PLUS TWO

MATHEMATICS REVISION QUESTIONS

CONTENTS

15.Prove that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

16.Evaluate $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

17.Solve for x. If $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$

(a) $|B|$ (b) $k\,|B|$

(c) $k^5|B|$ (d) $5\,|B|$

(ii) Prove that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ = (x-y) (y-z) (z-x)

(iii) Check the consistency of the following equations

$$2x + 3y + z = 6$$

$$x + 2y - z = 2$$

$$7x + y + 2z = 10$$

19. (a) Find the value of x in which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

(b) Using the property of determinants, show that the points

A (a, b + c), B (b, c + a), C (c, a + b) are collinear.

(c) Examine the consistency of system of following equations:

$$5x - 6y + 4z = 15$$

$$7x + 4y - 3z = 19$$

$$2x + y + 6z = 46$$

20. Consider a system of equation which is given below:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \text{ and}$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

(a) Express the above system in the matrix form AX = B

(b) Find A^{-1}, the inverse of A.

(c) Find x, y and z.

15.Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and passes through the point (2, 2, 1).

16.a) Find the vector equation of the line if its Cartesian equation is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

b) Find p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

c) Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

17.Consider the equations of lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{2} = \frac{y-4}{4} = \frac{z-5}{5}$

I.Write the equations in vector form.

II.Find the shortest distance between the lines.

III.Write the equation of the plane passing through the points (1, -2, 3) and normal to the line with direction ratios (1, 3, 2).

18.If θ is the angle between the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ then cos θ=________

19.Consider the Cartesian equation of a line $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-5}{-2}$

I.Find the vector equation.

II.Find its implementing point with the plane $5x + 2y - 6z - 7 = 0$

20.Consider the vector equation of two planes $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 3$ and $\vec{r}.(\hat{\imath} - \hat{\jmath} - \hat{k}) = 4$

I.Find the vector equation of any plane through the intersection of the above two planes.

II.Find the vector equation of the plane through the intersection of the above two planes and the point (1, 2, -1).

21.a) Find the equation of the plane through the points (3, -1, 2), (5, 2, 4) and (-1, -1, 6).

b) Find the perpendicular distance from the point (6, 5, 9) to this plane.

22.The foot of the perpendicular from the origin to a plane is P (4, -2, 5)

I.Write OP

II.Find the equation of the above plane in vector form and Cartesian form.

23.a) Find the angle between the lines having direction ratios <1, 1, 2> and $<\sqrt{3} - 1, -\sqrt{3} - 1, 4>$

b) If the lines $\frac{x-1}{3} = \frac{y-1}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular. Find the value of λ.

MODEL EXAMINATION

HSE II **MATHEMATICS** **Time: 2 hours**

Marks: 80

1. Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 \\ -1 & 4 & 2 \\ 6 & 0 & 8 \end{bmatrix}$$

   a) Find $A + A^T$ and show that it is symmetric

   b) Find $A - A^T$ and show that it is skew symmetric

   c) Express A as the sum of a symmetric & skew symmetric matrix.

2. a) If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4, then find P(A/B)

   b) Find the probability distribution of number of heads X in two tosses of a coin.

   c) Find expectation of X.

3. a) Find the principal value of $\cos^{-1}\left(-1/_2\right)$

   b) Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left[\frac{1-x}{1+x}\right]$

4. a) u = $(sinx)^{tanx}$, v = $(cosx)^{secx}$

   Find $\frac{du}{dx}$ and $\frac{dv}{dx}$

   b) Find $\frac{dy}{dx}$ if y = $(sinx)^{tanx} + (cosx)^{secx}$

   c) If y=$e^{a\cos^{-1}\theta}$, Show that $(1-x^2)y_2 - xy_1 - a^2y = 0$

5. Consider the function

$$f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

   a) Show that f(x) is discontinuous at x = 0

   b) Redefine the function in such a way that it becomes continuous at x= 0

6. Let $A = \begin{bmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 4 & 6 \end{bmatrix}$

   a) Find A^{-1}

   b) Hence solve the system of equations

$$5x - 6y + 4z = 15$$
$$7x + 4y - 3z = 14$$
$$2x + 4y + 6z = 46$$

7. If $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

   a) Perform $R_1 \rightarrow R_1 + R_2 + R_3$ on Δ

   b) Show that

$\Delta = (a+b+c)^3$

8. a) Find the slope of the curve $x^2 + 3y = 3$ at (1, 2)

   b) Find the equation of tangent to $x^2 + 3y = 3$ which is parallel to the line $y - 4x + 5 = 0$. Also find the equation of the normal to the curve at the point of contact.

9. An open box is made by removing squares of equal size from the corners of a tin sheet of size 16cm x 10cm and folding up sides. What is the maximum volume of such a box obtained?

OR

10. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Let (x, y) be a point on the curve nearest to (3, 2). Let S be the distance between these two points
    - I. Find S.
    - II. What is the minimum distance between the soldier and jet.

11. Integrate
    - I. $\int \frac{2x+1}{(x+1)(x-2)} dx$
    - II. $\int \frac{1}{\sqrt{7-6x-x^2}} dx$
    - III. $\int tan^{-1}x \, dx$

12. Find $\int_0^1 x^2 dx$ as the limit of a sum.

OR

13. Consider I = $\int_0^{\frac{\pi}{2}} \log sin\, x\, dx$
    - I. Prove that
      I = $\int_0^{\frac{\pi}{2}} \log cos\, x\, dx$
    - II. Show that
      I = $-\frac{\pi}{2} \log 2$

14. a) Draw the rough sketch of the curves y= x^2 and x=y^2
    b) Find the point of intersection of the two curves.
    c) Find the area bounded by the curves using integration.

15. Consider the differential equation $x\frac{dy}{dx} + y = x\, logx$
    - I. What is the degree of the differential equation
    - II. Find the integrating factor of the above differential equation.
    - III. Solve the above differential equation.

16. Consider the lines
    $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ And
    $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$
    - I. Express these lines in vector form as
      $\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$ and

$$\vec{r} = \overrightarrow{a_1} + \mu\overrightarrow{b_1}$$

II. Compute $\overrightarrow{b_1}$ x $\overrightarrow{b_2}$

III. Compute $\overrightarrow{a_2} - \overrightarrow{a_1}$

IV. Hence find the shortest distance between the lines.

OR

17.Consider the planes

$\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$ and

$\vec{r}.(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) = -5$

I. Find angle between the planes

II. Find the vector equation of the plane passing through the intersection of the above planes and the point (1, 1, 1).

18.Consider $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

I. Find $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

II. Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

19.Consider the points A (2, -1, 1), B (1, -3, -5), C (3, -4, -4)

I. Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$

II. Prove that the above points form a right angled triangle.

20.Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

I. All the five cards are spades?

II. Only 3 cards are spades

OR

21.A bag containing 4 red and 4 black balls, another bag containing 2 red and 6 black balls. One of the two bags is selected at random and a ball is selected from one bag which is found to be red. Find the probability that the ball is drawn from the first bag.

22.a) Find f∘g and g∘f if f(x) = $|x|$ and g(x) = $|5x - 2|$

b) Let A = N x N and '*' be binary operation on A defined by

(a, b) * (c, d) = (a + c, b + d)

I. Show that '*' is commutative and associative.

II. Find the identity element for '*' if any.

23.A company makes 2 products X and Y. The first requires 3 hours for assembling and 4 hours for packing. The second requires 4 hours for assembling and 2 hours for packing. The plant has 60 hours for assembling and 48 hours for packing. The profit margin for X is Rs.7/- and for Y is Rs.21/-

c) Convert this into a linear programming problem.

d) Sketch the graph to maximize the profit.