Binomial Theorem

* Use of Integration: This method is only applied when the numericals occur as the denominator of the binomial coefficient.

Solution: If $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$ then integrate both sides between suitable limits which gives required series. (i) If sum contains $C_0, C_1, C_2, \dots, C_n$ are all +ve signs, then integrate between 0 to 1. (ii) If sum contains alternate signs then between (-1 to 0). (iii) If sum contains odd coefficients (C_0, C_2, C_4) then between -1 to +1. (iv) If sum contains even coefficients $(C_1, C_3, \dots)$ then subtracting (ii) from (i) then dividing by 2. (v) If in denominator of binomial coefficient product of two numericals then integrate two times taken limits between first times, take suitable limits. 0 to x & second time,

* If $(1+x)^n = C_0 + C_1 x + C_2 x + \dots + C_n x^n$ then

$$C_0C_1 + C_1C_2 + \dots + C_{n-1}C_n = \frac{n!}{(n-1)!\,(n+1)!}$$

$$C_0C_2 + C_1C_3 + \dots + C_{n-2}C_n = \frac{2n!}{(n-2)!\,(n+2)!}$$

* If $n > 6$, then $\left(\frac{n}{3}\right)^n < n! < \left(\frac{n}{2}\right)^n$