

* Prove $e^{j\pi} = -1$

let us take $f(x) = y$

$$y = \cos(x) + j\sin(x)$$

$$\Rightarrow \frac{dy}{dx} = -\sin(x) + j\cos(x)$$

$$\Rightarrow \frac{dy}{dx} = j(\cos(x) + j\sin(x)) \quad [\because j^2 = -1]$$

$$\Rightarrow \frac{dy}{dx} = jy \quad [y = \cos(x) + j\sin(x)]$$

$$\Rightarrow \frac{dy}{y} = j dx$$

$$\Rightarrow \int \frac{dy}{y} = \int j dx$$

$$\Rightarrow \ln|y| = jx$$

$$\Rightarrow e^{\ln|y|} = e^{jx}$$