$$\int \cos^{n}(x)\,dx = \int \cos^{n-1}(x)\cos(x)\,dx = \left| \begin{array}{l} u = \cos^{n-1}(x) \\ \frac{du}{dx} = (n-1)\cos^{n-2}(x)\cdot(-\sin x) \\ dv = \cos x\,dx \\ v = \int \cos x\,dx \\ v = \sin x \end{array} \right| =$$

$$= \cos^{n-1}(x)\sin x + n-1 \int \sin^2 x \cos^{n-2} x\,dx =$$

$$= \cos^{n-1}(x)\sin x + n-1 \int (1-\cos^2 x)\cos^{n-2} x\,dx =$$

$$= \cos^{n-1}(x)\sin x + n-1 \int \cos^{n-2} x - \cos^{n} x\,dx =$$

$$= \cos^{n-1}(x)\sin x + n-1 \int \cos^{n-2} x\,dx - (n-1)\int \cos^{n} x\,dx =$$

$$\boxed{I = \int \cos^{n}(x)\,dx}$$

$$I = \cos^{n-1}(x)\sin x + n-1 \int \cos^{n-2} x\,dx - (n-1) I$$

$$I + (n-1) I = \cos^{n-1}(x)\sin x + n-1 \int \cos^{n-2} x\,dx$$

$$I(1+n-1) = \cos^{n-1}(x)\sin x + n-1 \int \cos^{n-2} x\,dx$$

$$I n = \cos^{n-1}(x)\sin x + n-1 \int \cos^{n-2} x\,dx$$

$$I = \frac{1}{n}\cos^{n-1}(x)\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \cos^{n}(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$