

Eg 8) Differentiate $\frac{1}{x + \sqrt{1+x^2}}$ w.r.t x .

$$\Rightarrow \text{Let } y = \frac{1}{x + \sqrt{1+x^2}} = \frac{1}{x + \sqrt{1+x^2}} \times \frac{x - \sqrt{1+x^2}}{x - \sqrt{1+x^2}}$$

$$= \frac{x - \sqrt{1+x^2}}{x^2 - (1+x^2)} = \frac{x - \sqrt{1+x^2}}{-1} \\ = -x + \sqrt{1+x^2}$$

$$\therefore \frac{dy}{dx} = -1 + \frac{2x}{2\sqrt{1+x^2}} = -1 + \frac{x}{\sqrt{1+x^2}}$$

Eg 9) If $y = (x + \sqrt{x^2 + a^2})^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

$\Rightarrow y = (x + \sqrt{x^2 + a^2})^n$
Differentiating both sides w.r.t x we get

$$\frac{dy}{dx} = n(x + \sqrt{x^2 + a^2})^{n-1} \cdot \frac{d}{dx} (x + \sqrt{x^2 + a^2})$$

$$\Rightarrow n(x + \sqrt{x^2 + a^2})^{n-1} \cdot \left(1 + \frac{2x}{2\sqrt{x^2 + a^2}}\right)$$

$$\Rightarrow n(x + \sqrt{x^2 + a^2})^{n-1} \cdot \left(1 + \frac{x}{\sqrt{x^2 + a^2}}\right)$$

$$\Rightarrow n(x + \sqrt{x^2 + a^2})^{n-1} \cdot \left(\frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}}\right)$$