Comparing the equations (1), (2), we get

$P = \cot x, \quad \& \quad Q = \sin 2x$

Integration factor

$e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

General solution

$y e^{\int p dx} = \int Q e^{\int p dx} dx + c$

$y \sin x = \int \sin 2x \sin x dx + c \qquad \because \sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

Here A=2x, B=x, substituting in the formula, we get

$y \sin x = \int \frac{1}{2}[\cos(2x-x) - \cos(2x+x)] dx + c$

$y \sin x = \frac{1}{2}\int[\cos(x) - \cos(3x)] dx + c$

$y \sin x = \frac{1}{2}[\frac{\sin x}{2} - \frac{\sin 3x}{3}] + c \qquad \left(\because \int \cos ax dx = \frac{\sin ax}{a}\right)$

Which is required solution.

Problem:-04

Solve $\frac{dy}{dx} + y \cot x = 4x \cos ecx$, given that y=0 when x=$\frac{\pi}{2}$

Solution:-

The given DE is $\frac{dy}{dx} + y \cot x = 4x \cos ecx$ -----(1)

It is of the form $\frac{dy}{dx} + P(x) y = Q(x)$ ----(2)

Given condition y(x)=0 when x=$\frac{\pi}{2}$-----(3) $\qquad [\because y = y(x)]$

Comparing the equations (1), (2), we get

$P = \cot x, \quad \& \quad Q = 4x \cos ecx$

Integration factor

$e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

$$y^{-n}\frac{dy}{dx}+P(x)y^{1-n}=Q(x).$$ ----(2)

Let $z=y^{1-n}$ (replacing the variable y into z)

$$\frac{dz}{dy}=(1-n)y^{1-n-1}$$

$$\frac{dz}{1-n}=y^{-n}dy$$

Update the values in equation(2), we have

$$\frac{y^{-n}dy}{dx}+P(x)y^{1-n}=Q(x).$$

$$\frac{1}{1-n}\frac{dz}{dx}+P(x)z=Q(x).$$

Multiply 1-n on both sides , we get

$$\frac{dz}{dx}+(1-n)P(x)z=(1-n)Q(x).$$

$$\frac{dz}{dx}+P_1(x)z=Q_1(x). \qquad \text{where } P_1(x)=(1-n)P(x) \ \& \ Q_1(x)=(1-n)Q(x)$$

This is the Leibnitz's linear equation in z.

This equation can be solved using following procedure

Integration factor

I.F= $e^{\int P_1(x)dx}$

General solution

$$ye^{\int P_1(x)dx}=\int Q_1(x)e^{\int P_1(x)dx}dx+c$$

Which gives the required solution.

Problem:-1

Solve $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$

Solution:-

The given DE is $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$ -----(1)