Theorem:(Principle Axis)

The following are equivalent for A, an $n \times n$ matrix:

- A is orthogonally diagonalizable
- A has a set of eigenvectors that form an orthonormal basis of $\mathbb{R}^n$
- $A^T = A$ (so A is symmetric).

Proving this is rather difficult, but we'll take a look at one bit. If A is orthogonally diagonalizable then:

$$A = PDP^T \quad \Longrightarrow \quad A^T = \left(PDP^T\right)^T = PD^T(P^T)^T = PDP^T = A,$$

so A has $A^T = A$ and is symmetric.

Procedure for orthogonally diagonalizing a matrix is fairly messy, but is much the same as regular diagonalizing. There's one extra element. The eigenvectors for different eigenvalues will be orthogonal, but those from the same eigenvalue will probably be only linearly independent. When you find an eigenvalue with multiple roots in the characteristic equation, you'll have to use Gram-Schmidt on the resulting vectors.

Example: Diagonalize $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (hint: 1 is an eigenvector).

Exercises:

Section 2.3: 2.bdf), 6.b), 7.bdfh), 8, 17.b)

Section 4.7: 2.bdf), 5.bc) (c is messy)