Now, multiplication, take $g(x)$ in V and $a \in \mathbb{R}$.

$$ag(1) = a(0) = 0$$

so yes, it is a subspace.

Spans

Definition: the Span of a (non-empty) set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_2, \dots, \mathbf{v}_n\} \in V$ is the set of all linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$. So:

$$\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\} = \{a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_n\mathbf{v}_n, \text{ for all } a_1, a_2, \dots, a_n \in \mathbb{R}\}.$$

Again, the set of ALL linear combinations. In essence, everything you can make out of the vectors v_1 to v_n, using vector addition and scalar multiplication.

Lets take a look at a few properties of spans.

First: the zero vector:

$$\mathbf{0} = 0\mathbf{v}_1 + 0\mathbf{v}_2 + \cdots + 0\mathbf{v}_n,$$

which is a perfectly reasonable linear combination.

Next: addition. Take two elements

$$\mathbf{a} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_n\mathbf{v}_n$$

and

$$\mathbf{b} = b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + \cdots + b_n\mathbf{v}_n.$$

Add them together for

$$\begin{aligned}\mathbf{a} + \mathbf{b} &= (a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_n\mathbf{v}_n) + (b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + \cdots + b_n\mathbf{v}_n) \\ &= (a_1 + b_1)\,\mathbf{v}_1 + (a_2 + b_2)\,\mathbf{v}_2 + \cdots + (a_n + b_n)\,v_n,\end{aligned}$$

another linear combination.

Next we check multiplication

$$\begin{aligned}c\,\mathbf{a} &= c\,(a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_n\mathbf{v}_n) \\ &= (ca_1)\mathbf{v}_1 + (ca_2)\mathbf{v}_2 + \cdots + (ca_n)\mathbf{v}_n,\end{aligned}$$

yet another linear combination.

So: we can say with certainty that

Theorem: The $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$, all $\mathbf{v}_k \in V$, is a Subspace of V.

This is saying something else, too. Namely: