S1 Probability — SUMMARY OF TECHNIQUES

Discrete random variables

These are variables which can take any discrete value within a range.

These have a **probability function** which dictates the probability for the variable to take a certain value.

They look like this:

$$\mathrm{P}(X=r)=\begin{cases} kr & \text{for r} = 1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

This means that the probability that X (the DRV) is equal to r (a certain value) is kr if r is equal to 1, 2, 3 or 4, and 0 if r is any other number.

When given the probability function, we can list out the probabilities for all the values. This must sum to 1.

Mean of DRVs

This is also known as the expected value. This is basically the weighted average of all the values of the DRV.

$$\mathrm{E}(X)=\sum r\mathrm{P}(X=r)$$

$\sum r\mathrm{P}(X=r)$ means the sum of all the DRV values, multiplied by their respective probabilities.

Variance of DRVs

This is similar to the mean squared deviation in the Data notes.

The variance of DRVs is the $\text{weighted mean of the squares} - \text{square of the mean}$. ie.:

$$\mathrm{Var}(X)=E(x^2)-E(x)^2$$

The number, X, of children per family in a certain city is modelled by the probability distribution:

$$\mathbf{P}(\boldsymbol{X}=\boldsymbol{r})=\boldsymbol{k}(\mathbf{6}-\boldsymbol{r})(\mathbf{1}+\boldsymbol{r})\ \textbf{for}\ \boldsymbol{r}=\mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}$$

Show that the value of k is 1/50. Also find E(X) and Var(X), and write down the probability that a randomly selected family in this city has more than the mean number of children.

1. List out the probabilities. Use the probability function to find the probabilities.
2. We know that the probabilities add to one, so now k can be found.
3. Put k back into the probabilities to get the actual values.

r	0	1	2	3	4
P(X=r)	6k	10k	12k	12k	10k

$$6k+10k+12k+12k+10k=1$$

$$k=\frac{1}{50}$$

r	0	1	2	3	4
P(X=r)	$\frac{6}{50}$	$\frac{10}{50}$	$\frac{12}{50}$	$\frac{12}{50}$	$\frac{10}{50}$

$$\mathrm{E}(X)=\sum r\mathrm{P}(X=r)$$

$$E(X)=0\times\frac{6}{50}+1\times\frac{10}{50}+2\times\frac{12}{50}+3\times\frac{12}{50}+4\times\frac{10}{50}=\frac{11}{5}$$

4. Use the formula to find the mean and variance.

$$\mathrm{Var}(X)=E(x^2)-E(x)^2$$

$$Var(x)=0^2\times\frac{6}{50}+1^2\times\frac{10}{50}+2^2\times\frac{12}{50}+3^2\times\frac{12}{50}+4^2\times\frac{10}{50}-\frac{11^2}{5}$$

$$Var(X)=\frac{42}{25}$$

5. The possibility is $\boldsymbol{P(X>1.72)}$

$$\boldsymbol{P(X>1.72)}=\frac{12}{50}+\frac{12}{50}+\frac{10}{50}=0.68$$