NCERT Solutions for Class 10 Maths Unit 7

Coordinate Geometry Class 10

Unit 7 Coordinate Geometry Exercise 7.1, 7.2, 7.3, 7.4 Solutions

Exercise 7.1 : Solutions of Questions on Page Number : **161**

Q1 :

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (-5, 7), (-1, 3) (iii) (*a*, *b*), (- *a*, - *b*)

Answer :

(i) Distance between the two points is given by

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Therefore, distance between $(2, 3)$ and $(4, 1)$ is given by

$$l=\sqrt{(2-4)^2+(3-1)^2}=\sqrt{(-2)^2+(2)^2}$$
$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

(ii) Distance between $(-5,7)$ and $(-1,3)$ is given by

$$l=\sqrt{(-5-(-1))^2+(7-3)^2}=\sqrt{(-4)^2+(4)^2}$$
$$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

(iii) Distance between (a,b) and $(-a,-b)$ is given by

$$l=\sqrt{(a-(-a))^2+(b-(-b))^2}$$
$$=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$$

Q2 :

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer :

Distance between points $(0,0)$ and $(36,15)$

$$x_1 = \frac{1\times(-2)+2\times 4}{1+2}, \quad y_1 = \frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } \mathrm{P}(x_1, y_1) = \left(2, -\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

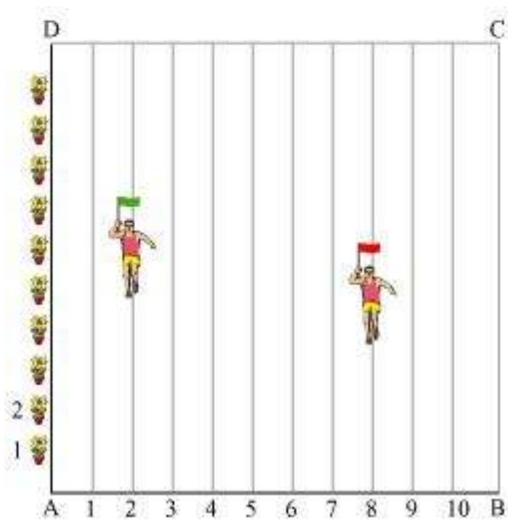
$$x_2 = \frac{2\times(-2)+1\times 4}{2+1}, \quad y_2 = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \quad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

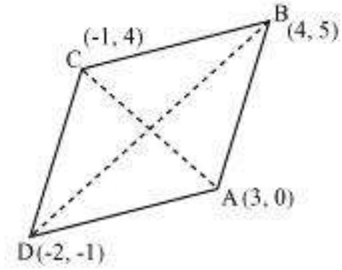
$$\mathrm{Q}(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Q5 :

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer :



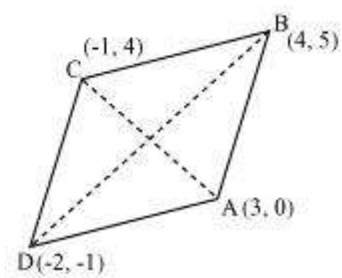
Let (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) are the vertices A, B, C, D of a rhombus ABCD.

$$\text{Length of diagonal AC} = \sqrt{\left[3-(-1)\right]^2 + (0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$
$$\text{Length of diagonal BD} = \sqrt{\left[4-(-2)\right]^2 + \left[5-(-1)\right]^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$
$$\text{Therefore, area of rhombus ABCD} = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ square units}$$

Q20 :

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) taken in order. [Hint: Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer :



Let (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) are the vertices A, B, C, D of a rhombus ABCD.

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\begin{aligned}\text{Area of the given triangle} &= \frac{1}{2}\left[2\{0-(-4)\} + (-1)\{(-4)-(3)\} + 2(3-0)\right]\\ &= \frac{1}{2}\{8+7+6\}\\ &= \frac{21}{2} \text{ square units}\end{aligned}$$

(ii) $$\text{Area of the given triangle} = \frac{1}{2}\left[(-5)\{(-5)-(2)\} + 3\left(2-(-1)\right) + 5\{-1-(-5)\}\right]$$

$$\begin{aligned}&= \frac{1}{2}\{35+9+20\}\\ &= 32 \text{ square units}\end{aligned}$$

Q3 :

In each of the following find the value of '*k*', for which the points are collinear.

(i) (7, - 2), (5, 1), (3, - *k*) (ii) (8, 1), (*k*, - 4), (2, - 5)

Answer :

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, - 2) (5, 1), and (3, *k*), area = 0

$$\begin{aligned}&\frac{1}{2}\left[7\{1-k\} + 5\{k-(-2)\} + 3\{(-2)-1\}\right] = 0\\ &7 - 7k + 5k + 10 - 9 = 0\\ &-2k + 8 = 0\\ &k = 4\end{aligned}$$

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (*k*, - 4), and (2, - 5), area = 0

$$\begin{aligned}&\frac{1}{2}\left[8\{-4-(-5)\} + k\{(-5)-(1)\} + 2\{1-(-4)\}\right] = 0\\ &8 - 6k + 10 = 0\\ &6k = 18\\ &k = 3\end{aligned}$$

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta\text{ABD} = \frac{1}{2}\left[(4)\{(-2)-(0)\} + (3)\{(0)-(-6)\} + (4)\{(-6)-(-2)\}\right]$$

$$= \frac{1}{2}(-8+18-16) = -3 \text{square units}$$

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

$$\text{Area of } \Delta\text{ADC} = \frac{1}{2}\left[(4)\{0-(2)\} + (4)\{(2)-(-6)\} + (5)\{(-6)-(0)\}\right]$$

$$= \frac{1}{2}(-8+32-30) = -3 \text{square units}$$

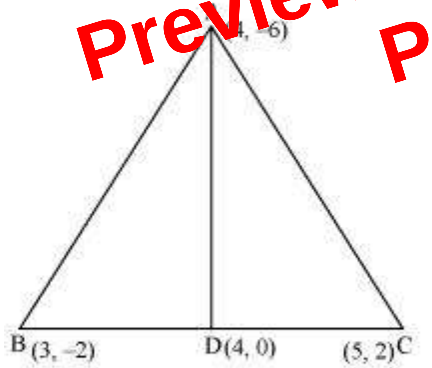
However, area cannot be negative. Therefore, area of ΔADC is 3 square units.

Clearly, median AD has divided ΔABC in two triangles of equal areas.

Q10 :

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, - 6), B (3, - 2) and C (5, 2).

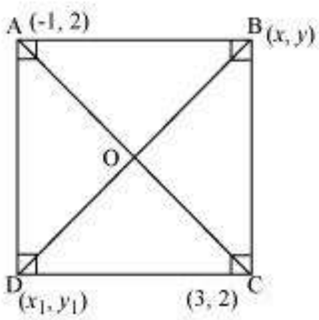
Answer :



Let the vertices of the triangle be A (4, - 6), B (3, - 2), and C (5, 2).

Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4,0)$$



Let ABCD be a square having (- 1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

∴ AB = BC

$$\Rightarrow \sqrt{(x+1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y-2)^2}$$
$$\Rightarrow x^2+2x+1+y^2-4y+4 = x^2+9-6x+y^2+4-4y$$
$$\Rightarrow 8x = 8$$
$$\Rightarrow x = 1$$

We know that in a square, all interior angles are of 90°.

In ΔABC,

$AB^2 + BC^2 = AC^2$

$$\Rightarrow \left(\sqrt{(1+1)^2+(y-2)^2}\right)^2 + \left(\sqrt{(1-3)^2+(y-2)^2}\right)^2 = \left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

⇒ $4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

⇒ $2y^2 + 16 - 8y = 16$

⇒ $2y^2 - 8y = 0$

⇒ $y(y - 4) = 0$

⇒ $y = 0$ or 4

We know that in a square, the diagonals are of equal length and bisect each other at 90°. Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

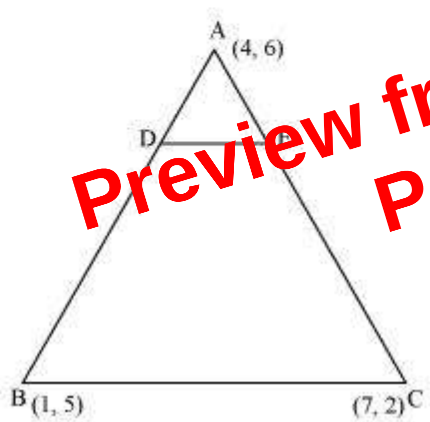
Therefore, ratio between the areas of ΔADE and ΔABC = $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Q11 :

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to

ratio of areas of two similar triangles)

Answer :



Given that, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC} = \frac{1}{4}$$
$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.