We need to fine values a_1 and a_2 such that

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} = a_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2a_1 + a_2 \\ 3a_1 + 2a_2 \end{bmatrix}.$$

To be true, that vector based equation has to match at both coordinates. The first coordinate value gives us the equation $-1 = 2a_1 + a_2$, which can be easily converted into $a_2 = -1 - 2a_1$. The second is

$$1 = 3a_1 + 2a_2 \implies 1 = 3a_1 - 2 - 4a_1 \implies 3 = -a_1$$

leading to $a_1 = -3$ and $a_2 = 5$. The final answer is, basically, 'yes', though it is good to include

$$-3 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -6+5 \\ -9+10 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Dot Products

A dot product between two vectors $\mathbf{x}$ and $\mathbf{y}$ is written simply as $\mathbf{x} \cdot \mathbf{y}$ and calculated like so:

$$\mathbf{x} \cdot \mathbf{y} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = x_1y_1 + x_2y_2 + x_3y_3 + \cdots + x_ny_n.$$

You multiply the equivalent terms in $\mathbf{x}$ and $\mathbf{y}$ then add them all up, resulting in a real number out of two vectors.

If we take the vector $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ in $\mathbb{R}^2$ we can calculate

$$\mathbf{v} \cdot \mathbf{v} = 4 \times 4 + 3 \times 3 = 16 + 9 = 25.$$

Why mention this? Well, consider a more geometric interpretation of $\mathbf{v}$. It is, in actual fact, the hypotenuse of a triangle with remaining lengths 4 and 3. By the Pythagorean theorem, that makes the length of $\mathbf{v}$ equal to the square root of

$$4^2 + 3^2 = 16 + 9 = 25,$$