- $\{1,2\} \notin \mathbf{Z}^+$. This actually makes sense. The set $\{1,2\}$ is an object in its own right, so could be an element of some set; however, $\{1,2\}$ is not a number, therefore is not an element of **Z**.
- $\varnothing \subseteq \varnothing$. Any set contains itself.
- $\varnothing \subset \varnothing$. No set can contain itself properly.

Cardinality

The ***cardinality*** of a set is the number of distinct elements in the set. $|S|$ denotes the cardinality of S.
Q: Compute each cardinality.
- $|\{1, -13, 4, -13, 1\}|$
- $|\{3, \{1,2,3,4\}, \varnothing\}|$
- $|\{\}|$
- $|\{ \{\}, \{\{\}\}, \{\{\{\}\}\} \}|$

Hint: After eliminating the redundancies just look at the number of top level commas and add 1 (except for the empty set).

A:
- $|\{1, -13, 4, -13, 1\}| = |\{1, -13, 4\}| = 3$
- $|\{3, \{1,2,3,4\}, \varnothing\}| = 3$. To see this, set $S = \{1,2,3,4\}$. Compute the cardinality of $\{3,S, \varnothing\}$
- $|\{\}| = |\varnothing| = 0$
- $|\{ \{\}, \{\{\}\}, \{\{\{\}\}\} \}| = |\{ \varnothing, \{\varnothing\}, \{\{\varnothing\}\}\}| = 3$

DEF: The set S is said to be ***finite*** if its cardinality is a nonnegative integer. Otherwise, S is said to be *infinite*.
EG: **N**, **Z**, $\mathbf{Z}^+$, **R**, **Q** are each infinite.

Note: We'll see later that not all infinities are the same. In fact, **R** will end up having a bigger infinity-type than **N**, but surprisingly, **N** has same infinity-type as **Z**, $\mathbf{Z}^+$, and **Q**.

Power Set

DEF: The ***power set*** of S is the set of all subsets of S.
Denote the power set by $P(S)$ or by 2^S.
The latter weird notation comes from the following.