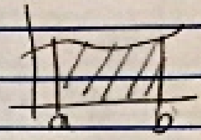
② If $f(x) \geq 0$ on $[a,b]$ then

$$\int_a^b f(x)\,dx = \text{" area under } f(x) \text{ for } a \leq x \leq b \text{"}$$



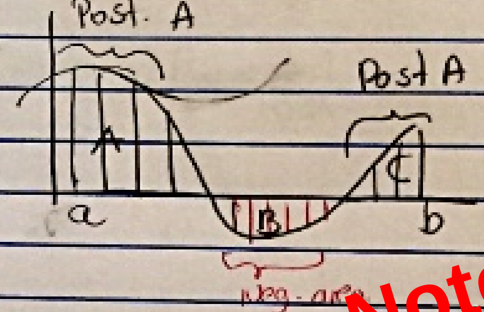
③ If f is both positive & negative on $a \leq x \leq b$

then :> Im repres. net area

(signed area)

$$\int_a^b f(x)\,dx = \text{"}\underline{\text{net area}}\text{ under } f(x) \text{ on } a \leq x \leq b\text{"}$$

Net area



$$\Rightarrow \int_a^b f(x)\,dx = \underbrace{A_A - A_B + A_C}_{\text{Net Area}}$$

④ Limit doesn't always exist but luckily, we have:

Thm:

If f is piecewise continuous on $[a,b]$

then

$$\int_a^b f(x)\,dx \text{ exists}$$

ie. any reasonable f can be integrated.

Q. How can we calculate definite Integrals $\int_a^b f(x)\,dx$ in general?

A for now our solution is hard and inefficient

① Calculate n^{th} Riemann Sum R_n (use magic formula) ✓

② Then $\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n$