So:

$$a_1\begin{bmatrix}1\\1\\0\end{bmatrix}+a_2\begin{bmatrix}1\\0\\1\end{bmatrix}+a_3\begin{bmatrix}2\\0\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\longrightarrow\left[\begin{array}{ccc|c}1&1&2&0\\1&0&0&1\\0&1&3&0\end{array}\right].$$

That reduces to $\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&-3\\0&0&1&1\end{array}\right]$, so

$$\begin{aligned}T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)&=T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}-3\begin{bmatrix}1\\0\\1\end{bmatrix}+\begin{bmatrix}2\\0\\3\end{bmatrix}\right)\\&=T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)-3T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)+T\left(\begin{bmatrix}2\\0\\3\end{bmatrix}\right)\\&=\begin{bmatrix}0\\0\end{bmatrix}-3\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}-2\\2\end{bmatrix}.\end{aligned}$$