor pressure at 25 deg C, psil	0.911	0 338 5	0.596		
Viscosity (abs) lbm/ft-sec	0.277	0.500 5	01020	134×10^{-1}	
Viscosity (abs), continoises ^a				0.02	
Sound velocity in gas m/sec				1.015	
Sound velocity in gas, in see					
THERMAL AND THERMO-					
DYNAMIC PROPERTIES					
Specific heat c Btu/lb-deg F					
or cal/a dea C	0.161		_		
Specific heat a L/kg/K	674				
Specific heat, c_p , $J/\text{Kg/K}$	1 001			1.66	
Gos constant P ft lb/lb/dag E	1.091		201	386	
Gas constant R, It-10/10 deg F		105		2 077	
Gas constant R, J/kg/deg C	· · · · · · · · · · · · · · · · · · ·			0.086	
Thermal conductivity, Btu/nr-ft-deg F				0.030	
Delling point (act 14.7 poin) deg E		14			
Boiling point (sat 14.7 psia), deg r		14.	25.0	- 4.52. A 22 K	
Boiling point (sat 760 mm), deg (52.4		127.1	4.22 K	
Latent heat of evap (arb) tu/lb	124 000	15 000	210.000	22 200	
Latent heat of two of the , kg	124 100.	15 000.	319 000.	23 300. h	
Free right atting, wint, deg F (1 atm)				-	
Friening (n. elting) point, deg C (1 arr)	- 10 .0			-	
atent heat of fusion on the				-	
Latent heat of fusion, and				-	
Critical temperature, dg F	176.		387.	- 450.3	
Critical temperature, deg C				5.2 K	
Critical pressure, psia	457.6			33.22	
Critical pressure, MN/m ²	3.155				
Critical volume, ft ³ /lb	0.026 1			0.231	
Critical volume, m3/kg	0.001 63	1		0.014 4	
Flammable (yes or no)	No	No	No	No	
Heat of combustion, Btu/ft ³		_	_	-	
Heat of combustion, Btu/lb	_		_	-	
Hand of nomburdies bille	_		_		

TABLE A.7 (continued) Chemical, Physical, and Thermal Properties of Gases: Gases and Vapors, Including Fuels and Refrigerants, English and Metric Units

^eFor N-sec/m² divide by 1 000. ^bHelium cannot be solidified at atmospheric pressure.

Common name(s)	Propane	Propylene (Propene)	Sulfur dioxide	Xenon
Chemical formula	C_3H_8	C_3H_6	SO ₂	Xe
Refrigerant number	290	1 270	764	_
CHEMICAL AND PHYSICAL PROPERTIES				
Molecular weight	44.097	42.08	64.06	131.30
Specific gravity, $air = 1$	1.52	1.45	2.21	4.53
Specific volume, ft ³ /lb	8.84	9.3	6.11	2.98
Specific volume, m ³ /kg	0.552	0.58		
Density of liquid (at atm bp), lb/ft ³	36.2	37.5	42.8	190.8
Density of liquid (at atm bp), kg/m ³	580.	601.	585.	3 060.
Vapor pressure at 25 deg C, psia	135.7	166.4	56.6	
Vapor pressure at 25 deg C, MN/m ²	0.936	1.147	0.390	
Viscosity (abs), lbm/ft-sec	53.8×10^{-6}	57.1×10^{-6}	8.74×10^{-6}	15.5×10^{-6}
Viscosity (abs), centipoises ^a	0.080	0.085	0.013	0.023
Sound velocity in gas, m/sec	253.	261.	220.	177.
THERMAL AND THERMO- DYNAMIC PROPERTIES				.\(
Specific heat, c_p , Btu/lb·deg F				
or cal/g deg C	0.39	0.36	0.11 👝	0.42
Specific heat, c_p , J/kg·K	1 630.	1 506.	60	481.
Specific heat ratio, c_p/c_v	1.2	1.16		1.67
Gas constant R , ft-lb/lb deg F	35.0	3617	4.1	11.8
Gas constant R , J/kg·deg C	188.		130.	63.5
Thermal conductivity, Btu/hr ft deg F	0.010	0.010	0. 06	0.003
Thermal conductivity, W/m·deg C		0.017	0.010	0.005 2
Boiling point (sat 14.7 psia), deg F 🛃 🚽		- 54.	1.0	-162.5
Boiling point (sat 760 mm), deg C	42.2	48.2	► - 10.	- 108.
Latent heat of wap (at to) Bu/lb	184.	88.2	155.5	41.4
Latent heat of even (ar bp, J/kg	428 00	42000.	362 000.	96 000.
Er e ҧ (n ling point, deg F (1 atm)	- 309.	- 301.	- 104.	- 220.
Feez (melting) point deg C (1 2 m)	189.9	- 185.	- 75.5	- 140.
Latent heat of fusion, tub	19.1		58.0	10.
Latent heat of fusion, Jeg	44 400.		135 000.	23 300.
Critical temperature, deg F	205.	197.	315.5	61.9
Critical temperature, deg C	96.	91.7	157.6	16.6
Critical pressure, psia	618.	668.	1 141.	852.
Critical pressure, MN/m ²	4.26	4.61	7.87	5.87
Critical volume, ft ³ /lb	0.073	0.069	0.03	0.014 5
Critical volume, m ³ /kg	0.004 5	0.004 3	0.001 9	0.000 90
Flammable (yes or no)	Yes	Yes	No	No
Heat of combustion, Btu/ft ³	2 450.	2 310.	—	-
Heat of combustion, Btu/lb	21 660.	21 500.	-	-
II Commission I. I. I.	60.240	50.000	1	1

TABLE A.7 (continued)Chemical, Physical, and Thermal Properties of Gases: Gases and Vapors, IncludingFuels and Refrigerants, English and Metric Units

"For N·sec/m² divide by 1 000.

TABLE A.8 Ideal Gas Properties of Air

Part a. SI Units

T	h	р,	u	υ,	5°	Ť	h	р,	u	υ,	5°
200	199.97	0.3363	142.56	1707.	1.29559	450	451.80	5.775	322.62	223.6	2.1116
210	209.97	0.3987	149.69	1512.	1.34444	460	462.02	6.245	329.97	211.4	2.134
220	219.97	0.4690	156.82	1346.	1.39105	470	472.24	6.742	337.32	200.1	2.156
230	230.02	0.5477	164.00	1205.	1.43557	480	482.49	7.268	344.70	189.5	2.1770
240	240.02	0.6355	171.13	1084.	1.47824	490	492.74	7.824	352.08	179.7	2.198
250	250.05	0.7329	178.28	979.	1.51917	500	503.02	8.411	359.49	170.6	2.219
260	260.09	0.8405	185.45	887.8	1.55848	510	513.32	9.031	366.92	162.1	2.239
270	270.11	0.9590	192.60	808.0	1.59634	520	523.63	9.684	374.36	154.1	2.259
280	280.13	1.0889	199.75	738.0	1.63279	530	533.98	10.37	381.84	146.7	2.279
285	285.14	1.1584	203.33	706.1	1.65055	540	544.35	11.10	389.34	139.7	2.299
290	290.16	1.2311	206.91	676.1	1.66802	550	554.74	11.86	396.86	133.1	2.318
295	295.17	1.3068	210.49	647.9	1.68515	560	565.17	12.66	404.42	127.0	2.336
300	300.19	1.3860	214.07	621.2	1.70203	570	575.59	13.50	411.97	121.2	2.355
305	305.22	1.4686	217.67	596.0	1.71865	580	586.04	14.38	419.55	115.7	2.373
310	310.24	1.5546	221.25	572.3	1.73498	590	596.52	15.31	427.15	110.6	2.391
315	315.27	1.6442	224.85	549.8	1.75106	600	607.02	16.28	434.78	105.8	2.409
320	320.29	1.7375	228.42	528.6	1.76690	610	617.53	17.30	442.42	101.2	2.426
325	325.31	1.8345	232.02	508.4	1.78249	620	628.07	18.36	450.09	96.92	2.443
330	330.34	1.9352	235.61	489.4	1.79783	630	638.63	19.84	457.78	92.84	2.40
340	340.42	2.149	242.82	454.1	1.82790	640	649.22	20.64	465.50	88.99	14
350	350.49	2.379	250.02	422.2	1.85708	650	659.84	21.86	473.25	8. 34	. 493
360	360.58	2.626	257.24	393.4	1.88543	660	670.47	23 3	181.0	.89	2.509
370	370.67	2.892	264.46	367.2	1.91313	670	681.14		488 31	78.61	2.525
380	380.77	3.176	271.69	343.4	1.94001	680 🔺	8,103	2 8	+96.62	75.50	2.541
390	390.88	3.481	278.93	321.5	1.96633	690	1057	27.29	504.45	72.56	2.557
400	400.98	3.806	286.16	301.6	1.99194		713.27	28	5 .37	69.76	2.572
410	411.12	4.153	293.43	283.3	01 99	710	724 94	30.3	52 .2	67.07	2.580
420	421.26	4.522	300.6	66.	20442	720	73-22	32.02	528.14	64.53	2.603
430	431.43	4.915	307.99		2.06533	730	7 5. 2	33.72	536.07	62.13	2.618
440	441.61	5 33	5.30	236.8	2.08870		6.44	35.50	544.02	59.82	2.632
7(ev	6		10	11						
			2								

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TABLE B.1 Properties of Liquid Water*

Symbols and Units:

- ρ = density, lbm/ft³. For g/cm³ multiply by 0.016018. For kg/m³ multiply by 16.018.
- c_p = specific heat, Btu/lbm·deg R = cal/g·K. For J/kg·K multiply by 4186.8
- μ = viscosity. For lbf·sec/ft² = slugs/sec·ft, multiply by 10⁻⁷. For lbm·sec·ft multiply by 10⁻⁷ and by 32.174. For g/sec·cm (poises) multiply by 10⁻⁷ and by 478.80. For N·sec/m² multiply by 10⁻⁷ and by 478.880.
- k = thermal conductivity, Btu/hr·ft·deg R. For W/m·K multiply by 1.7307.

Temp,	At 1 atm or 14.7 psia			At 1,000 psia				At 10,000 psia				
°F	ρ	c _p	μ	k	ρ	c _p	μ	k	ρ	c _p	μ	<i>k</i> †
32	62.42	1.007	366	0.3286	62.62	0.999	365	0.3319	64.5	0.937	357	0.3508
40	62.42	1.004	323	0.334	62.62	0.997	323	0.337	64.5	0.945	315	0.356
50	62.42	1.002	272	0.3392	62.62	0.995	272	0.3425	64.5	0.951	267	0.3610
60	62.38	1.000	235	0.345	62.58	0.994	235	0.348	64.1	0.956	233	0.366
70	62.31	0.999	204	0.350	62.50	0.994	204	0.353	64.1	0.960	203	0.371
80	62.23	0.998	177	0.354	62.42	0.994	177	0.358	64.1	0.962	176	0. 75
90	62.11	0.998	160	0.359	62.31	0.994	160	0.362	63.7	0.964	159	
100	62.00	0.998	142	0.3633	62.19	0.994	142	0.3666	63.7	0 105	42	0.3841
110	61.88	0.999	126	0.367	62.03	0.994	126	0.371	30	0. 🗲	126	0.388
120	61.73	0.999	114	0.371	61.88	0.995	114		.	0.967	114	0.391
							+ 6					
130	61.54	0.999	105	0.374	61.73	9. 9:		0.378	63.3	0.9 8	105	0.395
140	61.39	0.999	96	0.378	67.8	0., 6	96	0.381	4 8.3	0 60	98	0.398
150	61.20	1.000	89	0.200	6.3	0.996	89	0.3831	6 0	0.970	91	0.4003
160	61.01	1.001	83	023	61.20	0.997	- (0.386	62.9	0.971	85	0.403
170	60.79	-003	77	•0.386	60.98	0 998		0.389	62.5	0.972	79	0.405
	50.57	1.003	72	0.788	0.75	0.999	72	0.391	62.5	0.973	74	0.407
	60.35	1.004		0 190	60.53	1.001	68	0.393	62.1	0.974	70	0.409
200	60.10	1.00).3916	60.31	1.002	62.9	0.3944	62.1	0.975	65.4	0.4106
250		boilin po	int 212°1		59.03	1.001	47.8	0.3994	60.6	0.981	50.6	0.4158
300		-			57.54	1.024	38.4	0.3993	59.5	0.988	41.3	0.4164
					55.02	1 0 4 4	22.1	0.2044	50.1	0.000	25.1	0.4122
350					53.83	1.044	32.1	0.3944	58.1	0.999	35.1	0.4132
400					53.91	1.0/2	27.6	0.3849	52.0	1.011	30.6	0.4064
500					49.11	1.181 :1:	21.0	0.3508	32.9	1.051	24.8	0.3830
600						bouing poi	ini 544.58	s r	40.3	1.118	21.0	0.5495

†At 7,500 psia.

* From: "1967 ASME Steam Tables", American Society of Mechanical Engineers, Tables 9, 10, and 11 and Figures 6, 7, 8, and 9. The ASME compilation is a 330-page book of tables and charts, including a 2½ × 3½-ft Mollier chart. All values have been computed in accordance with the 1967 specifications of the International Formulation Committee (IFC) and are in conformity with the 1963 International Skeleton Tables. This standardization of tables began in 1921 and was extended through the International Conferences in London (1929), Berlin (1930), Washington (1934), Philadelphia (1954), London (1956), New York (1963) and Glasgow (1966). Based on these world-wide standard data, the 1967 ASME volume represents detailed computer output in both tabular and graphic form. Included are density and volume, enthalpy, entropy, specific heat, viscosity, thermal conductivity, Prandtl number, isentropic exponent, choking velocity, p-v product, etc., over the entire range (to 1500 psia 1500°F). English units are used, but all conversion factors are given.

TABLE B.2 Physical and Thermal Properties of Common Liquids

Part a. SI Units

(At 1.0 Atm Pressure (0.101 325 MN/m²), 300 K, except as noted.)

Common name	Density, kg/m ³	Specific heat, kJ/kg·K	Viscosity, N·s/m ²	Thermal conductivity, W/m·K	Freezing point, K	Latent heat of fusion, kJ/kg	Boiling point, K	Latent heat of evapora- tion, kJ/kg	Coefficient of cubical expansion per K
Acetic acid	1 049	2.18	.001 155	0.171	290	181	391	402	0.001 1
Acetone	784.6	2.15	.000 316	0.161	179.0	98.3	329	518	0.001 5
Alcohol, ethyl	785.1	2.44	.001 095	0.171	158.6	108	351.46	846	0.001 1
Alcohol, methyl	786.5	2.54	.000 56	0.202	175.5	98.8	337.8	1 100	0.001 4
Alcohol, propyl	800.0	2.37	.001 92	0.161	146	86.5	371	779	
Ammonia (aqua)	823.5	4.38		0.353					
Benzene	873.8	1.73	.000 601	0.144	278.68	126	353.3	390	0.001 3
Bromine		.473	.000 95		245.84	66.7	331.6	193	0.001 2
Carbon disulfide	1 261	.992	.000 36	0.161	161.2	57.6	319.40	351	0.001 3
Carbon tetrachloride	1 584	.866	.000 91	0.104	250.35	174	349.6	194	0.001 3
Castor oil	956.1	1.97	.650	0.180	263.2				
Chloroform	1 465	1.05	.000 53	0.118	209.6	77.0	334.4	247	0.001 3
Decane	726.3	2.21	.000 859	0.147	243.5	201	447.2	263	
Dodecane	754.6	2.21	.001 374	0.140	247.18	216	489.4	~ (
Ether	713.5	. 2.21	.000 223	0.130	157	96.2	17	72	0.001 6
Ethylene glycol	1 097	2.36	.016 2	0.258	2602	50	+70	800	
Fluorine									
refrigerant R-11	1 476	.870ª	.000 42	0. 14	162		27.0	<u>s0</u> ¢	
Fluorine									
refrigerant R-12	1 311	97		0.071	1	34.4	243.4	165 ^b	
Fluorine				07					
refrigerant 72	4	1.26ª		69	113	183	232.4	232°	
Gielin	1 259	2.62	.950	0.287	264.8	200	563.4	974	0.000 54
Heptane	0.00	64	.000 376	0.128	182.54	140	371.5	318	
Hexane	6	26	.000 297	0.124	178.0	152	341.84	365	
Iodine		2.15			386.6	62.2	457.5	164	
Kerosene	820.1	2.09	.001 64	0.145				251	
Linseed oil	929.1	1.84	.033 1		253		560		
Mercury		.139	.001 53		234.3	11.6	630	295	0.000 18
Octane	698.6	2.15	.000 51	0.131	216.4	181	398	298	0.000 72
Phenol	1 072	1.43	.008 0	0.190	316.2	121	455		0.000 90
Propane	493.5	2.41	.000 11		85.5	79.9	231.08	428 ^b	
Propylene	514.4	2.85	.000 09		87.9	71.4	225.45	342	
Propylene glycol	965.3	2.50	.042		213		460	914	
Sea water	1 025	3.76-			270.6				
Toluene	862.3	1.72	.000 550	0.133	178	71.8	383.6	363	
T	868.2	1 78	001 375	0.121	214	1	433	293	0.000.99
Turpentine	000.2	1.70	.001 515	0.121	217			275	0.000 //

"At 297 K, liquid. "At .101 325 meganewtons, saturation temperature.

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Greek Letter		Greek Name	English Equivalent	Greek Letter	Greek Name	English Equivalent	
Αα		Alpha	а	Νν	Nu	n	
Β β		Beta	b	Ξξ	Xi	х	
Γγ		Gamma	g	Оо	Omicron	0	
$\Delta \delta$		Delta	d	Ππ	Pi	р	
Ε ε		Epsilon	e	Ρρ	Rho	r	
Ζζ		Zeta	Z	Σσς	Sigma	S	
Ηη		Eta	e	Ττ	Tau	t	
Θθ	θ	Theta	th	Υ υ	Upsilon	u	
Iι		Iota	i	Φφφ	Phi	ph	
Кκ		Kappa	k	Χχ	Chi	ch	
Λ λ		Lambda	1	ΨΨ	Psi	ps	
Μμ		Mu	m	Ωω	Omega	0	

Appendix D. SI Units and Conversion Factors

Greek Alphabet

International System of Units (SI)

The International System of units (SI) was adopted by the 11th General Conference on Weights and Measures (CGPM) in 1960. It is a coherent system of units built for Goven SI base units, one for each of the seven dimensionally independent base domines the meter, kilogram second, ampere, kelvin, mole, and candela, for the dimensions bach, moss time, electric current ther not ynamic temperature, amount of substance, and lumin unit ensuy, respectively. The definitions of the SI base units are given below. The *SI derived polits* are expressed as products of powers of the base units, analogous to the correspondent the forst between physical qualities but with numerical factors equal to unity.

Oh the international System the is only one SI unit for each physical quantity. This is either the appropriate SI base the international expropriate SI derived unit. However, any of the approved decimal prefixes, called *SI prefixes*, may be used to construct decimal multiples or submultiples of SI units.

It is recommended that only SI units be used in science and technology (with SI prefixes where appropriate). Where there are special reasons for making an exception to this rule, it is recommended always to define the units used in terms of SI units. This section is based on information supplied by IUPAC.

Definitions of SI Base Units

Meter: The meter is the length of path traveled by light in vacuum during a time interval of 1/299 792 458 of a second (17th CGPM, 1983).

Kilogram: The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram (3rd CGPM, 1901).

Second: The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (13th CGPM, 1967).

Ampere: The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length (9th CGPM, 1958).

Kelvin: The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (13th CGPM, 1967).

Mole: The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, or other particles, or specified groups of such particles (14th CGPM, 1971). Examples of the use of the mole: