So:

$$2\begin{vmatrix} -1 & 1 & 0 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & -8 & 9 \\ 0 & 0 & 0 & -1 \end{vmatrix} = 2(-1)(1)(-8)(-1) = -16$$

Things to note:

$$\det(bA) = a^n \det(B), \qquad a \in \mathbb{R}, \quad B \text{ is } n \times n.$$

$$\det(A^T) = \det(A).$$

$$\det(A) + \det(B) \neq \det(A+B), \qquad \text{at least not very often.}$$

Example:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \text{ has } \det(A+B) = 0 \qquad \det(A) + \det(B) = 0.$$

Implication 2: none of the officially recognized Row Operations will multiply the determinant by zero. As a result: if A, B are Row Equivalent then

$$\det(A) = 0 \Longleftrightarrow \det(B) = 0 \quad , \quad \det(A) \neq 0 \Longleftrightarrow \det(B) \neq 0.$$

Here's something else: $\det(I) = 1$. So:

$$\det(A)\det(A^{-1}) = \det(AA^{-1}) = \det(I) = 1$$

Property: $\det(A^{-1}) = \frac{1}{\det(A)}$.

This implies:

$$\det(A) = 0 \quad \Longrightarrow \quad A \text{ not invertible.}$$

Furthermore, if A is not invertible then (using the $\left[\begin{array}{c|c} A & I \end{array}\right]$ algorithm) does not reduce to I on the left. As a result, it reduces to something with a row or column of zeros, so determinant zero. As a result:

$$A \text{ not invertible} \quad \Longrightarrow \quad \det(A) = 0$$

Theorem:

$$A \text{ not invertible} \quad \Longleftrightarrow \quad \det(A) = 0.$$

$$A \text{ invertible} \quad \Longleftrightarrow \quad \det(A) \neq 0.$$

We can also say that $\det(A) = 0$ if and only if A has a missing pivot (as in, not n of them) etc.