EXAMPLE: Find the reference angle for

(a) $\theta = \dfrac{5\pi}{3}$ (b) $\theta = 870°$

Solution:

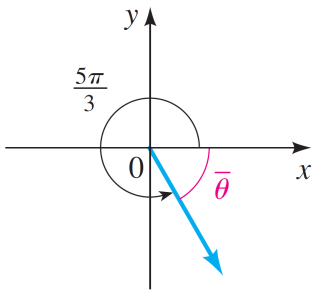
(a) Since

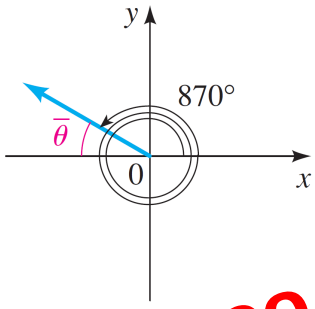
$$\frac{5\pi}{3} = \frac{6\pi - \pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = 2\pi - \frac{\pi}{3}$$

the reference number for $5\pi/3$ is $\pi/3$ and the terminal point of $5\pi/3$ is in Quadrant IV.

(b) The angles $870°$ and $150°$ are coterminal [because $870 - 2(360) = 150$]. Thus, the terminal side of this angle is in Quadrant II (see the second Figure below). So the reference angle is

$$\overline{\theta} = 180° - 150° = 30°$$





EXAMPLE: Find the reference angle for

(a) $\theta = \dfrac{17\pi}{6}$ (b) $\theta = \dfrac{13\pi}{7}$

Solution:

(a) We have $\dfrac{17\pi}{6} = \dfrac{18\pi - \pi}{6} = \dfrac{18\pi}{6} - \dfrac{\pi}{6} = 3\pi - \dfrac{\pi}{6}$, therefore $\overline{\theta} = \dfrac{\pi}{6}$ (Quadrant II).

(b) We have $\dfrac{13\pi}{7} = \dfrac{14\pi - \pi}{7} = \dfrac{14\pi}{7} - \dfrac{\pi}{7} = 2\pi - \dfrac{\pi}{7}$, therefore $\overline{\theta} = \dfrac{\pi}{7}$ (Quadrant IV).

Evaluating Trigonometric Functions for Any Angle

To find the values of the trigonometric functions for any angle θ, we carry out the following steps.

1. Find the reference angle $\overline{\theta}$ associated with the angle θ.
2. Determine the sign of the trigonometric function of θ by noting the quadrant in which θ lies.
3. The value of the trigonometric function of θ is the same, except possibly for sign, as the value of the trigonometric function of $\overline{\theta}$.

EXAMPLE: Find

(a) $\sin 240°$ (b) $\cot 495°$