M2 Mechanics SUMMARY OF TECHNIQUES

Work, Energy and Power

The following are the equations needed for this section:

$$Work = Fd \qquad KE = \frac{1}{2}mv^2 \qquad GPE = mgh \qquad P = \frac{Work}{t} = Fv$$

For any movement there are generally 3 parts to the energy change: work against a resistance, change in KE and change in GPE.

A car of 800kg is travelling at a steady speed of 25ms^{-1} along a straight, level road. The car engine is working at the rate of 40 kW. Calculate:

a) The resistance to motion

1. We have the power and the speed, and thus can find the driving force.
   It is at a steady speed so is in equilibrium.

$$P = Fv$$
$$40000 = 25F$$
$$F = 1600N$$

The car reaches a slope with an angle of $\sin^{-1}\frac{1}{10}$ which it climbs at the same speed against the same resistance.

b) What extra power must the engine produce?

2. Find the resistance acting on the car.
3. Use this new force to find the new power for the engine.
4. The extra power needed is the difference between the two powers.

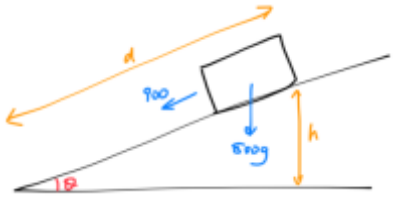
$$F = 1600 + 800g\sin\left(\arcsin\frac{1}{10}\right)$$
$$F = 2384$$
$$P = Fv$$
$$P = 2384(25) = 59.6\,kW$$

Extra power: $59.6 - 40 = 19.6\,kW$

Whilst the car is climbing the slope at a speed of 25ms^{-1}, the power is suddenly removed and the car slows down and comes to rest. Whilst the car is slowing down the resistance may be taken to have a constant rate of 900N.

c) How far along the slope does the car travel whilst slowing down to come to rest?

5. Draw a diagram of the motion (and only over the range of motion).



Consider each part of the energy change:
Note that the change in KE is a loss (it slows down).

6. Since energy is conserved, we know that if we add them all we will get 0.

Work done against resistance:

$$Fd = 900d$$

Change in GPE:

$$\Delta GPE = mgh$$
$$\Delta GPE = 800 \times 9.8 \times d\sin\left(\sin^{-1}\frac{1}{10}\right)$$
$$= 784d$$

Change in KE:

$$\Delta KE = \frac{1}{2}mv^2$$
$$\Delta KE = 0 - \frac{1}{2} \times 800 \times 25^2 = -250000$$

Energy of the system:

$$900d + 784d - 250000 = 0$$
$$d = 148m\ (3sf)$$