Proof that :-

$y = C_1 \cos 2x + C_2 \sin 2x$ is a general solution for $y'' + 4y = 0$ then find the solution that satisfies $y(0) = 2$, $y'(0) = -4$

$y' = -2C_1 \sin 2x + 2C_2 \cos 2x$

$y'' = -4C_1 \cos 2x - 4C_2 \sin 2x$

$4y = 4C_1 \cos 2x + 4C_2 \sin 2x$

$\therefore y'' + 4y = 0$

* First order & first degree differential equations:

1- Seperation:-

$$\int f(x)\,dx = \int g(y)\,dy$$

Ex:-

* $xy' = y$

$\rightarrow x\frac{dy}{dx} = y$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln y = \ln x + C$

* $y'x(1-y^2) + (1+x^2)y = 0$

$\rightarrow y' = \frac{-y(1+x^2)}{x(1-y^2)}$

$\frac{dy}{dx} = \frac{-y(1+x^2)}{x(1-y^2)}$

$\int \frac{(1-y^2)}{y}\,dy = -\int \frac{1+x^2}{x}\,dx$

$\ln y - \frac{y^2}{2} = -\ln x - \frac{x^2}{2} + C$

* $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

$\rightarrow \int \frac{dx}{1+x^2} = \int \frac{dy}{1+y^2}$

$\tan^{-1} x = \tan^{-1} y + C$

Remember That :-

$\rightarrow \int \frac{f'(x)}{a^2 + f^2(x)}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + C$

$\rightarrow \int \frac{f'(x)}{\sqrt{a^2 - f^2(x)}}\,dx = \sin^{-1}\frac{f(x)}{a} + C$

$\rightarrow \int \frac{f'(x)}{f(x)\sqrt{f^2(x) - a^2}}\,dx = \frac{1}{a}\sec^{-1}\frac{f(x)}{a} + C$