IN THIS CHAPTER you will find that functions are part of our everyday thinking: converting from degrees Celsius to degrees Fahrenheit, DNA testing in forensic science, determining stock values, and the sale price of a shirt. We will develop a more complete, thorough understanding of functions. First, we will establish what a relation is, and then we will determine whether a relation is a function. We will discuss common functions, domain and range of functions, and graphs of functions. We will determine whether a function is increasing or decreasing on an interval and calculate the average rate of change of a function. We will perform operations on functions and composition of functions. We will discuss one-to-one functions and inverse functions. Finally, we will model applications with functions using variation.

FUNCTIONS AND THEIR GRAPHS

3.1 Functions

- Relations and Functions
- Functions Defined by Equations
- Function Notation
- Domain of a Function

3.2 Graphs of Functions; Piecewise-Defined Functions; Increasing and Decreasing Functions; Average Rate of Change

- Recognizing and Classifying Functions
- Increasing and Decreasing Functions
- Average Rate of Change
- Piecewise-Defined Functions

3.3 Graphing Techniques: Transformations

- Horizontal and Vertical Shifts
- Reflection about the Axes
- Stretching and Compressing

3.4 Operations on Functions and Composition of Functions

- Adding, Subtracting, Multiplying, and Dividing Functions
- Composition of Functions

3.5 One-to-One Functions and Inverse Functions

- Determine Whether a Function Is One-to-One
- Inverse Functions
- Graphical Interpretation of Inverse Functions
- Finding the Inverse Function

3.6 Modeling Functions Using Variation

- Direct Variation
- Inverse Variation
- Joint Variation and Combined Variation

LEARNING OBJECTIVES

- Find the domain and range of a function.
- Sketch the graphs of common functions.
- Sketch graphs of general functions employing translations of common functions.
- Perform composition of functions.
- Find the inverse of a function.
- Model applications with functions using variation.

All of the examples we have discussed thus far are **discrete** sets in that they represent a countable set of distinct pairs of (x, y). A function can also be defined algebraically by an equation.

Functions Defined by Equations

Let's start with the equation $y = x^2 - 3x$, where x can be any real number. This equation assigns to each x-value exactly one corresponding y-value.

x	$y = x^2 - 3x$	y
1	$y = (1)^2 - 3(1)$	-2
5	$y = (5)^2 - 3(5)$	10
$-\frac{2}{3}$	$y = \left(-\frac{2}{3}\right)^2 - 3\left(-\frac{2}{3}\right)$	$\frac{22}{9}$
1.2	$y = (1.2)^2 - 3(1.2)$	-2.16

Since the variable y *depends* on what value of x is selected, we denote y as the **dependent variable**. The variable x can be any number in the domain; therefore, we denote x as the **independent variable**.

Although functions are defined by equations, it is important to recognize that *not all equations are functions*. The requirement for an equation to define a function is that each element in the domain corresponds to exactly one element in the range. Throughout the ensuing discussion, we assume x to be the independent variable and y to be the dependent variable.

CAUTION

Not all equations are functions.

Equations that represent functions of x: $y = x^2$ $\quad y = |x|$ $\quad y = x^3$

Equations that do not represent functions of x: $x = y^2$ $\quad x^2 + y^2 = 1$ $\quad x = |y|$

Study Tip

We say that $x = y^2$ is not a function of x. However, if we reverse the independent and dependent variables, then $x = y^2$ is a function of y.

In the "equations that represent functions of x," every x-value corresponds to exactly one y-value. Some ordered pairs that correspond to these functions are

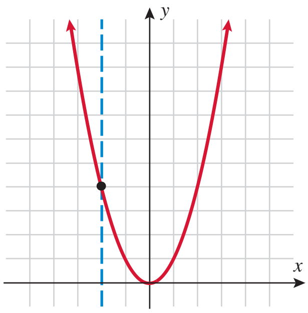
$$y = x^2: \quad (-1, 1)\ (0, 0)\ (1, 1)$$
$$y = |x|: \quad (-1, 1)\ (0, 0)\ (1, 1)$$
$$y = x^3: \quad (-1, -1)\ (0, 0)\ (1, 1)$$

The fact that $x = -1$ and $x = 1$ both correspond to $y = 1$ in the first two examples does not violate the definition of a function.

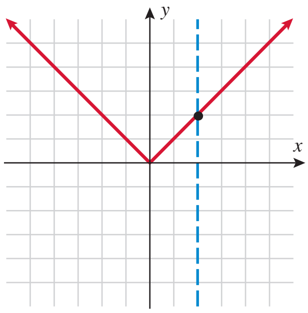
In the "equations that do not represent functions of x," some x-values correspond to *more than one* y-value. Some ordered pairs that correspond to these equations are

Relation	Solve Relation for y	Points That Lie on the Graph	
$x = y^2$	$y = \pm x$	$(1, -1)\ (0, 0)\ (1, 1)$	$x = 1$ maps to **both** $y = -1$ and $y = 1$
$x^2 + y^2 = 1$	$y = \pm\sqrt{1 - x^2}$	$(0, -1)\ (0, 1)\ (-1, 0)\ (1, 0)$	$x = 0$ maps to **both** $y = -1$ and $y = 1$
$x = \lvert y \rvert$	$y = \pm x$	$(1, -1)\ (0, 0)\ (1, 1)$	$x = 1$ maps to **both** $y = -1$ and $y = 1$

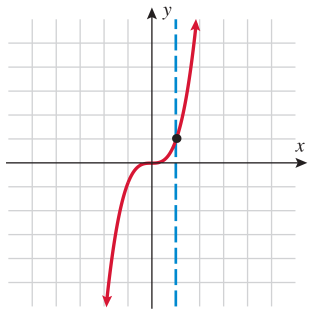
Let's look at the graphs of the three **functions of *x***:



$y = x^2$

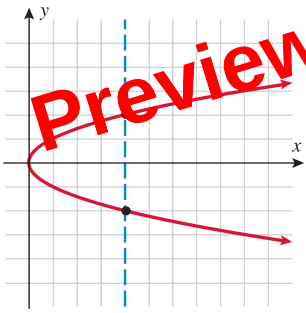


$y = |x|$

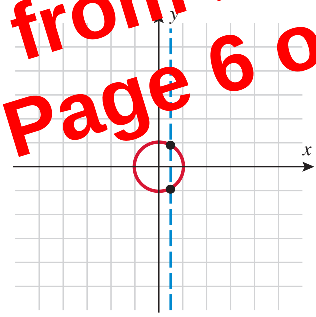


$y = x^3$

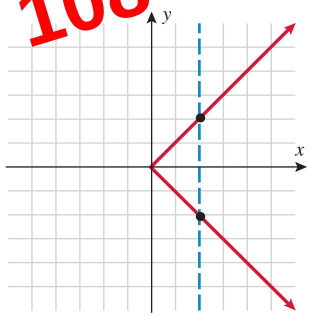
Let's take any value for x, say $x = a$. The graph of $x = a$ corresponds to a vertical line. A function of x maps each x-value to exactly one y-value; therefore, there should be at most one point of intersection with any vertical line. We see in the three graphs of the functions above that if a vertical line is drawn at any value of x on any of the three graphs the vertical line only intersects the graph in one place. Look at the graphs of the three equations that do **not** represent **functions of *x***.



$x = y^2$



$x^2 + y^2 = 1$



$x = |y|$

A vertical line can be drawn on any of the three graphs such that the vertical line will intersect each of these graphs at two points. Thus, there are two y-values that correspond to some x-value in the domain, which is why these equations do not define y as a function of x.

DEFINITION **Vertical Line Test**

Given the graph of an equation, if any vertical line that can be drawn intersects the graph at no more than one point, the equation defines y as a function of x. This test is called the **vertical line test**.

Study Tip

If any x-value corresponds to more than one y-value, then y is **not** a function of x.

Classroom Example 3.1.3
Let $f(x) = -x^4 - x - 1$.
Compute:
a. $f(-1)$ **b.** $-2f(1)$

Answer:
a. -1 **b.** 6

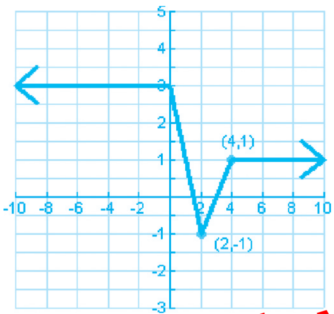
EXAMPLE 3 Evaluating Functions by Substitution

Given the function $f(x) = 2x^3 - 3x^2 + 6$, find $f(-1)$.

Solution:

Consider the independent variable x to be a placeholder.	$f(\square) = 2(\square)^3 - 3(\square)^2 + 6$
To find $f(-1)$, substitute $x = -1$ into the function.	$f(-1) = 2(-1)^3 - 3(-1)^2 + 6$
Evaluate the right side.	$f(-1) = -2 - 3 + 6$
Simplify.	$\boxed{f(-1) = 1}$

Classroom Example 3.1.4
Consider this graph:



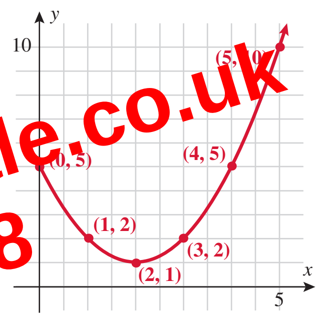
a. Find $f(-3)$.
b. Find $f(2)$.
c.* Find x such that $f(x) = 0$.

Answer:
a. 3 **b.** -1 **c.** 1.5, 3

EXAMPLE 4 Finding Function Values from the Graph of a Function

The graph of f is given on the right.

a. Find $f(0)$.
b. Find $f(1)$.
c. Find $f(2)$.
d. Find $4f(3)$.
e. Find x such that $f(x) = 10$.
f. Find x such that $f(x) = 2$.



Solution (a): The value $x = 0$ corresponds to the value $y = 5$. $\boxed{f(0) = 5}$

Solution (b): The value $x = 1$ corresponds to the value $y = 2$. $\boxed{f(1) = 2}$

Solution (c): The value $x = 2$ corresponds to the value $y = 1$. $\boxed{f(2) = 1}$

Solution (d): The value $x = 3$ corresponds to the value $y = 2$. $4f(3) = 4 \cdot 2 = \boxed{8}$

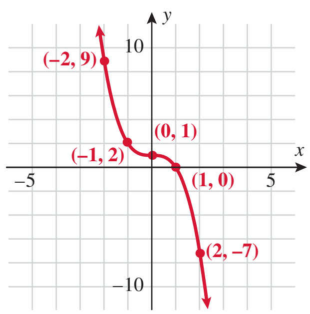
Solution (e): The value $y = 10$ corresponds to the value $\boxed{x = 5}$.

Solution (f): The value $y = 2$ corresponds to the values $\boxed{x = 1}$ and $\boxed{x = 3}$.

■ **Answer:** **a.** $f(-1) = 2$
b. $f(0) = 1$
c. $3f(2) = -21$
d. $x = 1$

■ **YOUR TURN** For the following graph of a function, find:

a. $f(-1)$ **b.** $f(0)$ **c.** $3f(2)$
d. the value of x that corresponds to $f(x) = 0$



In Exercises 65–96, find the domain of the given function. Express the domain in interval notation.

65. $f(x) = 2x - 5$

66. $f(x) = -2x - 5$

67. $g(t) = t^2 + 3t$

68. $h(x) = 3x^4 - 1$

69. $P(x) = \dfrac{x + 5}{x - 5}$

70. $Q(t) = \dfrac{2 - t^2}{t + 3}$

71. $T(x) = \dfrac{2}{x^2 - 4}$

72. $R(x) = \dfrac{1}{x^2 - 1}$

73. $F(x) = \dfrac{1}{x^2 + 1}$

74. $G(t) = \dfrac{2}{t^2 + 4}$

75. $q(x) = \sqrt{7 - x}$

76. $k(t) = \sqrt{t - 7}$

77. $f(x) = \sqrt{2x + 5}$

78. $g(x) = \sqrt{5 - 2x}$

79. $G(t) = \sqrt{t^2 - 4}$

80. $F(x) = \sqrt{x^2 - 25}$

81. $F(x) = \dfrac{1}{\sqrt{x - 3}}$

82. $G(x) = \dfrac{2}{\sqrt{5 - x}}$

83. $f(x) = \sqrt[3]{1 - 2x}$

84. $g(x) = \sqrt[5]{7 - 5x}$

85. $P(x) = \dfrac{1}{\sqrt[5]{x + 4}}$

86. $Q(x) = \dfrac{x}{\sqrt[3]{x^2 - 9}}$

87. $R(x) = \dfrac{x + 1}{\sqrt[4]{3 - 2x}}$

88. $p(x) = \dfrac{x^2}{\sqrt{25 - x^2}}$

89. $H(t) = \dfrac{t}{\sqrt{t^2 - t - 6}}$

90. $f(t) = \dfrac{t - 3}{\sqrt[4]{t^2 + 9}}$

91. $f(x) = (x^2 - 16)^{1/2}$

92. $g(x) = (2x - 5)^{1/3}$

93. $r(x) = x^2(3 - 2x)^{-1/2}$

94. $p(x) = (x - 1)^2 (x^2 - 9)^{-3/5}$

95. $f(x) = \frac{2}{5}x - \frac{2}{4}$

96. $g(x) = \frac{2}{3}x^2 - \frac{1}{6}x - \frac{3}{4}$

97. Let $g(x) = x^2 - 2x - 5$ and find the values of x that correspond to $g(x) = 3$.

98. Let $g(x) = \frac{5}{6}x - \frac{3}{4}$ and find the value of x that corresponds to $g(x) = \frac{2}{3}$.

99. Let $f(x) = 2x(x - 5)^3 - 12(x - 5)^2$ and find the values of x that correspond to $f(x) = 0$.

100. Let $f(x) = 3x(x + 3)^2 - 6(x + 3)^3$ and find the values of x that correspond to $f(x) = 0$.

APPLICATIONS

101. Budget: Event Planning. The cost associated with a catered wedding reception is \$45 per person for a reception for more than 75 people. Write the cost of the reception in terms of the number of guests and state any domain restrictions.

102. Budget: Long-Distance Calling. The cost of a local home phone plan is \$35 for basic service and \$.10 per minute for any domestic long-distance calls. Write the cost of monthly phone service in terms of the number of monthly long-distance minutes and state any domain restrictions.

103. Temperature. The average temperature in Tampa, Florida, in the springtime is given by the function $T(x) = -0.7x^2 + 16.8x - 10.8$, where T is the temperature in degrees Fahrenheit and x is the time of day in military time and is restricted to $6 \le x \le 18$ (sunrise to sunset). What is the temperature at 6 A.M.? What is the temperature at noon?

104. Falling Objects: Firecrackers. A firecracker is launched straight up, and its height is a function of time, $h(t) = -16t^2 + 128t$, where h is the height in feet and t is the time in seconds with $t = 0$ corresponding to the instant it launches. What is the height 4 seconds after launch? What is the domain of this function?

105. Collectibles. The price of a signed Alex Rodriguez baseball card is a function of how many are for sale. When Rodriguez was traded from the Texas Rangers to the New York Yankees in 2004, the going rate for a signed baseball card on eBay was $P(x) = 10 + \sqrt{400{,}000 - 100x}$, where x represents the number of signed cards for sale. What was the value of the card when there were 10 signed cards for sale? What was the value of the card when there were 100 signed cards for sale?

Be careful, though, because functions that are combinations of even- and odd-degree polynomials can turn out to be neither even nor odd, as we will see in Example 1.

EXAMPLE 1 Determining Whether a Function Is Even, Odd, or Neither

Determine whether the functions are even, odd, or neither.

a. $f(x) = x^2 - 3$ **b.** $g(x) = x^5 + x^3$ **c.** $h(x) = x^2 - x$

Classroom Example 3.2.1
Determine whether these functions are even, odd, or neither.

a. $f(x) = 3x^3 + x^5$
b. $f(x) = 3x^3 - x^5$
c.* $f(x) = -(3x^3 + x^5)$
d. $f(x) = 3x^3 + x^5 + 1$
e.* $f(x) = (3x^3 + x^5)^2$

Answer:
a. odd **b.** odd **c.** odd
d. neither **e.** even

Solution (a):

Original function. $f(x) = x^2 - 3$

Replace x with $-x$. $f(-x) = (-x)^2 - 3$

Simplify. $f(-x) = x^2 - 3 = f(x)$

Because $f(-x) = f(x)$, we say that $\boxed{f(x) \text{ is an } even \text{ function}}$.

Solution (b):

Original function. $g(x) = x^5 + x^3$

Replace x with $-x$. $g(-x) = (-x)^5 + (-x)^3$

Simplify. $g(-x) = -x^5 - x^3 = -(x^5 + x^3) = -g(x)$

Because $g(-x) = -g(x)$, we say that $\boxed{g(x) \text{ is an } odd \text{ function}}$.

Solution (c):

Original function. $h(x) = x^2 - x$

Replace x with $-x$. $h(-x) = (-x)^2 - (-x)$

Simplify. $h(-x) = x^2 + x$

$h(-x)$ is neither $-h(x)$ nor $h(x)$; therefore the function $h(x)$ is $\boxed{\text{neither even nor odd}}$.

In parts (a), (b), and (c), we classified these functions as either even, odd, or neither, using the algebraic test. Look back at them now and reflect on whether these classifications agree with your intuition. In part (a), we combined two functions: the square function and the constant function. Both of these functions are even, and adding even functions yields another even function. In part (b), we combined two odd functions: the fifth-power function and the cube function. Both of these functions are odd, and adding two odd functions yields another odd function. In part (c), we combined two functions: the square function and the identity function. The square function is even, and the identity function is odd. In this part, combining an even function with an odd function yields a function that is neither even nor odd and, hence, has no symmetry with respect to the vertical axis or the origin.

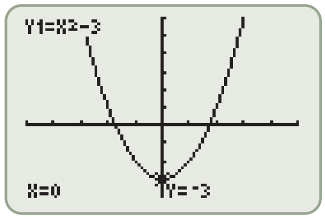
YOUR TURN Classify the functions as even, odd, or neither.

a. $f(x) = |x| + 4$ **b.** $f(x) = x^3 - 1$

Answer: a. even **b.** neither

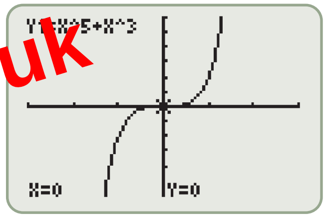
Technology Tip

a. Graph $y_1 = f(x) = x^2 - 3$.



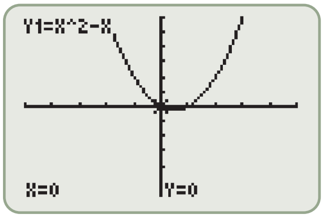
Even; symmetric with respect to the y-axis.

b. Graph $y_1 = g(x) = x^5 + x^3$.



Odd; symmetric with respect to origin.

c. Graph $y_1 = h(x) = x^2 - x$.



No symmetry with respect to y-axis or origin.

84. **Phone Cost: Long-Distance Calling.** A phone company charges \$.39 per minute for the first 10 minutes of an international long-distance phone call and \$.12 per minute every minute after that. Find the cost function $C(x)$ as a function of the length of the phone call x in minutes.

85. **Event Planning.** A young couple are planning their wedding reception at a yacht club. The yacht club charges a flat rate of \$1000 to reserve the dining room for a private party. The cost of food is \$35 per person for the first 100 people and \$25 per person for every additional person beyond the first 100. Write the cost function $C(x)$ as a function of the number of people x attending the reception.

86. **Home Improvement.** An irrigation company gives you an estimate for an eight-zone sprinkler system. The parts are \$1400, and the labor is \$25 per hour. Write a function $C(x)$ that determines the cost of a new sprinkler system if you choose this irrigation company.

87. **Sales.** A famous author negotiates with her publisher the monies she will receive for her next suspense novel. She will receive \$50,000 up front and a 15% royalty rate on the first 100,000 books sold, and 20% on any books sold beyond that. If the book sells for \$20 and royalties are based on the selling price, write a royalties function $R(x)$ as a function of total number x of books sold.

88. **Sales.** Rework Exercise 87 if the author receives \$35,000 up front, 15% for the first 100,000 books sold, and 25% on any books sold beyond that.

89. **Profit.** Some artists are trying to decide whether they will make a profit if they set up a Web-based business to market and sell stained glass that they make. The costs associated with this business are \$[illegible]00 per month for the website and \$700 per month for the studio they rent. The materials cost \$35 for each work in stained glass, and the artists charge \$100 for each unit they sell. Write the monthly profit as a function of the number of stained-glass units they sell.

90. **Profit.** Philip decides to host a shrimp boil at his house as a fundraiser for his daughter's AAU basketball team. He orders gulf shrimp to be flown in from New Orleans. The shrimp costs \$5 per pound. The shipping costs \$30. If he charges \$10 per person, write a function $F(x)$ that represents either his loss or profit as a function of the number of people x that attend. Assume that each person will eat 1 pound of shrimp.

91. **Postage Rates.** The following table corresponds to first-class postage rates for the U.S. Postal Service. Write a piecewise-defined function in terms of the greatest integer function that models this cost of mailing flat envelopes first class.

WEIGHT LESS THAN (OUNCES)	FIRST-CLASS RATE (FLAT ENVELOPES)
1	\$0.80
2	\$0.97
3	\$1.14
4	\$1.31
5	\$1.48
6	\$1.65
7	\$1.82
8	\$1.99
9	\$2.16
10	\$2.33
11	\$2.50
12	\$2.67
13	\$2.84

92. **Postage Rates.** The following table corresponds to first-class postage rates for the U.S. Postal Service. Write a piecewise-defined function in terms of the greatest integer function that models this cost of mailing parcels first class.

WEIGHT LESS THAN (OUNCES)	FIRST-CLASS RATE (PARCELS)
1	\$1.13
2	\$1.30
3	\$1.47
4	\$1.64
5	\$1.81
6	\$1.98
7	\$2.15
8	\$2.32
9	\$2.49
10	\$2.66
11	\$2.83
12	\$3.00
13	\$3.17

CONCEPTUAL

In Exercises 105–108, determine whether each statement is true or false.

105. The identity function is a special case of the linear function.

106. The constant function is a special case of the linear function.

107. If an odd function has an interval where the function is increasing, then it also has to have an interval where the function is decreasing.

108. If an even function has an interval where the function is increasing, then it also has to have an interval where the function is decreasing.

CHALLENGE

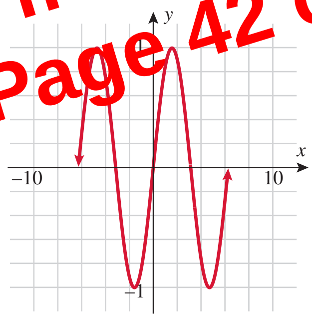
In Exercises 109 and 110, for *a* and *b* real numbers, can the function given ever be a continuous function? If so, specify the value for *a* and *b* that would make it so.

109. $f(x) = \begin{cases} ax & x \leq 2 \\ bx^2 & x > 2 \end{cases}$

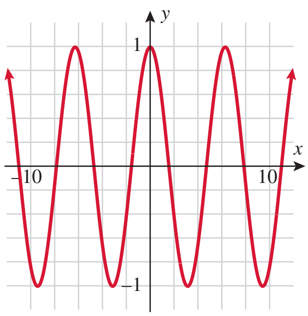
110. $f(x) = \begin{cases} -\dfrac{1}{x} & x < a \\ \dfrac{1}{x} & x \geq a \end{cases}$

TECHNOLOGY

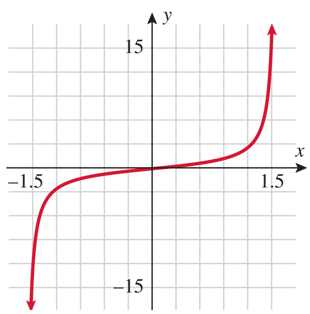
111. In trigonometry you will learn about the sine function, sin x. Plot the function $f(x) = \sin x$, using a graphing utility. It should look like the graph on the right. Is the sine function even, odd, or neither?



112. In trigonometry you will learn about the cosine function, cos x. Plot the function $f(x) = \cos x$, using a graphing utility. It should look like the graph on the right. Is the cosine function even, odd, or neither?



113. In trigonometry you will learn about the tangent function, tan x. Plot the function $f(x) = \tan x$, using a graphing utility. If you restrict the values of x so that $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the graph should resemble the graph below. Is the tangent function even, odd, or neither?



114. Plot the function $f(x) = \dfrac{\sin x}{\cos x}$. What function is this?

115. Graph the function $f(x) = [[3x]]$ using a graphing utility. State the domain and range.

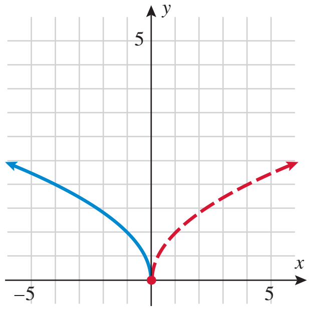
116. Graph the function $f(x) = \left[\left[\frac{1}{3}x\right]\right]$ using a graphing utility. State the domain and range.

Note that if the graph of $f(x) = x^2$ is reflected about the x-axis, the result is the graph of $g(x) = -x^2$. Also note that the function $g(x)$ can be written as the negative of the function $f(x)$; that is, $g(x) = -f(x)$. In general, **reflection about the x-axis** is produced by multiplying a function by -1.

Let's now investigate reflection about the y-axis. To sketch the graphs of $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$ start by listing points that are on each of the graphs and then connecting the points with smooth curves.

x	$f(x)$
0	0
1	1
4	2
9	3

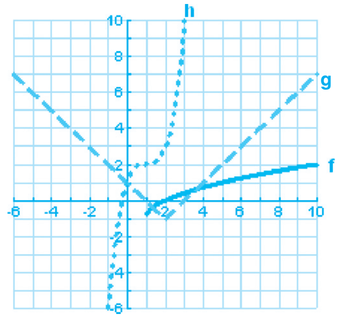
x	$g(x)$
−9	3
−4	2
−1	1
0	0



Note that if the graph of $f(x) = \sqrt{x}$ is reflected about the y-axis, the result is the graph of $g(x) = \sqrt{-x}$. Also note that the function $g(x)$ can be written as $g(x) = f(-x)$. In general, **reflection about the y-axis** is produced by replacing x with $-x$ in the function. Notice that the domain of f is $[0, \infty)$, whereas the domain of g is $(-\infty, 0]$.

Classroom Example 3.3.3
Graph using translation and state the domain and range.
a. $f(x) = \sqrt{x-1} - 1$
b.* $g(x) = |x-2| - 1$
c. $h(x) = (x-1)^3 + 2$

Answer: The graphs are given by:



a. Domain: $[1, \infty)$, Range: $[-1, \infty)$
b. Domain: $(-\infty, \infty)$, Range: $[-1, \infty)$
c. Domain: $(-\infty, \infty)$, Range: $(-\infty, \infty)$

Reflection about the Axes

The graph of $-f(x)$ is obtained by reflecting the graph of $f(x)$ about the x-axis.
The graph of $f(-x)$ is obtained by reflecting the graph of $f(x)$ about the y-axis.

Example 4 Sketching the Graph of a Function Using Both Shifts and Reflections

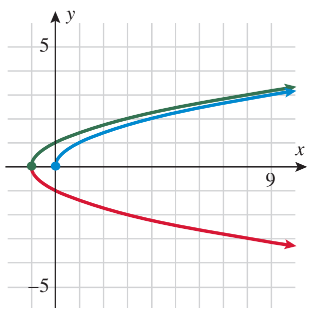
Sketch the graph of the function $G(x) = -\sqrt{x+1}$.

Solution:

Start with the square root function. $f(x) = \sqrt{x}$

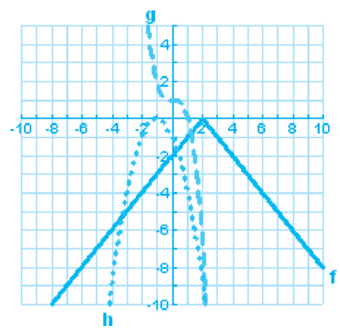
Shift the graph of $f(x)$ to the left one unit to arrive at the graph of $f(x+1)$. $f(x+1) = \sqrt{x+1}$

Reflect the graph of $f(x+1)$ about the x-axis to arrive at the graph of $-f(x+1)$. $-f(x+1) = -\sqrt{x+1}$



Classroom Example 3.3.4
Graph:
a. $f(x) = -|x-2|$
b. $g(x) = -x^3 + 1$
c. $h(x) = -(x+1)^2$

Answer: The graphs are given by:



In Exercises 75–80, transform the function into the form $f(x) = c(x - h)^2 + k$, where c, k, and h are constants, by completing the square. Use graph-shifting techniques to graph the function.

75. $y = x^2 - 6x + 11$

76. $f(x) = x^2 + 2x - 2$

77. $f(x) = -x^2 - 2x$

78. $f(x) = -x^2 + 6x - 7$

79. $f(x) = 2x^2 - 8x + 3$

80. $f(x) = 3x^2 - 6x + 5$

APPLICATIONS

81. Salary. A manager hires an employee at a rate of $10 per hour. Write the function that describes the current salary of the employee as a function of the number of hours worked per week, x. After a year, the manager decides to award the employee a raise equivalent to paying him for an additional 5 hours per week. Write a function that describes the salary of the employee after the raise.

82. Profit. The profit associated with St. Augustine sod in Florida is typically $P(x) = -x^2 + 14{,}000x - 48{,}700{,}000$, where x is the number of pallets sold per year in a normal year. In rainy years Sod King gives away 10 free pallets per year. Write the function that describes the profit of x pallets of sod in rainy years.

83. Taxes. Every year in the United States each working American typically pays in taxes a percentage of his or her earnings (minus the standard deduction). Karen's 2011 taxes were calculated based on the formula $T(x) = 0.22(x - 6500)$. That year the standard deduction was $6500 and her tax bracket paid 22% in taxes. Write the function that will determine her 2012 taxes, assuming she receives the raise that places her in the 33% bracket.

84. Medication. The amount of medication that an infant requires is typically a function of the baby's weight. The number of milliliters of an antiseizure medication A is given by $A(x) = \sqrt{x} + 2$, where x is the weight of the infant in ounces. In emergencies there is often not enough time to weigh the infant, so nurses have to estimate the baby's weight. What is the function that represents the actual amount of medication the infant is given if his weight is overestimated by 3 ounces?

For Exercises 85 and 86, refer to the following:

Body Surface Area (BSA) is used in physiology and medicine for many clinical purposes. BSA can be modeled by the function

$$BSA = \sqrt{\frac{wh}{3600}}$$

where w is weight in kilograms and h is height in centimeters. Since BSA depends on weight and height, it is often thought of as a function of both weight and height. However, for an individual adult height is generally considered constant; thus BSA can be thought of as a function of weight alone.

85. Health/Medicine. (a) If an adult female is 162 centimeters tall, find her BSA as a function of weight. (b) If she loses 3 kilograms, find a function that represents her new BSA.

86. Health/Medicine. (a) If an adult male is 180 centimeters tall, find his BSA as a function of weight. (b) If he gains 5 kilograms, find a function that represents his new BSA.

CATCH THE MISTAKE

In Exercises 87–90, explain the mistake that is made.

87. Describe a procedure for graphing the function $f(x) = \sqrt{x - 3} + 2$.

Solution:

a. Start with the function $f(x) = \sqrt{x}$.

b. Shift the function to the left three units.

c. Shift the function up two units.

This is incorrect. What mistake was made?

88. Describe a procedure for graphing the function $f(x) = -\sqrt{x + 2} - 3$.

Solution:

a. Start with the function $f(x) = \sqrt{x}$.

b. Shift the function to the left two units.

c. Reflect the function about the y-axis.

d. Shift the function down three units.

This is incorrect. What mistake was made?

The previous examples involved polynomials. The domain of any polynomial is the set of all real numbers. Adding, subtracting, and multiplying polynomials result in other polynomials, which have domains of all real numbers. Let's now investigate operations applied to functions that have a restricted domain.

The domain of the sum function, difference function, or product function is the *intersection* of the individual domains of the two functions. The quotient function has a similar domain in that it is the intersection of the two domains. However, any values that make the denominator zero must also be eliminated.

Function	Notation	Domain
Sum	$(f+g)(x) = f(x) + g(x)$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$
Difference	$(f-g)(x) = f(x) - g(x)$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$
Quotient	$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$	$\{\text{domain of } f\} \cap \{\text{domain of } g\} \cap \{g(x) \neq 0\}$

We can think of this in the following way: Any number that is in the domain of *both* the functions is in the domain of the combined function. The exception to this is the quotient function, which also eliminates values that make the denominator equal to zero.

Classroom Example 3.4.1
Let $f(x) = 1 + 2\sqrt[3]{x}$ and $g(x) = -4\sqrt[3]{x}$. Compute $f + g, f - g, fg$, and $\dfrac{f}{g}$. State the domains.

Answer:
$f + g = 1 - 2\sqrt[3]{x}$
Domain: $(-\infty, \infty)$,

$f - g = 1 + 6\sqrt[3]{x}$
Domain: $(-\infty, \infty)$,

$fg = -4\sqrt[3]{x} - 8\sqrt[3]{x^2}$
Domain: $(-\infty, \infty)$,

$\dfrac{f}{g} = \dfrac{1 + 2\sqrt[3]{x}}{-4\sqrt[3]{x}}$
Domain: $x \neq 0$.

EXAMPLE 1 Operations on Functions: Determining Domains of New Functions

For the functions $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{4-x}$, determine the sum function, difference function, product function, and quotient function. State the domain of these four new functions.

Solution:

Sum function: $f(x) + g(x) = \sqrt{x-1} + \sqrt{4-x}$

Difference function: $f(x) - g(x) = \sqrt{x-1} - \sqrt{4-x}$

Product function:
$$f(x) \cdot g(x) = \sqrt{x-1} \cdot \sqrt{4-x} = \sqrt{(x-1)(4-x)} = \sqrt{-x^2 + 5x - 4}$$

Quotient function:
$$\frac{f(x)}{g(x)} = \frac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\frac{x-1}{4-x}}$$

The domain of the square root function is determined by setting the argument under the radical greater than or equal to zero.

$$\text{Domain of } f(x)\text{: } [1, \infty)$$

$$\text{Domain of } g(x)\text{: } (-\infty, 4]$$

The domain of the sum, difference, and product functions is

$$[1, \infty) \cap (-\infty, 4] = [1, 4]$$

The quotient function has the additional constraint that the denominator cannot be zero. This implies that $x \neq 4$, so the domain of the quotient function is $[1, 4)$.

YOUR TURN Given the function $f(x) = \sqrt{x+3}$ and $g(x) = \sqrt{1-x}$, find $(f+g)(x)$ and state its domain.

Answer:
$(f+g)(x) = \sqrt{x+3} + \sqrt{1-x}$
Domain: $[-3, 1]$

In Exercises 51–60, show that $f(g(x)) = x$ and $g(f(x)) = x$.

51. $f(x) = 2x + 1, \quad g(x) = \dfrac{x - 1}{2}$

52. $f(x) = \dfrac{x - 2}{3}, \quad g(x) = 3x + 2$

53. $f(x) = \sqrt{x - 1}, \quad g(x) = x^2 + 1$ for $x \geq 1$

54. $f(x) = 2 - x^2, \quad g(x) = \sqrt{2 - x}$ for $x \leq 2$

55. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x}$ for $x \neq 0$

56. $f(x) = (5 - x)^{1/3}, \quad g(x) = 5 - x^3$

57. $f(x) = 4x^2 - 9, \quad g(x) = \dfrac{\sqrt{x + 9}}{2}$ for $x \geq 0$

58. $f(x) = \sqrt[3]{8x - 1}, \quad g(x) = \dfrac{x^3 + 1}{8}$

59. $f(x) = \dfrac{1}{x - 1}, \quad g(x) = \dfrac{x + 1}{x}$ for $x \neq 0,\ x \neq 1$

60. $f(x) = \sqrt{25 - x^2}, \quad g(x) = \sqrt{25 - x^2}$ for $0 \leq x \leq 5$

In Exercises 61–66, write the function as a composite of two functions f and g. (More than one answer is correct.)

61. $f(g(x)) = 2(3x - 1)^2 + 5(3x - 1)$

62. $f(g(x)) = \dfrac{1}{1 + x^2}$

63. $f(g(x)) = \dfrac{2}{|x - 3|}$

64. $f(g(x)) = \sqrt{1 - x^2}$

65. $f(g(x)) = \dfrac{3}{\sqrt{x + 1} - 2}$

66. $f(g(x)) = \dfrac{\sqrt{x}}{3\sqrt{x} + 2}$

APPLICATIONS

Exercises 67 and 68 depend on the relationship between degrees Fahrenheit, degrees Celsius, and kelvins:

$$F = \frac{9}{5}C + 32 \qquad C = K - 273.15$$

67. Temperature. Write a composite function that converts kelvins into degrees Fahrenheit.

68. Temperature. Convert the following degrees Fahrenheit to kelvins: 32°F and 212°F.

69. Dog Run. Suppose that you want to build a *square* fenced-in area for your dog. Fencing is purchased in linear feet.

a. Write a composite function that determines the area of your dog pen as a function of how many linear feet are purchased.
b. If you purchase 100 linear feet, what is the area of your dog pen?
c. If you purchase 200 linear feet, what is the area of your dog pen?

70. Dog Run. Suppose that you want to build a *circular* fenced-in area for your dog. Fencing is purchased in linear feet.

a. Write a composite function that determines the area of your dog pen as a function of how many linear feet are purchased.
b. If you purchase 100 linear feet, what is the area of your dog pen?
c. If you purchase 200 linear feet, what is the area of your dog pen?

71. Market Price. Typical supply and demand relationships state that as the number of units for sale increases, the market price decreases. Assume that the market price p and the number of units for sale x are related by the demand equation:

$$p = 3000 - \frac{1}{2}x$$

Assume that the cost $C(x)$ of producing x items is governed by the equation

$$C(x) = 2000 + 10x$$

and the revenue $R(x)$ generated by selling x units is governed by

$$R(x) = 100x$$

a. Write the cost as a function of price p.
b. Write the revenue as a function of price p.
c. Write the profit as a function of price p.

72. Market Price. Typical supply and demand relationships state that as the number of units for sale increases, the market price decreases. Assume that the market price p and the number of units for sale x are related by the demand equation:

$$p = 10{,}000 - \frac{1}{4}x$$

Assume that the cost $C(x)$ of producing x items is governed by the equation

$$C(x) = 30{,}000 + 5x$$

and the revenue $R(x)$ generated by selling x units is governed by

$$R(x) = 1000x$$

a. Write the cost as a function of price p.
b. Write the revenue as a function of price p.
c. Write the profit as a function of price p.

In Exercises 73 and 74, refer to the following:

The cost of manufacturing a product is a function of the number of hours t the assembly line is running per day. The number of products manufactured n is a function of the number of hours t the assembly line is operating and is given by the function $n(t)$. The cost of manufacturing the product C measured in thousands of dollars is a function of the quantity manufactured, that is, the function $C(n)$.

73. Business. If the quantity of a product manufactured during a day is given by

$$n(t) = 50t - t^2$$

and the cost of manufacturing the product is given by

$$C(n) = 10n + 1375$$

a. Find a function that gives the cost of manufacturing the product in terms of the number of hours t the assembly line was functioning, $C(n(t))$.

b. Find the cost of production on a day when the assembly line was running for 16 hours. Interpret your answer.

74. Business. If the quantity of a product manufactured during a day is given by

$$n(t) = 100t - 4t^2$$

and the cost of manufacturing the product is given by

$$C(n) = 8n + 2375$$

a. Find a function that gives the cost of manufacturing the product in terms of the number of hours t the assembly line was functioning, $C(n(t))$.

b. Find the cost of production on a day when the assembly line was running for 24 hours. Interpret your answer.

In Exercises 75 and 76, refer to the following:

Surveys performed immediately following an accidental oil spill at sea indicate the oil moved outward from the source of the spill in a nearly circular pattern. The radius of the oil spill r measured in miles is a function of time t measured in days from the start of the spill, while the area of the oil spill is a function of radius, that is, the function $A(r)$.

75. Environment: Oil Spill. If the radius of the oil spill is given by

$$r(t) = 10t - 0.2t^2$$

and the area of the oil spill is given by

$$A(r) = \pi r^2$$

a. Find a function that gives the area of the oil spill in terms of the number of days since the start of the spill, $A(r(t))$.

b. Find the area of the oil spill to the nearest square mile 7 days after the start of the spill.

76. Environment: Oil Spill. If the radius of the oil spill is given by

$$r(t) = 8t - 0.1t^2$$

and the area of the oil spill is given by

$$A(r) = \pi r^2$$

a. Find a function that gives the area of the oil spill in terms of the number of days since the start of the spill, $A(r(t))$.

b. Find the area of the oil spill to the nearest square mile 5 days after the start of the spill.

77. Environment: Oil Spill. An oil spill makes a circular pattern around a ship such that the radius in feet grows as a function of time in hours $r(t) = 150\sqrt{t}$. Find the area of the spill as a function of time.

78. Pool Volume. A 20 foot by 10 foot rectangular pool has been built. If 50 cubic feet of water is pumped into the pool per hour, write the water-level height (feet) as a function of time (hours).

79. Fireworks. A family is watching a fireworks display. If the family is 2 miles from where the fireworks are being launched and the fireworks travel vertically, what is the distance between the family and the fireworks as a function of height above ground?

80. Real Estate. A couple are about to put their house up for sale. They bought the house for \$172,000 a few years ago, and when they list it with a realtor they will pay a 6% commission. Write a function that represents the amount of money they will make on their home as a function of the asking price p.

CATCH THE MISTAKE

In Exercises 81–86, for the functions $f(x) = x + 2$ and $g(x) = x^2 - 4$, find the indicated function and state its domain. Explain the mistake that is made in each problem.

81. $\dfrac{g}{f}$

Solution:

$$\frac{g(x)}{f(x)} = \frac{x^2 - 4}{x + 2} = \frac{(x - 2)(x + 2)}{x + 2} = x - 2$$

Domain: $(-\infty, \infty)$

This is incorrect. What mistake was made?

82. $\dfrac{f}{g}$

Solution:

$$\frac{f(x)}{g(x)} = \frac{x + 2}{x^2 - 4} = \frac{x + 2}{(x - 2)(x + 2)} = \frac{1}{x - 2} = \frac{1}{x - 2}$$

Domain: $(-\infty, 2) \cup (2, \infty)$

This is incorrect. What mistake was made?

5. $\{(0, 1), (1, 2), (2, 3), (3, 4)\}$

6. $\{(0, -2), (2, 0), (5, 3), (-5, -7)\}$

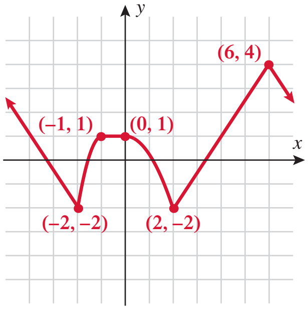
7. $\{(0, 0), (9, -3), (4, -2), (4, 2), (9, 3)\}$

8. $\{(0, 1), (1, 1), (2, 1), (3, 1)\}$

9. $\{(0, 1), (1, 0), (2, 1), (-2, 1), (5, 4), (-3, 4)\}$

10. $\{(0, 0), (-1, -1), (-2, -8), (1, 1), (2, 8)\}$

11.



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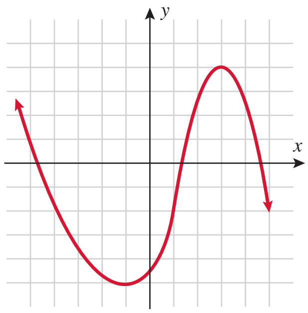

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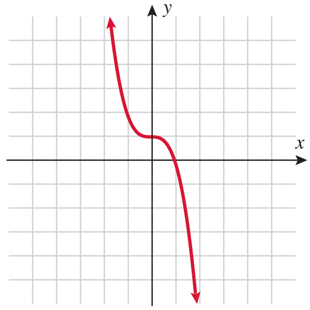

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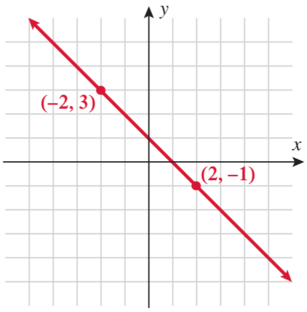

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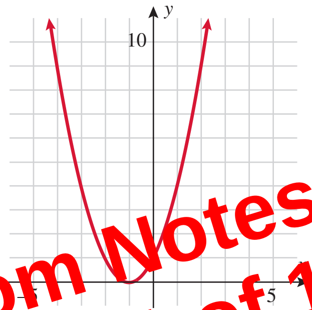

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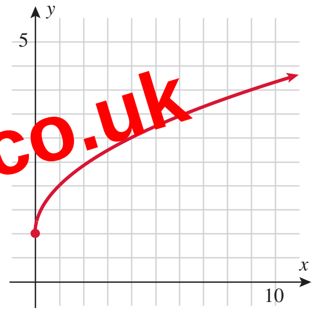


In Exercises 17–24, determine algebraically and graphically whether the function is one-to-one.

17. $f(x) = |x - 3|$

18. $f(x) = (x - 2)^2 + 1$

19. $f(x) = \dfrac{1}{x - 1}$

20. $f(x) = \sqrt[3]{x}$

21. $f(x) = x^2 - 4$

22. $f(x) = \sqrt{x + 1}$

23. $f(x) = x^3 - 1$

24. $f(x) = \dfrac{1}{x + 2}$

In Exercises 25–34, verify that the function $f^{-1}(x)$ is the inverse of $f(x)$ by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. Graph $f(x)$ and $f^{-1}(x)$ on the same axes to show the symmetry about the line $y = x$.

25. $f(x) = 2x + 1;\ f^{-1}(x) = \dfrac{x - 1}{2}$

26. $f(x) = \dfrac{x - 2}{3};\ f^{-1}(x) = 3x + 2$

27. $f(x) = \sqrt{x - 1}, x \geq 1;\ f^{-1}(x) = x^2 + 1, x \geq 0$

28. $f(x) = 2 - x^2, x \geq 0;\ f^{-1}(x) = \sqrt{2 - x}, x \leq 2$

29. $f(x) = \dfrac{1}{x};\ f^{-1}(x) = \dfrac{1}{x}, x \neq 0$

30. $f(x) = (5 - x)^{1/3};\ f^{-1}(x) = 5 - x^3$

31. $f(x) = \dfrac{1}{2x + 6}, x \neq -3;\ f^{-1}(x) = \dfrac{1}{2x} - 3, x \neq 0$

32. $f(x) = \dfrac{3}{4 - x}, x \neq 4;\ f^{-1}(x) = 4 - \dfrac{3}{x}, x \neq 0$

33. $f(x) = \dfrac{x + 3}{x + 4}, x \neq -4;\ f^{-1}(x) = \dfrac{3 - 4x}{x - 1}, x \neq 1$

34. $f(x) = \dfrac{x - 5}{3 - x}, x \neq 3;\ f^{-1}(x) = \dfrac{3x + 5}{x + 1}, x \neq -1$

An example of a **joint variation** is simple interest (Section 1.2), which is defined as

$$I = Prt$$

where

- I is the interest in dollars
- P is the principal (initial) dollars
- r is the interest rate (expressed in decimal form)
- t is time in years

The interest earned is proportional to the product of three quantities (principal, interest rate, and time). Note that if the interest rate increases, then the interest earned also increases. Similarly, if either the initial investment (principal) or the time the money is invested increases, then the interest earned also increases.

An example of **combined variation** is the combined gas law in chemistry,

$$P = k\frac{T}{V}$$

where

- P is pressure
- T is temperature (kelvins)
- V is volume
- k is a gas constant

This relation states that the pressure of a gas is directly proportional to the temperature and inversely proportional to the volume containing the gas. For example, as the temperature increases, the pressure increases, but when the volume decreases, pressure increases.

As an example, the gas in the headspace of a soda bottle has a fixed volume. Therefore, as temperature increases, the pressure increases. Compare the different pressures of opening a twist-off cap on a bottle of soda that is cold versus one that is hot. The hot one feels as though it "releases more pressure."

Classroom Example 3.6.4*
Write an equation describing the following situation: E is directly proportional to m and the square of c. $E = 27.1803$ when $m = 3$ and $c = 3.01$.

Answer: $E = mc^2$

EXAMPLE 4 Combined Variation

The gas in the headspace of a soda bottle has a volume of 9.0 ml, pressure of 2 atm (atmospheres), and a temperature of 298 K (standard room temperature of 77°F). If the soda bottle is stored in a refrigerator, the temperature drops to approximately 279 K (42°F). What is the pressure of the gas in the headspace once the bottle is chilled?

Solution:

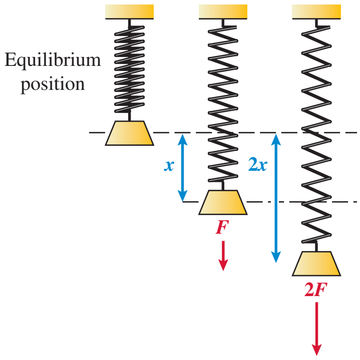
Write the combined gas law.	$P = k\frac{T}{V}$
Let $P = 2$ atm, $T = 298$ K, and $V = 9.0$ ml.	$2 = k\frac{298}{9}$
Solve for k.	$k = \frac{18}{298}$
Let $k = \frac{18}{298}$, $T = 279$, and $V = 9.0$ in $P = k\frac{T}{V}$.	$P = \frac{18}{298} \cdot \frac{279}{9} \approx 1.87$
Since we used the same physical units for both the chilled and room-temperature soda bottles, the pressure is in atmospheres.	$P = 1.87$ atm

For Exercises 43 and 44, refer to the following:

Hooke's law in physics states that if a spring at rest (equilibrium position) has a weight attached to it, then the distance the spring stretches is directly proportional to the force (weight), according to the formula:

$$F = kx$$

where F is the force in Newtons (N), x is the distance stretched in meters (m), and k is the spring constant (N/m).



43. Physics. A force of 30 N will stretch the spring 10 centimeters. How far will a force of 72 N stretch the spring?

44. Physics. A force of 30 N will stretch the spring 10 centimeters. How much force is required to stretch the spring 18 centimeters?

45. Business. A cell phone company develops a pay-as-you-go cell phone plan in which the monthly cost varies directly as the number of minutes used. If the company charges $17.70 in a month when 236 minutes are used, what should the company charge for a month in which 500 minutes are used?

46. Economics. Demand for a product varies inversely with the price per unit of the product. Demand for the product is 10,000 units when the price is $5.75 per unit. Find the demand for the product (to the nearest hundred units) when the price is $6.50.

47. Sales. Levi's makes jeans in a variety of price ranges for juniors. The Flare 519 jeans sell for about $20, whereas the 646 Vintage Flare jeans sell for $300. The demand for Levi's jeans is inversely proportional to the price. If 300,000 pairs of the 519 jeans were bought, approximately how many of the Vintage Flare jeans were bought?

48. Sales. Levi's makes jeans in a variety of price ranges for men. The Silver Tab Baggy jeans sell for about $30, whereas the Offender jeans sell for about $160. The demand for Levi's jeans is inversely proportional to the price. If 400,000 pairs of the Silver Tab Baggy jeans were bought, approximately how many of the Offender jeans were bought?

For Exercises 49 and 50, refer to the following:

In physics, the inverse square law states that any physical quantity or strength is inversely proportional to the square of the distance from the source of that physical quantity. In particular, the intensity of light radiating from a point source is inversely proportional to the square of the distance from the source. Below is a table of average distances from the Sun:

Planet	Distance to the Sun
Mercury	58,000 km
Earth	150,000 km
Mars	228,000 km

49. Solar Radiation. The solar radiation on the Earth is approximately 1400 watts per square meter (w/m^2). How much solar radiation is there on Mars? Round to the nearest hundred watts per square meter.

50. Solar Radiation. The solar radiation on the Earth is approximately 1400 watts per square meter. How much solar radiation is there on Mercury? Round to the nearest hundred watts per square meter.

51. Investments. Marilyn receives a $25,000 bonus from her company and decides to put the money toward a new car that she will need in two years. Simple interest is directly proportional to the principal and the time invested. She compares two different banks' rates on money market accounts. If she goes with Bank of America, she will earn $750 in interest, but if she goes with the Navy Federal Credit Union, she will earn $1500. What is the interest rate on money market accounts at both banks?

52. Investments. Connie and Alvaro sell their house and buy a fixer-upper house. They made $130,000 on the sale of their previous home. They know it will take 6 months before the general contractor will start their renovation, and they want to take advantage of a 6-month CD that pays simple interest. What is the rate of the 6-month CD if they will make $3250 in interest?

53. Chemistry. A gas contained in a 4 milliliter container at a temperature of 300 K has a pressure of 1 atmosphere. If the temperature decreases to 275 K, what is the resulting pressure?

54. Chemistry. A gas contained in a 4 milliliter container at a temperature of 300 K has a pressure of 1 atmosphere. If the container changes to a volume of 3 millileters, what is the resulting pressure?

MODELING OUR WORLD

The U.S. National Oceanic and Atmospheric Association (NOAA) monitors temperature and carbon emissions at its observatory in Mauna Loa, Hawaii. NOAA's goal is to help foster an informed society that uses a comprehensive understanding of the role of the oceans, coasts, and atmosphere in the global ecosystem to make the best social and economic decisions. The data presented in this chapter is from the Mauna Loa Observatory, where historical atmospheric measurements have been recorded for the last 50 years. You will develop linear models based on this data to predict temperature and carbon emissions in the future.

The following table summarizes average yearly temperature in degrees Fahrenheit °F and carbon dioxide emissions in parts per million (ppm) for Mauna Loa, Hawaii.

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005
Temperature (°F)	44.45	43.29	43.61	43.35	46.66	45.71	45.53	47.53	45.86	46.23
CO_2 Emissions (ppm)	316.9	320.0	325.7	331.1	338.7	345.9	354.2	360.6	369.4	379.7

1. Plot the temperature data with time on the horizontal axis and temperature on the vertical axis. Let $t = 0$ correspond to 1960.

2. Find a *linear function* that models the temperature in Mauna Loa.
- **a.** Use data from 1965 and 1995.
- **b.** Use data from 1960 and 1990.
- **c.** Use linear regression and all data given.

3. Predict what the temperature will be in Mauna Loa in 2020.
- **a.** Apply the line found in Exercise 2(a).
- **b.** Apply the line found in Exercise 2(b).
- **c.** Apply the line found in Exercise 2(c).

4. Predict what the temperature will be in Mauna Loa in 2100.
- **a.** Apply the line found in Exercise 2(a).
- **b.** Apply the line found in Exercise 2(b).
- **c.** Apply the line found in Exercise 2(c).

5. Do you think your models support the claim of "global warming"? Explain.

6. Plot the carbon dioxide emissions data with time on the horizontal axis and carbon dioxide levels on the vertical axis. Let $t = 0$ correspond to 1960.

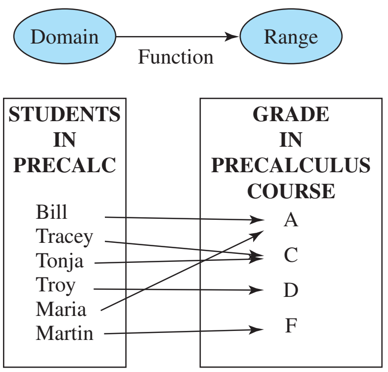
7. Find a *linear function* that models the CO_2 emissions (ppm) in Mauna Loa.
- **a.** Use data from 1965 and 1995.
- **b.** Use data from 1960 and 1990.
- **c.** Use linear regression and all data given.

8. Predict the expected CO_2 levels in Mauna Loa in 2020.
- **a.** Apply the line found in Exercise 7(a).
- **b.** Apply the line found in Exercise 7(b).
- **c.** Apply the line found in Exercise 7(c).

9. Predict the expected CO_2 levels in Mauna Loa in 2100.
- **a.** Apply the line found in Exercise 7(a).
- **b.** Apply the line found in Exercise 7(b).
- **c.** Apply the line found in Exercise 7(c).

10. Do you think your models support the claim of the "greenhouse effect"? Explain.

92.



93. $\{(2, 3), (-1, 2), (3, 3), (-3, -4), (-2, 1)\}$

94. $\{(-3, 9), (5, 25), (2, 4), (3, 9)\}$

95. $\{(-2, 0), (4, 5), (3, 7)\}$

96. $\{(-8, -6), (-4, 2), (0, 3), (2, -8), (7, 4)\}$

97. $y = \sqrt{x}$ **98.** $y = x^2$ **99.** $f(x) = x^3$ **100.** $f(x) = \frac{1}{x^2}$

Verify that the function $f^{-1}(x)$ is the inverse of $f(x)$ by showing that $f(f^{-1}(x)) = x$. Graph $f(x)$ and $f^{-1}(x)$ on the same graph and show the symmetry about the line $y = x$.

101. $f(x) = 3x + 4;\ f^{-1}(x) = \frac{x - 4}{3}$

102. $f(x) = \frac{1}{4x - 7};\ f^{-1}(x) = \frac{1 + 7x}{4x}$

103. $f(x) = \sqrt{x + 4};\ f^{-1}(x) = x^2 - 4 \quad x \geq 0$

104. $f(x) = \frac{x + 2}{x - 7};\ f^{-1}(x) = \frac{7x + 2}{x - 1}$

The function f is one-to-one. Find its inverse and check your answer. State the domain and range of both f and f^{-1}.

105. $f(x) = 2x + 1$ **106.** $f(x) = x^5 + 2$

107. $f(x) = \sqrt{x + 4}$ **108.** $f(x) = (x + 4)^2 + 3 \quad x \geq -4$

109. $f(x) = \frac{x + 6}{x + 3}$ **110.** $f(x) = 2\sqrt[3]{x - 5} - 8$

Applications

111. Salary. A pharmaceutical salesperson makes $22,000 base salary a year plus 8% of the total products sold. Write a function $S(x)$ that represents her yearly salary as a function of the total dollars worth of products sold x. Find $S^{-1}(x)$. What does this inverse function tell you?

112. Volume. Express the volume V of a rectangular box that has a square base of length s and is 3 feet high as a function of the square length. Find V^{-1}. If a certain volume is desired, what does the inverse tell you?

3.6 Modeling Functions Using Variation

Write an equation that describes each variation.

113. C is directly proportional to r. $C = 2\pi$ when $r = 1$.

114. V is directly proportional to both l and w. $V = 12h$ when $w = 6$ and $l = 2$.

115. A varies directly with the square of r. $A = 25\pi$ when $r = 5$.

116. F varies inversely with both λ and L. $F = 20\pi$ when $\lambda = 10\ \mu$m and $L = 10$ km.

Applications

117. Wages. Cole and Dickson both work at the same museum and have the following paycheck information for a certain week. Find an equation that shows their wages (W) varying directly with the number of hours (H) worked.

Employee	Hours Worked	Wage
Cole	27	$229.50
Dickson	30	$255.00

118. Sales Tax. The sales tax in two neighboring counties differs by 1%. A new resident knows the difference but doesn't know which county has the higher tax rate. The resident lives near the border of the two counties and wants to buy a new car. If the tax on a $50.00 jacket is $3.50 in County A and the tax on a $20.00 calculator is $1.60 in County B, write two equations (one for each county) that describe the tax (T), which is directly proportional to the purchase price (P).

Technology Exercises

Section 3.1

119. Use a graphing utility to graph the function and find the domain. Express the domain in interval notation.

$$f(x) = \frac{1}{\sqrt{x^2 - 2x - 3}}$$

120. Use a graphing utility to graph the function and find the domain. Express the domain in interval notation.

$$f(x) = \frac{x^2 - 4x - 5}{x^2 - 9}$$

REVIEW EXERCISES