Solving Fractional Equations

$$\frac{x+3}{3-x} + \frac{11x+3}{x^2-9} = \frac{1-5x}{x+3}$$

To identify a fractional (rational) equation …

- Look at the denominators of the equation.
- If there is a VARIABLE in the denominator, then it is a fractional equation.

Special Consideration:

- The denominator of a fraction CANNOT EQUAL ZERO because that would make the term UNDEFINED.
- Therefore, when there is a variable in the denominator, there are values that the variable CANNOT EQUAL.
- DOMAIN:
  - The values that the variable CAN EQUAL.
- When we state the domain at the beginning of the problem, we are noting what the variable CAN EQUAL, which in turn tells us what it cannot equal.

EXTRANEOUS ROOT:

- Is a root of the manipulated equation which does NOT satisfy the original equation.
- For a fractional equation, there is an extraneous root if the root is NOT A MEMBER OF THE DOMAIN.

$$\frac{x+3}{3-x} + \frac{11x+3}{x^2-9} = \frac{1-5x}{x+3}$$

$$(x-3)(x+3)\left[\frac{-x-3}{(x-3)} + \frac{11x+3}{(x+3)(x-3)} = \frac{-5x+1}{(x+3)}\right]$$

$$(-x-3)(x+3) + 11x+3 = (-5x+1)(x-3)$$

$$-x^2-6x-9+11x+3 = -5x^2+15x+x-3$$

$$-x^2+5x-6 = -5x^2+16x-3$$

$$4x^2-11x-3 = 0$$

$$(4x+1)(x-3) = 0$$

$x = -\frac{1}{4}$ $x = 3$ ← extraneous

Domain: $\{x : x \neq \pm 3\}$

$\{-\frac{1}{4}\}$

STEPS:

(1) Find the LCD.

- May need to factor the denominators.
- May need to get rid of negative exponents.

LCD: $(x+3)(x-3)$

(2) Multiply each term of the equation by the LCD.

- Cancel all denominators.

(3) Solve the resulting polynomial equation.

(4) Compare your solution to the domain you have stated.

- Note if any of the roots are extraneous.
- Only the roots that are NOT extraneous are considered solutions.
  - Those roots go in the solution set!