And making equal intercepts on the axes.

Solution

(i) Let $l_1:\ 2x+5y-8=0$

$l_2:\ 3x-4y-6=0$

Equation of line passing through point of intersection of l_1 and l_2 is

$$2x+5y-8+k\left(3x-4y-6\right)=0 \ldots \text{ (i)}$$

Since $\left(2,-9\right)$ lies on (i) therefore put $x=2$ and $y=-9$ in (i)

$$2(2)+5(-9)-8+k\left(3(2)-4(-9)-6\right)=0$$

$$\Rightarrow\ 4-45-8+k\left(6+36-6\right)=0$$

$$\Rightarrow\ -49+36k=0$$

$$\Rightarrow\ 36k=49 \qquad \Rightarrow\ k=\frac{49}{36}$$

Putting value of k in (i)

$$2x+5y-8+\frac{49}{36}\left(3x-4y-6\right)=0$$

$$\Rightarrow\ 72x+180y-288+49\left(3x-4y-6\right)=0 \qquad \times\text{ing by } 36$$

$$\Rightarrow\ 72x+180y-288+147x-196y-294=0$$

$$\Rightarrow\ 219x-16y-582=0 \quad \text{is the required equation.}$$

(ii) Let $l_1:\ x-y-4=0$

$l_2:\ 7x+y+20=0$

$l_3:\ 6x+y-14=0$

Let l_4 be a line passing through point of intersection of l_1 and l_2, then

$$l_4:\ l_1+k\,l_2=0$$

$$\Rightarrow\ x-y-4+k\left(7x+y+20\right)=0 \ldots \quad \text{(i)}$$

$$\Rightarrow\ \left(1+7k\right)x+\left(-1+k\right)y+\left(-4+20k\right)=0$$

Slope of $l_4 = m_1 = -\dfrac{1+7k}{-1+k}$

Slope of $l_3 = m_2 = -\dfrac{6}{1} = -6$

(a) If l_3 and l_4 are parallel then

$$m_1=m_2$$

$$\Rightarrow\ -\frac{1+7k}{-1+k}=-6$$

$$\Rightarrow\ 1+7k=6\left(-1+k\right) \qquad \Rightarrow\ 1+7k=-6+6k$$

$$\Rightarrow\ 7k-6k=-6-1 \qquad \Rightarrow\ k=-7$$

Putting value of k in (i)

$$x-y-4-7\left(7x+y+20\right)=0$$

$$\Rightarrow\ x-y-4-49x-7y-140=0$$

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Now equation of $\perp$ bisector having slope -1 through $D(-3,2)$

$$y-2=-1(x+3)$$
$$\Rightarrow\ y-2=-x-3 \quad \Rightarrow\ x+3+y-2=0$$
$$\Rightarrow\ x+y+1=0 \ \ldots\ldots\ldots\ldots\ (\text{iii})$$

Now equation of $\perp$ bisector having slope $-\dfrac{7}{4}$ through $E\left(-\dfrac{1}{2},3\right)$

$$y-3=-\frac{7}{4}\left(x+\frac{1}{2}\right) \quad \Rightarrow\ 4y-12=-7x-\frac{7}{2}$$
$$\Rightarrow\ 7x+\frac{7}{2}+4y-12=0 \quad \Rightarrow\ 7x+4y-\frac{17}{2}=0$$
$$\Rightarrow\ 14x+8y-17=0 \ \ldots\ldots\ldots\ldots\ldots\ (\text{iv})$$

For point of intersection of (iii) and (iv)

$$\frac{x}{-17-8}=\frac{-y}{-17-14}=\frac{1}{8-14}$$
$$\Rightarrow\ \frac{x}{-25}=\frac{-y}{-31}=\frac{1}{-6}$$
$$\Rightarrow\ \frac{x}{-25}=\frac{1}{-6} \quad \text{and} \quad \frac{-y}{-31}=\frac{1}{-6}$$
$$\Rightarrow\ x=\frac{-25}{-6}=\frac{25}{6} \quad \text{and} \quad y=-\frac{31}{6}$$

Hence $\left(\dfrac{25}{6},-\dfrac{31}{6}\right)$ is the circumcentre of the triangle.

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Now to check $(-1,3)$, $\left(-\dfrac{34}{3},\dfrac{58}{3}\right)$ and $\left(\dfrac{25}{6},-\dfrac{31}{6}\right)$ are collinear, let

$$\begin{vmatrix} -1 & 3 & 1 \\ -34/3 & 58/3 & 1 \\ 25/6 & -31/6 & 1 \end{vmatrix}$$
$$=-1\left(\frac{58}{3}+\frac{31}{6}\right)-3\left(-\frac{34}{3}-\frac{25}{6}\right)+1\left(\frac{1054}{18}-\frac{1450}{18}\right)$$
$$=-1\left(\frac{49}{2}\right)-3\left(-\frac{31}{2}\right)+1(-22)$$
$$=-\frac{49}{2}+\frac{93}{2}-22=0$$

Hence centroid, orthocentre and circumcentre of triangle are collinear.

Question # 8

Check whether the lines $4x-3y-8=0$; $3x-4y-6=0$; $x-y-2=0$ are concurrent. If so, find the point where they meet.

$$\Rightarrow \frac{x}{85}=\frac{-y}{153}=\frac{1}{17}$$

$$\Rightarrow \frac{x}{85}=\frac{1}{17} \quad \text{and} \quad \frac{-y}{153}=\frac{1}{17}$$

$$\Rightarrow x=\frac{85}{17}=5 \quad \text{and} \quad y=-\frac{153}{17}=-9$$

$\Rightarrow (5,-9)$ is the point of intersection of l_2 and l_3.

For point of intersection of l_1 and l_3

$$\frac{x}{-3+20}=\frac{-y}{21+30}=\frac{1}{14+3}$$

$$\Rightarrow \frac{x}{17}=\frac{-y}{51}=\frac{1}{17}$$

$$\Rightarrow \frac{x}{17}=\frac{1}{17} \quad \text{and} \quad \frac{-y}{51}=\frac{1}{17}$$

$$\Rightarrow x=\frac{17}{17}=1 \quad \text{and} \quad y=-\frac{51}{17}=3$$

$\Rightarrow (1,-3)$ is the point of intersection of l_1 and l_3.

Now area of triangle having vertices $(3,11)$, $(5,-9)$ and $(1,-3)$ is given by

$$\frac{1}{2}\begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left|3(-9+3)-11(5-1)+1(-15+9)\right|$$

$$=\frac{1}{2}\left|3(-6)-11(4)+1(-6)\right| \quad =\frac{1}{2}\left|-18-44-6\right|$$

$$=\frac{1}{2}\left|-68\right| \quad =\frac{1}{2}(68) \quad =34 \text{ sq. unit}$$

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Question # 15

The vertices of a triangle are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$. Find the centre of the circum centre of the triangle?

Solution *Same Question # 7(c)*

Question # 16

Express the given system of equations in matrix form. Find in each case whether in lines are concurrent.

(a) $x+3y-2=0$; $2x-y+4=0$; $x-11y+14=0$

(b) $2x+3y+4=0$; $x-2y-3=0$; $3x+y-8=0$

(c) $3x-4y-2=0$; $x+2y-4=0$; $3x-2y+5=0$