

But we have yet again hit another ceiling, because once we reach omega and go past it to omega plus omega we can find another infinite set. And that would guarantee that aleph-null exists. Will we have to add a new symbol every time we describe something new? No, fortunately we can use the scheme of replacement. This assumption states that if you take a set like the set of all natural numbers and replace each element by something else the thing you are left with is also a set. So let's take the set of all the natural numbers and replace it by omega plus the natural number. You will get omega plus one until omega plus omega, which can be replaced by omega times two. Using this we can make jumps of any size as long as we only use numbers we've already reached. Using this principle you can achieve omega squared. We can go even further until omega to the omega to the omega to the omega and so on.

And that is called epsilon-nought. And we can continue from here, but now there are all of these ordinals, they are all well ordered so they have an order type, but after an infinite amount of ordinals there is one ordinal that comes after all of them. In this case that ordinal is called omega one, not omega plus one, omega one. Because by definition omega one comes after every single order type of aleph-null things so it must describe an arrangement of more things than the last aleph. The cardinal number describing the amount of things used to make an arrangement with order type omega one is aleph-one. But going back to the power set of aleph-null, it's not in between aleph-null and aleph-one, because there are no cardinals in between those two, it might be the same as aleph-one but we just don't know. We won't get an answer yet but in the mean time we can go even higher and higher using the scheme of replacement. Reaching aleph-omega and aleph-omega-omega-omega-etc. We can reach bigger and bigger infinities each time and wherever you land there will be a place of even bigger numbers allowing you to make even bigger jumps than before. There is no end.

Let's take all of this in mind and focus on the following, we have been using these numbers like there's no problem with them but if (at any point down the line of natural numbers) you can always add one, can we really talk about this endless process as a totality and then follow it with something. Well, of course we can. We are not dealing with science we are dealing with math. When you assume something is true in math it's called axioms. With axioms it's true because we say it is. Mathematicians don't fit their theories to some physical universe. Mathematicians create a universe themselves. And all you have to do to achieve omega is to accept and say that omega exists. If you just read this entire piece and still refuse to accept it that's okay, because that makes you a finitist, but if you accept it (like most mathematicians do) you can go very far. Beyond all the natural numbers and reach aleph-null and omega and many more. That is true because people with that mind set of accepting that there might be something beyond public beliefs can achieve far greater than those who can't, just because they accept that things are the way they are.

As stated above there is no end. There is nothing beyond them, right? That's what we said about getting past the finite numbers to omega, why not accept that (as an axiom) there exist some next number, so big no amount of replacement could ever get you there. Such a number is called an inaccessible cardinal, because you are unable to reach it. Some people think that aleph-null is an inaccessible cardinal, because you can't reach it from below. But I disagree, because if something really is an inaccessible cardinal it must also be uncountable, which is not the case with aleph-null. I will leave it with this, the conceptual jump from nothing to the first infinity is equal to the jump from the first infinity to the inaccessible cardinal.