

Around the neighbourhood of $x=x_0$ and $f(x)$ is continuous at x_0 , if $f(x)$ changes orientation from decreasing to increasing at x_0 , then $x=x_0$ is a local minimum point. (19)



Around the neighbourhood of $x=x_0$ and $f(x)$ is continuous at x_0 , and if $f(x)$ does not change orientation at x_0 , then $x=x_0$ is neither a local minimum nor a local maximum point.

Inflection Points (20)

Inflection points may occur when

- i) $f''(x) = 0$ or
- ii) $f''(x)$ does not exist.

If $f''(x) > 0$ on an interval I , this indicates that $f(x)$ is concave up on I .

If $f''(x) < 0$ on an interval I , this indicates that $f(x)$ is concave down on I .

Around the neighbourhood of $x=x_0$, and $f(x)$ and $f'(x)$ are continuous at x_0 , and if $f(x)$ changes concavity at x_0 , then $x=x_0$ is an inflection point. (21)

Around the neighbourhood of $x=x_0$, and $f(x)$ and $f'(x)$ are continuous at x_0 , and if $f(x)$ does not change concavity at x_0 , then $x=x_0$ is not an inflection point.

- (a) If $f'(x_0) = 0$ and $f''(x_0) < 0$, then $f(x)$ has a local maximum at x_0 . (22)
- (b) If $f'(x_0) = 0$ and $f''(x_0) > 0$, then $f(x)$ has a local minimum at x_0 .
- (c) If $f(x)$ has an inflection point at $x=x_0$, then $f''(x_0) = 0$ or $f''(x_0)$ does not exist.
- (d) However, if $f''(x_0) = 0$ or $f''(x_0)$ does not exist, this test fails and we cannot conclude if $x=x_0$ is a local minimum or a local maximum or an inflection point.

Guidelines for curve sketching can be found on page 249 of your textbook. (23)

Example :-

Sketch the graph of $f(x) = 3x^4 + 8x^3 + 8x^2$