James Gregory (1637-1675) derived the well known (to us) formula

$$\int_0^x \frac{dt}{1+t^2} = \arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots,$$

now called Gregory's formula. Evaluating at $x = 1$, we get

$$\frac{\pi}{4} = 1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \cdots$$

This series was used by **Abraham Sharp** (1651-1742) under instructions from Edmund Halley (1656-1743) (of comet fame) to calculate π to 72 places. (Note the convergence is very slow. How many terms are required for 10^{-72} accuracy?)

John Machin (1680-1751), a professor of astronomy at Gresham College in London obtained 100 places of accuracy for the computation of π using Gregory's formula

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$$

Even **Issac Newton** in 1665-1666 calculated π from the formula

$$\begin{aligned}\pi &= \frac{3\sqrt{3}}{4} + 24\left(\frac{1}{12} - \frac{1}{5 \cdot 2^5} - \frac{1}{28 \cdot 2^7} + \cdots\right) \\ &= \frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{x - x^2}\, dx\end{aligned}$$

Said Newton:

"I am ashamed to tell you to how many figures I carried these computations, having not other business at the time."

In 1874, Machin's formula was used by **William Shanks** (1812-1882) to achieve 707 places of accuracy for π. Because the series is alternating and almost geometric, the convergence is very, very fast. Only about 510 terms are required to achieve this accuracy.

Germany. Perhaps the greatest mathematician that ever lived, **Leonhard Euler** (1707-1783), was born in Basel and studied under James Bernoulli. He eventually found a position at St. Petersburg, originally to serve on the faculty of medicine and physiology.

In 1730, due in part to the death of his colleague and friend, Nicolas Bernoulli, he found himself in the chair of natural philosophy—not medicine. His contributions to mathematics have been well documented and are prodigious.

The Current Record. 20th September 1999

Japan The current record is held by Dr. Yasumasa KANADA of the Computer Centre, University of Tokyo Dear folks,
Our latest record was established as the followings;
Declared record: 206,158,430,000 decimal digits
Two independent calculation based on two different algorithms generated $206,158,430,208(=3*2^{3}6)$ decimal digits of pi and comparison of two generated sequences matched up to 206,158,430,163 decimal digits, e.g., 45 decimal digits difference. Then we are declaring 206,158,430,000 decimal digits as the new world record.
Optimized Main program run:
Job start : 18th September 1999 19:00:52 (JST)
Job end : 20th September 1999 08:21:56 (JST)
Elapsed time : 37:21:04
Main memory : 865 GB (= 6.758 GB * 128)
Algorithm : Gauss-Legendre algorithm

Optimized Verification program run:
Job start : 26th June 1999 01:22:50 (JST)
Job end : 27th June 1999 23:30:40 (JST)
Elapsed time : 46:07:10
Main memory : 817 GB (= 6.383 GB * 128)
Algorithm : Borwein's 4-th order convergent algorithm

Some of interesting digits sequences of pi:
01234567890 : from 53,217,681,704-th of pi
01234567890 : from 148,425,641,592-th of pi
01234567891 : from 26,852,899,245-th of pi
01234567891 : from 41,952,536,161-th of pi
01234567891 : from 99,972,955,571-th of pi
01234567891 : from 102,081,851,717-th of pi
01234567891 : from 171,257,652,369-th of pi
432109876543 : from 149,589,314,822-th of pi
543210987654 : from 197,954,994,289-th of pi
98765432109 : from 123,040,860,473-th of pi
98765432109 : from 133,601,569,485-th of pi
98765432109 : from 150,339,161,883-th of pi
98765432109 : from 183,859,550,237-th of pi
09876543210 : from 42,321,758,803-th of pi
09876543210 : from 57,402,068,394-th of pi
09876543210 : from 83,358,197,954-th of pi
10987654321 : from 89,634,825,550-th of pi
10987654321 : from 137,803,268,208-th of pi