

5.5 Properties of Integrals and Average Value

I) Even / odd functions

Let a be positive real number and let f be an integrable function on the interval $[-a, a]$

• If f is even $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

Note: Even function $f(x) = f(-x)$

Evaluate $\int_{-5}^5 x^2 dx = 2 \int_0^5 x^2 dx = 2 \left. \frac{x^3}{3} \right|_0^5 = \frac{2 \cdot (5)^3}{3} - 0$

$$\int_{-\pi/4}^{\pi/4} \cos(x) dx = 2 \int_0^{\pi/4} \cos(x) dx$$

• If f is an odd function then $\int_{-a}^a f(x) dx = 0$

Odd function $f(-x) = -f(x)$



$$\int_{-5}^5 x^3 dx = 0$$

$$\int_{-\pi/3}^{\pi/3} \sin(x) dx = 0$$

Definition

Average value of a function on $[a, b]$

$$\frac{1}{b-a} \int_a^b f(x) dx$$

Example: $f(x) = 2x$ $[1, 5]$

$$\frac{1}{5-1} \int_1^5 2x dx = \left. \frac{1}{4} x^2 \right|_1^5 = \frac{1}{4} (5)^2 - \frac{1}{4} (1)^2 = \boxed{6}$$