In other words, x can't approach 3 from the left (through numbers less than 3) because $\sqrt{x^2 - 9}$ is undefined for $x < 3$. Hence, the limit is undefined.

Later on, I'll show that if x approaches 3 *from the right*, then the limit is indeed 0. □

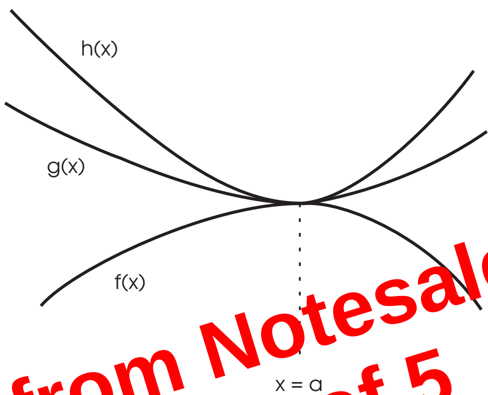
The next result is often called the **Sandwich Theorem** (or the **Squeeze Theorem**). It is different from the other computational rules in that it produces an answer in an indirect way.

The **Sandwich Theorem** is an intuitively obvious result about limits. Suppose you have three functions $f(x)$, $g(x)$, $h(x)$, and you're trying to compute the limit of $g(x)$ as x approaches a.

Suppose you know that:

1. $\lim\limits_{x \to a} f(x) = L$ and $\lim_{x \to a} h(x) = L$.

2. $f(x) \le g(x) \le h(x)$ (at least for x's in some interval around a).



Then

$$\lim_{x \to a} g(x) = L.$$

The result in reasonable because g is "sandwiched" between f and h.

Example. $\lim\limits_{x \to 0} x^2 \sin \dfrac{1}{x}$

As $x \to 0$, $x^2 \to 0$, but $\sin \dfrac{1}{x}$ oscillates. And at $x = 0$, $\sin \dfrac{1}{x}$ is undefined. There's no "algebraic" rule which would allow you to compute the limit, but the Sandwich Theorem makes it easy.

sin(anything) always lies between -1 and 1:

$$-1 \le \sin \frac{1}{x} \le 1.$$

Multiply through by x^2:

$$-x^2 \le x^2 \sin \frac{1}{x} \le x^2.$$

Now

$$\lim_{x \to 0} (-x^2) = 0 \quad \text{and} \quad \lim_{x \to 0} x^2 = 0.$$

Hence, by the Sandwich Theorem

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0.$$