They're orthogonal, and so linearly independent, so they span $\mathbb{R}^2$. We just use the inner product:

$$= \frac{-1}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} + \frac{-3}{6}\begin{bmatrix}-1\\1\\2\end{bmatrix} + \frac{6}{3}\begin{bmatrix}1\\-1\\1\end{bmatrix}$$

$$\text{Check: } = \begin{bmatrix}-1/2\\-1/2\\0\end{bmatrix} + \begin{bmatrix}1/2\\-1/2\\-1\end{bmatrix} + \begin{bmatrix}2\\-2\\2\end{bmatrix} = \begin{bmatrix}2\\-3\\1\end{bmatrix}.$$

The helpful thing here is that we can actually expand this into projections, projections on to subspaces (U) and orthogonal complements ($U^\perp$). The result is

$$\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2, \qquad \mathbf{x}_1 \in U \quad \mathbf{x}_2 \in U^\perp.$$

with $\mathbf{x}_1 \cdot \mathbf{x}_2 = 0$, and so on.

Theorem: If $\mathbf{x} \in \mathbb{R}^n$ and $V \subseteq \mathbb{R}^n$ has orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_m\}$ then

$$\text{Proj}_V(\mathbf{x}) = \text{Proj}_{\mathbf{v}_1}(\mathbf{x}) + \text{Proj}_{\mathbf{v}_2}(\mathbf{x}) + \cdots + \text{Proj}_{\mathbf{v}_m}(\mathbf{x}).$$

Proving that this is true is surprisingly easy. Recall the section about projections. The value $\mathbf{z}$ is a projection of $\mathbf{x}$ onto $\mathbf{y}$ if $(\mathbf{x} - \mathbf{z})$ is orthogonal to $\mathbf{y}$ (this means that $\mathbf{z}$ has ALL of the $\mathbf{x}$ component in the $\mathbf{y}$ direction, all that's left is at right angles). We use the same principle. This time we need to confirm that

$$\left(\text{Proj}_{\mathbf{v}_1}(\mathbf{x}) + \text{Proj}_{\mathbf{v}_2}(\mathbf{x}) + \cdots + \text{Proj}_{\mathbf{v}_m}(\mathbf{x}) - \mathbf{x}\right)$$

is orthogonal to all elements in V (which will simultaneously prove we've found the projection onto $V^\perp$!).

Recall that V is spanned by the $\mathbf{v}$ vectors. If we get orthogonality for all of them, we get orthogonality for V. So:

$$\begin{aligned}
&\left(\text{Proj}_{\mathbf{v}_1}(\mathbf{x}) + \text{Proj}_{\mathbf{v}_2}(\mathbf{x}) + \cdots + \text{Proj}_{\mathbf{v}_m}(\mathbf{x}) - \mathbf{x}\right) \cdot \mathbf{v}_i \\
&= \text{Proj}_{\mathbf{v}_i}(\mathbf{x}) \cdot \mathbf{v}_i - \mathbf{x} \cdot \mathbf{v}_i \\
&= \left(\frac{\mathbf{v}_i \cdot \mathbf{x}}{\mathbf{v}_i \cdot \mathbf{v}_i}\mathbf{v}_i\right) \cdot \mathbf{v}_i - \mathbf{x} \cdot \mathbf{v}_i \\
&= \frac{\mathbf{v}_i \cdot \mathbf{x}}{\mathbf{v}_i \cdot \mathbf{v}_i}\mathbf{v}_i \cdot \mathbf{v}_i - \mathbf{x} \cdot \mathbf{v}_i \\
&= \mathbf{x} \cdot \mathbf{v}_i - \mathbf{x} \cdot \mathbf{v}_i = 0
\end{aligned}$$

and done.

Example Questions:

Section 4.5: 2.b), 5.b), 6.b) (Expansion Theorem: the one right above, about using projections to figure out the coefficients).

Section 4.6: 1.df),2, 9.bdf) 14, 16