For instance, $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$, as is $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. Those are clearly not in the same direction. They'll do. Recall that we need to match the right hand side. *Any* combination of those two will not change the total on the left. We just need a single point that matches. So,

$$x = \frac{1}{2} \implies \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}, \qquad y = -1 \implies \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \qquad z = 2 \implies \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}.$$

So, any will do, here's one final set

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} s.$$

Example: Convert the scalar equation $x - 2z = 0$ into a parametric form. We only need the orthogonal vectors, since we have a zero on the right. There is *no y term*, so we can use $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ as an orthogonal vector. The remaining term is $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$. Final answer

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} s.$$

Note: To get the normal vector for the equation, just use the cross product on the two vectors in the parametric form, then get a starting point. As usual, *any* point output by the parametric equation will do.

Example: Convert

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} t + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} s$$

into the scalar equation form.

First, we use the cross product to calculate a normal vector

$$\mathbf{n} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+2 \\ -6-2 \\ -3+1 \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}.$$

This give us an equation of the form

$$4x_1 - 8x_2 - 2x_3 = \text{ something}$$

so we need to find a right hand side value.

As usual, we just need to make it match at one point. If we use $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, the left hand side will be

$$4(1) - 8(0) - 2(0) = 2$$