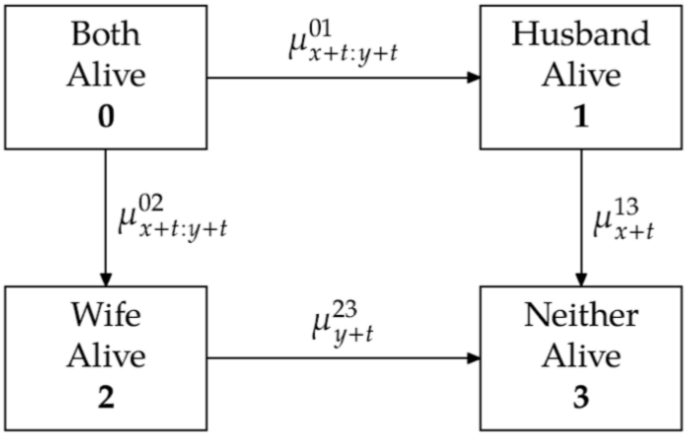
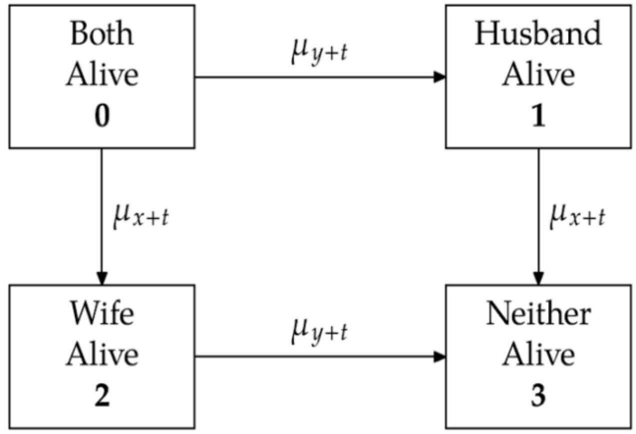
Multiple Lives

Markov chain model





- If the two lives are independent: $\mu^{01}_{x+t:y+t} = \mu^{23}_{y+t}$, $\mu^{02}_{x+t:y+t} = \mu^{13}_{x+t}$
- Notation:

Actuarial notation	Markov chain notation
${}_tp_{xy}$	${}_tp^{00}_{xy}$
${}_tq_{xy}$	${}_tp^{0\cdot}_{xy}$ or ${}_tp^{01}_{xy} + {}_tp^{02}_{xy} + {}_tp^{03}_{xy}$ $p^{0\cdot}$: the probability of transitioning from state 0 to any other state.

- Formula: (xy) is a status that fails when either life dies

$${}_tp_{xy} + {}_tq_{xy} = 1$$
$${}_{t|u}q_{xy} = {}_tp_{xy}\ {}_uq_{x+t:y+t} = {}_tp_{xy} - {}_{t+u}p_{xy}$$
$${}_{t+u}p_{xy} = {}_tp_{xy}\ {}_up_{x+t:y+t}$$
$${}_tp_{xy} = {}_tp^{00}_{xy} = e^{-\int_0^t \mu^{0\cdot}_{x+s:y+s}ds}$$

Independent lives

1. With the assumption of independence, ${}_tp_{xy} = {}_tp_x\ {}_tp_y$, but ${}_tq_{xy} \neq {}_tq_x\ {}_tq_y$, this generalises to any number of lives.
$$1 - {}_tq_{xy} = (1 - {}_tq_x)(1 - {}_tq_y)$$
$${}_tq_{xy} = 1 - (1 - {}_tq_x)(1 - {}_tq_y) = {}_tq_x + {}_tq_y - {}_tq_x\ {}_tq_y$$
2. Under independence, define μ_{xy} as the force of leaving state 0.
$$\mu_{xy} = \mu_x + \mu_y$$
3. If mortality is uniformly distributed, $\mu_x = \frac{1}{\omega - x}$
4. If mortality has a beta distribution, $\mu_x = \frac{\alpha}{\omega - x}$
5. $\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t}$
6. ${}_tp_{xy} = e^{-\int_0^t(\mu_{x+s} + \mu_{y+s})ds}$

Gompertz equivalent age

$${}_tp_{xy} = e^{-Bc^x\left(\frac{c^t-1}{lnc}\right) - Bc^y\left(\frac{c^t-1}{lnc}\right)} = e^{-B(c^x+c^y)\left(\frac{c^t-1}{lnc}\right)}$$

$c^z = c^x + c^y$ or $z = \frac{\ln(c^x+c^y)}{lnc}$