▪Johnny *is at least* 5 *inches taller than* Debbie.
Q: What universes of discourse are involved?
A: Again, many correct answers. The most obvious answers are:
▪For "Johnny *is taller than* Debbie" the universe of discourse of **both** variables is all people in the universe
▪For "17 *is greater than one of* 12, 45" the universe of discourse of all three variables is **Z** (the set of integers)
▪For "Johnny *is at least* 5 *inches taller than* Debbie" the first and last variable have people as their universe of discourse, while the second variable has **R** (the set of real numbers).

Multivariable Propositional Functions
The multivariable predicates, together with their variables create ***multivariable propositional functions***. In the above examples, we have the following generalizations:
▪x is taller than y
▪a is greater than one of b, c
▪x is at least n inches taller than y

Quantifiers
There are two quantifiers
◆Existential Quantifier
"∃" reads "there exists"
Existential quantifiers synonyms:
" some x " ; " there is an x " ; " there is at least x " ; " for some x "

◆Universal Quantifier
"∀" reads "for all"
Universal quantifiers synonyms:
" for every x " ; " all of x " ; " for each x " ; " given any x " ; " for arbitrary x "

Each is placed in front of a propositional function and ***binds*** it to obtain a proposition with semantic value.
Existential Quantifier
▪"$\exists x \; P(x)$" is true when *an* instance can be found which when plugged in for x makes $P(x)$ true
▪Like disjunctioning over entire universe $\exists x\, P(x) \Leftrightarrow P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots$

Existential Quantifier. Example
Consider a universe consisting of
◆Leo: a lion
◆Jan: an octopus with all 8 tentacles
◆Bill: an octopus with only 7 tentacles
And recall the propositional functions
▪$P(x)$ = "x is an octopus"
▪$Q(x)$ = "x has 8 limbs"

$\exists x \; (P(x) \rightarrow Q(x))$
Q: Is the proposition true or false?
A: True. Proposition is equivalent to
$(P(\text{Leo}) \rightarrow Q(\text{Leo})) \vee (P(\text{Jan}) \rightarrow Q(\text{Jan})) \vee (P(\text{Bill}) \rightarrow Q(\text{Bill}))$
P (Leo) is false because Leo is a Lion, not an octopus, therefore the conditional
$P(\text{Leo}) \rightarrow Q(\text{Leo})$ is true, and the disjunction is true.
Leo is called a *positive example*.

The Universal Quantifier
▪"$\forall x \; P(x)$" true when *every* instance of x makes $P(x)$ true when plugged in
▪Like conjunctioning over entire universe $\forall x\, P(x) \Leftrightarrow P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots$
Example: Consider the same universe and propositional functions as before.
$\forall x \; (P(x) \rightarrow Q(x))$
Q: Is the proposition true or false?
A: False. The proposition is equivalent to
$(P(\text{Leo}) \rightarrow Q(\text{Leo})) \wedge (P(\text{Jan}) \rightarrow Q(\text{Jan})) \wedge (P(\text{Bill}) \rightarrow Q(\text{Bill}))$
Bill is the ***counter-example***, i.e. a value making an instance –and therefore the whole universal quantification– false.
P (Bill) is true because Bill is an octopus, while Q (Bill) is false because Bill only has 7 tentacles, not 8. Thus the conditional $P(\text{Bill}) \rightarrow Q(\text{Bill})$ is false since T→F gives F, and the conjunction is false.

Illegal Quantifications