For the rarer case of right truncated data, the likelihood of x is

$$\frac{f(x)}{\Pr(X > d)} = \frac{f(x)}{F(u)}$$

Where for the expression on the right side we are assuming that X is a continuous random variable.

- Combination of censoring and truncation:

  Data that are both left truncated and right censored have likelihood $\frac{S(u)}{S(d)}$.

  Grouped data that are between d and c_j in the presence of truncation at d has likelihood $\frac{F(c_j)-F(d)}{1-F(d)}$.

Likelihood formulas for the uncombined cases

Discrete distribution, individual data	p_x
Continuous distribution, individual data	$f(x)$
Grouped data	$F(c_j) - F(c_{j-1})$
Individual data censored from above at u	$1 - F(u)$ for censored observations
Individual data censored from below at d	$F(d)$ for censored observations
Individual data truncated from above at u	$\frac{f(x)}{F(u)}$
Individual data truncated from below at d	$\frac{f(x)}{1 - F(d)}$

Preview from Notesale.co.uk
Page 2 of 3

Kaplan-Meier estimator (Product limit estimator)

1. Formula

$$S_n(t) = \prod_{i=1}^{j-1} \left(1 - \frac{s_i}{r_i}\right), \qquad y_{j-1} \le t \le y_j$$

$$p(x) = \begin{cases} S_n(y_{j-1}) - S_n(y_j), & x = y_j,\ 1 \le i \le k \\ 0, & otherwise \end{cases}$$

- Discrete failure rate function: For a discrete random variable that has nonzero probability only for a set of values y_j,

$$h(y_j) = \Pr(Y = y_j | Y \ge y_j) = \frac{p(y_j)}{S(y_{j-1})}, \qquad 1 \le j \le k$$

- Hazard rate function for a continuous distribution,

$$h(x) = \frac{f(x)}{S(x)}$$

$$p(y_j) = S(y_{j-1}) - S(y_j)$$

$$h(y_j) = 1 - \frac{S(y_j)}{S(y_{j-1})}$$

$$S(y_j) = \prod_{i=1}^{j} \left(1 - h(y_j)\right)$$

$$p(y_j) = S(y_{j-1}) - S(y_j) = \prod_{i=1}^{j-1} \left(1 - h(y_j)\right) - \prod_{i=1}^{j} \left(1 - h(y_j)\right)$$

$$= \prod_{i=1}^{j-1} \left(1 - h(y_j)\right)\left(1 - \left(1 - h(y_j)\right)\right) = h(y_j) \prod_{i=1}^{j-1} \left(1 - h(y_j)\right)$$