C4 Vectors SUMMARY OF TECHNIQUES

Vector equation of a line

Every line in vector form consists of the start point and the direction vector.

The equation of the line through $\binom{a}{b}$ in direction $\binom{c}{d}$ is

$$\mathbf{r} = \binom{a}{b} + \lambda \binom{c}{d}$$

This can also be expressed as

$$\mathbf{r} = \overrightarrow{\mathrm{OA}} + \lambda\, \overrightarrow{\mathrm{AB}}$$

where A and B are two points on the line.

λ is basically how far along the point $\mathbf{r}$ is.

For example:

The vector passes through $\mathbf{A} = \binom{1}{3}$ and $\mathbf{B} = \binom{5}{8}$

1. Find $\overrightarrow{\mathbf{AB}}$, which in this case is $\binom{5}{8} - \binom{1}{3} = \binom{4}{5}$
2. You can now write down the vector equation:

$$\mathbf{r} = \binom{1}{3} + \lambda \binom{4}{5}$$

Vector to Cartesian equations

Since any column vector $\binom{a}{b}$ represents both the horizontal and vertical quantities of a vector, the following becomes true:

$$\mathbf{r} = \binom{a}{b} + \lambda \binom{c}{d}$$

$$\binom{x}{y} = \binom{a}{b} + \lambda \binom{c}{d}$$

$$\mathbf{r} = \binom{1}{3} + \lambda \binom{1}{2}$$

$$\binom{x}{y} = \binom{1}{3} + \lambda \binom{1}{2}$$

The Cartesian equation can now be found with simultaneous equations.

$x = 1 + \lambda$

$\lambda = x - 1$

$y = 3 + 2\lambda$

$y = 3 + 2(x - 1)$

Final answer: $y = 2x + 1$

Cartesian to vector equations

Going the other way is even more straightforward.

For any line $y = mx + c$, the y-intercept can be read off as $\binom{0}{c}$, and the direction of the line is basically the reciprocal of the gradient (as gradient is $\frac{\Delta y}{\Delta x}$).

Therefore if we make the y-intercept the starting point:

$$y = mx + c$$

$$\mathbf{r} = \binom{0}{c} + \lambda \binom{1}{m}$$

$$y = 2x + 3$$

$$\mathbf{r} = \binom{0}{3} + \lambda \binom{1}{2}$$