The Rectangular (or uniform) Distribution

A continuous random variable X having a probability density function f(x) such that

$$f(x) = \frac{1}{b-a} \quad \text{for } a \le x \le b$$

where a and b are constants, is said to follow a **rectangular** (or uniform) **distribution**.

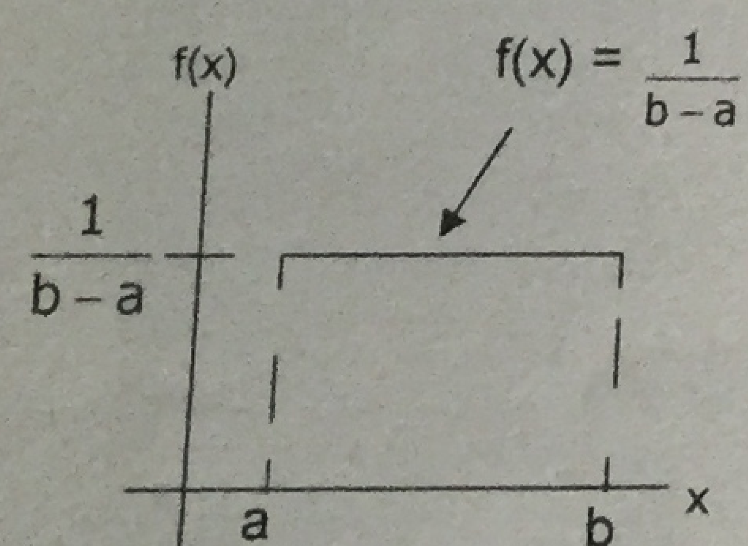
If X is distributed in this way, we write

$$X \sim R(a, b)$$

The graph of y = f(x) is a straight line parallel to the x-axis.

The area of the rectangle must equal 1 and therefore we can write,

$$P(a \le X \le b) = 1$$



$$\text{Area} = \frac{1}{b-a}(b-a) = 1$$

Also,

$$\int_{\text{all } x} f(x)\,dx = \int_a^b \frac{1}{b-a}\,dx = \frac{1}{b-a}[x]_a^b = \frac{1}{b-a}(b-a) = 1$$

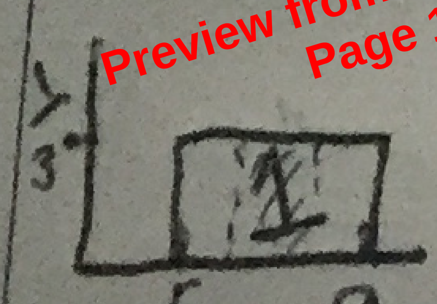
Obviously, the graph for f(x) depends on the values of a and b.

For example, if $X \sim R(0, 4)$ then $f(x) = \frac{1}{4}$ if $0 \le x \le 4$.

Example 1 If $X \sim R(6, 9)$ find $P(7.2 \le x \le 8.4)$

$X \sim R(6, 9)$

$6 - 9 = 3$



← Area = Square (1), So $3 \times x = 1$,

$x = \frac{1}{3}$

$8.4 - 7.2 = 1.2$

$1.2 \times \frac{1}{3} = \underline{0.4}$

Simple area of a Square

Mean and variance of a continuous uniform distribution

If $X \sim R(a, b)$ then

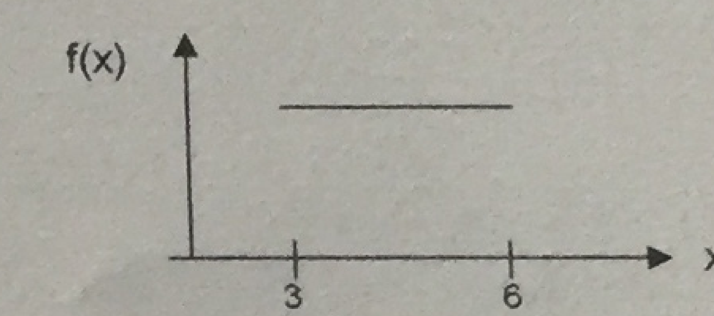
$$E[X] = \frac{1}{2}(a+b)$$

$$Var[X] = \frac{1}{12}(b-a)^2$$

Quotable from the formula booklet

By symmetry, E[X] is half-way between a and b so $E[X] = \frac{1}{2}(a+b)$.

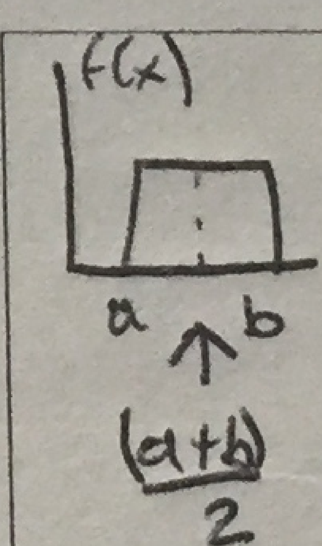
Example 1 Given the graph of the continuous random variable X is



Find,

a) the p.d.f. of X
b) E[X]
c) Var[X]
d) P(X > 5)

Solution



a) P.D.F $= \frac{1}{b-a}$

$\rightarrow \frac{1}{6-3} = \frac{1}{3}$, for $3 \le x \le 6$

b) $E(x) = \frac{1}{2}(a+b) = \frac{1}{2}(6+3) = \underline{4.5}$

c) $Var(x) = \frac{1}{12}(b-a)^2 = \frac{1}{12}(6-3)^2 = \underline{\frac{3}{4}}$

d) $P(x > 5) = (6-5) \times \frac{1}{3} = \underline{\frac{1}{3}}$

PROOF FOR MEAN AND VARIANCE OF RECTANGULAR DISTRIBUTION

To come – may be asked for in exam.