M1 Mechanics SUMMARY OF TECHNIQUES

Vectors

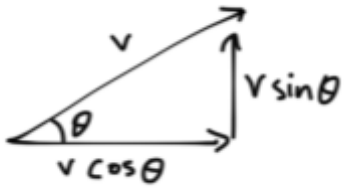
For vectors $\mathbf{a} = \overrightarrow{OA} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\mathbf{b} = \overrightarrow{OB} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$,

$|\mathbf{a}| = \sqrt{(a_1)^2 + (a_2)^2}$ $\quad$ $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\quad$ $\mathbf{a} + \mathbf{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$ $\quad$ Direction of $\mathbf{a} = \tan^{-1}\left(\frac{a_2}{a_1}\right)$

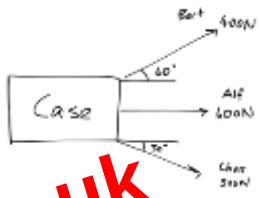
Vectors can be split into their horizontal and vertical components as per the diagram on the right.

The bearing is the angle measured clockwise from north (vertically upwards).

Parallel vectors are multiples (or ratios, same thing) of each other.



A heavy packing case is on a rough, horizontal floor. Alf, Bert and Chas pull with the forces shown in the diagram. The packing case does not move.



Calculate: a) the components of the combined pulling force, b) the magnitude of the force, and c) the angle between the force and the $\mathbf{i}$ direction.

1. Resolve all the forces.

Alf: $\begin{pmatrix} 600 \\ 0 \end{pmatrix}$
Bert: $\begin{pmatrix} 400\cos 60° \\ 400\sin 60° \end{pmatrix}$
Chas: $\begin{pmatrix} 500\cos 30° \\ 500\sin 30° \end{pmatrix}$

2. Add to get the components.
Remember that Chas is pulling down relative to everyone else, therefore his y-component is negative.

$$\begin{pmatrix} 600 \\ 0 \end{pmatrix} + \begin{pmatrix} 400\cos 60° \\ 400\sin 60° \end{pmatrix} + \begin{pmatrix} 500\cos 30° \\ -500\sin 30° \end{pmatrix} = \begin{pmatrix} 1233 \\ 96.4 \end{pmatrix}$$

3. Find the magnitude.

$$\sqrt{1233^2 + 96.4^2} = 1236.8$$

4. Find the angle.

$$\tan^{-1}\frac{96.4}{1233} = 4.47°$$

Dave now joins the other three and pulls with a force of $(\mathbf{100i} + \boldsymbol{p}\mathbf{j})$N. Now the box moves at a steady speed in the direction of $(\mathbf{12i} + \mathbf{j})$. Find p.

5. Set $\boldsymbol{v}$ as the speed and solve.

$$\begin{pmatrix} 1233 \\ 96.4 \end{pmatrix} + \begin{pmatrix} 100 \\ p \end{pmatrix} = v\begin{pmatrix} 12 \\ 1 \end{pmatrix}$$
$$1233 + 100 = 12v$$
$$v = 111$$
$$96.4 + p = 111$$
$$p = 14.6$$