(Q14.) $\int \cos^2(2x)\, dx$

$$= \frac{1}{2}\int (1+\cos 4x)\, dx$$

$$= \frac{1}{2}(x + \frac{1}{4}\sin 4x)$$

$$= \boxed{\frac{1}{2}x + \frac{1}{8}\sin 4x + C}$$

<u>Aside</u>

$$\cos^2 x = \frac{1}{2}(1+\cos(2x))$$

$$\cos^2 2x = \frac{1}{2}(1+\cos(4x))$$

(Q15.) $\int \frac{1}{x^3+1}\, dx$

$$= \frac{1}{3}\int \frac{1}{x+1}\, dx - \frac{1}{3}\int \frac{x-2}{x^2-x+1}\, dx$$

$$= \frac{1}{3}\int \frac{1}{x+1}\, dx - \frac{1}{3}\,\frac{1}{2}\int \frac{2(x-1-3)}{x^2-x+1}\, dx$$

$$= I_1 - \frac{1}{6}\int \left(\frac{2x-1}{x^2-x+1} - \frac{3}{(x^2+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right) dx$$

$$= \frac{1}{3}\ln(x+1) - \frac{1}{6}(\ln(x^2-x+1)) + \frac{1}{2}\,\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

$$= \boxed{\frac{1}{3}\ln(x+1) - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) + C}$$

<u>Aside</u>

$$(x^3+1) = (x+1)(x^2-x+1)$$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$x+1 = 0$ when $x = -1$

substituting $x = -1$ into $x^2 - x + 1 = 3$

$$A = \frac{1}{3}$$

used x = -1, now we sub x = 0

$$1 = \frac{\frac{1}{3}}{0+1} + \frac{B\times 0 + C}{0^2 - 0 + 1}$$

$$1 = \frac{1}{3} + C$$

$$C = \frac{2}{3}$$

use x = 1

$$\frac{1}{(1+1)(1^2-1+1)} = \frac{\frac{1}{3}}{1+1} + \frac{B\times 1 + \frac{2}{3}}{1^2-1+1}$$

$$\frac{1}{2} = \frac{1}{6} + B + \frac{2}{3}$$

$$B = -\frac{1}{3}$$

$$\frac{d}{dx}(x^2-x+1) = 2x-1$$

$$x^2 - x + 1 = x^2 - x + \frac{1}{4} + \frac{3}{4}$$

(Q16.) $\int x\sin^2 x\,dx$

$= \frac{1}{2}\int x\,(1-\cos(2x))\,dx$

$= \frac{1}{2}(\int x\,dx + \int -x\cos(2x)\,dx)$

$= \frac{1}{2}(\frac{1}{2}x^2 - \frac{1}{2}x\sin(2x) - \frac{1}{4}\cos(2x))$

$= \boxed{\frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x) + C}$

Aside

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$

	D	I
$+$	$-x$	$\cos^{2x}$
$-$	-1	$\frac{1}{2}\sin 2x$
$+$	0	$-\frac{1}{4}\cos 2x$

(Q17.) $\int \left(x+\frac{1}{x}\right)^2\,dx$

$= \int x^2 + 2 + x^{-2}\,dx$

$= \boxed{\frac{1}{3}x^3 + 2x - \frac{1}{x} + C}$

(Q18.) $\int \frac{3}{x^2+4x+29}\,dx$

$=3\int \frac{1}{(x+2)^2 + 5^2}\,dx$

$= \boxed{\frac{3}{5}\tan^{-1}(\frac{x+2}{5}) + C}$

Aside

$x^2+4x+29 = x^2+4x+4+25$

$= (x+2)^2 + 5^2$

(Q19.) $\int \cot^5 x\,dx$

$= \int \frac{\cos^5 x}{\sin^5 x}\,dx$

$= \int \frac{(1-\sin^2 x)^2\cos x}{\sin^5 x}\,dx$

$= \int \frac{(1-u^2)^2}{u^5}\,du$

$= \int u^{-5} - 2u^{-3} + u^{-1}\,du$

$= \boxed{-\frac{1}{4}\csc^4 x + \csc^2 x + \ln|\sin x| + C}$

Aside

$\cos^4 x = (1-\sin^2 x)^2$

Let $u = \sin x$

$du = \cos x$

$\frac{1}{\sin x} = \csc x$

(Q20.) $\int_{-1}^{1} \frac{\tan x}{x^4 - x^2 + 1}\, dx$

$= \boxed{0}$

Aside

$\frac{\text{odd } f(x)}{\text{even } f(x)} = \text{odd } f(x)$

$\int_{-a}^{a} \text{odd } f(x)\, dx = 0$

(Q21.) $\int \sin^3 x \cos^2 x\, dx$

$= \int (1 - \cos^2 x) \cos^2 x \sin x\, dx$

$= - \int (1 - u^2)\, u^2\, du$

$= - \int (u^2 - u^4)\, du$

$= \boxed{-\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C}$

Aside

$\sin^3 x = \sin^2 x \times \sin x$

$= (1 - \cos^2 x) \times \sin x$

Let $u = \cos x$

$du = -\sin x\, dx$

(Q22.) $\int \frac{1}{x^2 \sqrt{x^2 + 1}}\, dx$

$= \int \frac{x^{-3}}{\sqrt{1 + x^{-2}}}\, dx$

$= -\frac{1}{2} \int \frac{1}{\sqrt{u}}\, du$

$= -\frac{1}{2} \int u^{-\frac{1}{2}}\, du$

$= \boxed{-\sqrt{1 + \frac{1}{x^2}} + C}$

Aside

$x^2 \sqrt{x^2 + 1} = x^2 \sqrt{x^2 (\frac{x^2}{x^2} + \frac{1}{x^2})}$

$= x^2 x \sqrt{1 + \frac{1}{x^2}}$

$= \frac{x^{-3}}{\sqrt{1 + x^{-2}}}$

Let $u = 1 + x^{-2}$

$du = 2x^{-3} dx$

(Q23.) $\int \sin x \sec x \tan x\, dx$

$= \int \tan^2 x\, dx$

$= \int (\sec^2 x - 1)\, dx$

$= \boxed{\tan x - x + C}$

Aside

$\sin x \sec x = \frac{\sin x}{\cos x}$

$= \tan x$

(Q58.) $\int \frac{1-\cos x}{1+\cos x}\,dx$

$= \int \frac{2\sin^2(\frac{x}{2})}{2\cos^2(\frac{x}{2})}\,dx$

$= \int \tan^2(\frac{x}{2})\,dx$

$= \int (\sec^2(\frac{x}{2}) - 1)\,dx$

$= \boxed{2\tan(\frac{x}{2}) - x + C}$

Aside

$\cos^2\frac{x}{2} = \frac{1}{2}(1+\cos x)$

$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)$

(Q59.) $\int x^2\sqrt{x+4}\,dx$

$= \int (u-4)^2\sqrt{u}\,du$

$= \int (u^2 - 8u + 16)u^{\frac{1}{2}}\,du$

$= \int (\frac{2}{7}u^{\frac{5}{2}+1} - \frac{2}{5}8^{\frac{3}{2}+1} + 16\times\frac{2}{3}u^{\frac{1}{2}+1})\,du$

$= \boxed{\frac{2}{7}(x+4)^{\frac{7}{2}} - \frac{16}{5}(x+4)^{\frac{5}{2}} + \frac{2}{3}(x+4)^{\frac{3}{2}} + C}$

Aside

Let $u = x+4$

$du = dx$

$x = u - 4$

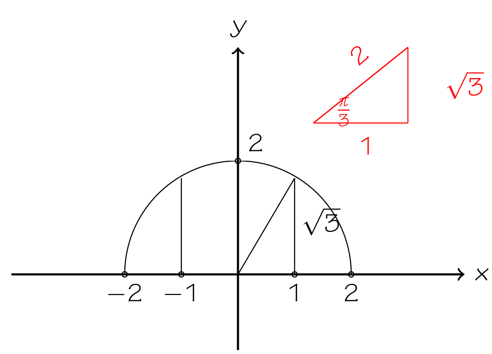
(Q60.) $\int_{-1}^{1}\sqrt{4-x^2}\,dx$

$=2\int_{0}^{1}\sqrt{4-x^2}\,dx$

$=2(\frac{1}{2}\times 1\times\sqrt{3} + \frac{1}{2}2^2\times\frac{\pi}{6})$

$= \boxed{\sqrt{3} + \frac{2\pi}{3}}$

Aside



$\theta = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

Area of Sector $= \frac{1}{2}r^2\theta$

(Q70.) $\int_{\frac{1}{e}}^{e} \frac{\sqrt{1-(\ln x)^2}}{x}\, dx$

$= \int_{-1}^{1} \frac{\sqrt{1-u^2}}{\not{x}}\, \not{x}\, du$

$= \boxed{\frac{\pi}{2}}$

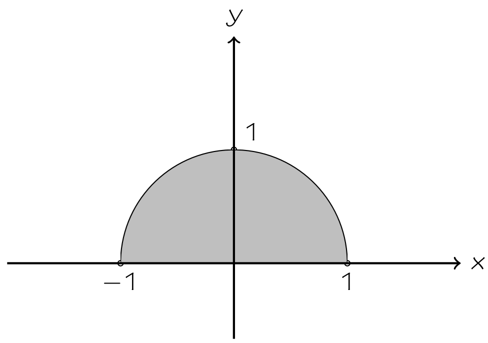
Aside

Let $u = \ln x$

$du = \frac{1}{x}\, dx$

When $x = \frac{1}{e}, u = -1$

When $x = e, u = 1$



Area $= \frac{\pi r^2}{2}$

$= \frac{\pi}{2}$

(Q71.) $\int \frac{1}{\sqrt[3]{x}+1}\, dx$

$= \int \frac{1}{u}\, 3(u-1)^2\, du$

$= 3 \int \frac{(u-1)^2}{u}\, du$

$= 3 \int \frac{u^2-2u+1}{u}\, du$

$= 3 \int (u - 2 + \frac{1}{u})\, du$

$= 3(\frac{1}{2}u^2 - 2u + \ln|u|)$

$= \boxed{\frac{3}{2}(\sqrt[3]{x}+1)^2 - 6(\sqrt[3]{x}+1) + \ln|\sqrt[3]{x}+1| + C}$

Aside

Let $u = \sqrt[3]{x} + 1$

$x = (u-1)^3$

$dx = 3(u-1)^2$