Mathematics IGCSE notes

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{also use the intranet revision course of question papers and answers by topic }

Questions.

(a) Water needs to be removed from an underground chamber before work can commence. When the water was at a depth of 3m, five suction pipes were used and emptied the chamber in 4 hours. If the water is now at a depth of 5m (same cross-section), and you want to empty the chamber in 10 hours time, how many pipes need to be used?

(b) y is proportional to x^2 and when x is 5 y is 6. Find
(i) y when x is 25 (ii) x when y is 8.64

(c) The time t seconds taken for an object to travel a certain distance from rest is inversely proportional to the square root of the acceleration a. When a is 4m/s^2, t is 2s.
What is the value of a if the time taken is 5 seconds?

Answers

(a) No. of pipes = $5 \times \dfrac{5}{3} \times \dfrac{4}{10} = 3\tfrac{1}{3}$, so it would be necessary to use 4 pipes to be sure of emptying within 10 hours.

(b) $y \propto x^2$ and we know when x is 5, y is 6, so
$y = kx^2$

$6 = k \times 5^2$, so $k = \tfrac{6}{25}$ and we can write the relationship as
$y = \tfrac{6}{25} x^2$

(i) When x is 25, $y = \tfrac{6}{25} \times 25^2 = 150$.
(ii) When y is 8.64, $8.64 = \tfrac{6}{25} x^2$, so

$x^2 = \dfrac{25 \times 8.64}{6}$ no, don't reach for the calculator yet!

$x^2 = 25 \times 1.44$, so $x = 5 \times 1.2 = 6$.

(c) $t \propto \dfrac{1}{\sqrt{a}}$,

So $t = \dfrac{k}{\sqrt{a}}$. Substituting given values:

$2 = \dfrac{k}{\sqrt{4}}$, so $k = 4$, ie

$t = \dfrac{4}{\sqrt{a}}$.

When t = 5, $5 = \dfrac{4}{\sqrt{a}}$, so $\sqrt{a} = \dfrac{4}{5}$, and $a = \dfrac{16}{25}$ or 0.64 m/s^2.

5. Fractions and ratios

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(a) Fractions

(i) Adding/subtracting: e.g. $3\frac{1}{6}-1\frac{2}{3}$. Convert to vulgar form first: $\frac{19}{6}-\frac{5}{3}$, then find the lowest common denominator, in this case 6. Then

$$\frac{19}{6}-\frac{5}{3}=\frac{19-2\times5}{6} = \frac{9}{6} = 1\frac{1}{2}.$$

(ii) Multiplying/dividing: e.g. $5\frac{1}{3}\times\frac{7}{8}$. Convert to vulgar form: $\frac{16}{3}\times\frac{7}{8}$, and then always cancel any factor in the numerator with a factor in the denominator if possible, before multiplying together:

$$\frac{16}{3}\times\frac{7}{8} = \frac{{}^{2}\cancel{16}}{3}\times\frac{7}{{}^{1}\cancel{8}} = \frac{2\times7}{3\times1} = \frac{14}{3}.$$

To divide, turn the $\div$ into a $\times$ and invert the second fraction.

(iii) Converting to and from decimals: e.g. what is $\frac{3}{40}$ as a decimal?

$$\begin{array}{r} 0.075 \\ 40\overline{)3.000} \end{array}$$ so $\frac{3}{40}$ is 0.075.

But what is 0.075 as a fraction? 0.075 means $\frac{75}{1000}$, then cancel down to $\frac{3}{40}$.

(b) Ratios

(iv) To divide a quantity into 3 parts in the ratio 3: 4:5, call the divisions 3 parts, 4 parts and 5 parts. There are 12 parts altogether, so find 1 part, and hence the 3 portions.

(v) To find the ratio of several quantities, express in the same units then cancel or multiply up until in lowest terms e.g. what is the ratio of 3.0m to 2.25m to 75cm?
Perhaps metres is the best unit to use here, so the ratio is 3 :2.25: 0.75.
Multiplying up by 4 (or 100 if you really insist) will render all numbers integer. So the ratio is 12 : 9 : 3, and we can now cancel down to 4:3:1

7. **Rationals and irrational numbers**

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A rational number is one which can be expressed as $\frac{a}{b}$ where a and b are integers. An irrational number is one which can't. Fractions, integers, and recurring decimals are rational. Examples of rationals: $\frac{2}{3}$, 1, 0.25, $\sqrt[3]{8}$.
Examples of irrationals: π, $\sqrt{2}$, 0.1234....(not recurring).

(i) Converting rationals to the form $\frac{a}{b}$ (to confirm they really are rational)

A terminating decimal: $0.125 = \frac{125}{1000} = \frac{1}{8}$

A recurring decimal: $0.\dot{1}2\dot{3}$. Call the number x, so $x = 0.123123123......$
Multiply by a suitable power of 10 so the recurring decimal appears exactly again: $1000x = 123.123123.....$ $= 123 + 0.123123....$

so $1000x = 123 + x$, then $999x = 123$ and $x = \frac{123}{999} = \frac{41}{333}$.

(ii) rationalising a denominator:

$3\frac{\sqrt{2}}{\sqrt{3}}$ has a $\sqrt{3}$ in the denominator, so multiply top and bottom by $\sqrt{3}$ (which does not change the value of the expression, only the shape):

$3\frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3\frac{\sqrt{6}}{3} = \sqrt{6}$.

(iii)

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$
$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

and the same with cube roots, etc.
To simplify expressions using these:

$\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

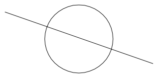
(iv) Finding irrational numbers in a given area:
e.g. find an irrational number between 5 and 6. Note that most square roots are irrational (except for $\sqrt{16}$, $\sqrt{\frac{4}{9}}$, etc) are irrational, so as $5 = \sqrt{25}$ and $6 = \sqrt{36}$, pick a root in between, e.g. $\sqrt{28}$. (Or say $\pi + 2$ for example).

one linear, one quadratic:

$x^2 + y^2 = 25$

$x + y = 0.8$

Rearrange the linear equation and substitute into the quadratic:
$y = 0.8 - x$, so $x^2 + (0.8 - x)^2 = 25$. Multiply out, and solve the quadratic in *x*. Note that each *x* answer will then produce a *y* answer, and this gives two pairs, as it should because the equations represent the intersection of :



Questions

(a) Solve $\frac{x}{3} - \frac{1-x}{2} = 1$

(b) Solve $x^2 + 2x = 15$

(c) Solve $2x^2 + x - 6 = 0$

(d) Solve $x - \frac{1}{x} = 2$

(e) Solve the simultaneous equations $\begin{aligned} x + 2y &= 5 \\ x^2 - y^2 &= -3 \end{aligned}$

Answers

(a) $\frac{x}{3} - \frac{1-x}{2} = 1$ [×6]

$2x - 3(1-x) = 6 \quad \rightarrow 2x - 3 + 3x = 6$

$5x = 9$, so $x = \frac{9}{5}$.

(b) $x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5,\ 3$.

(c) $2 \times -6 = -12$, so look for two numbers which multiply to –12 and add to 1. These are 4, -3.

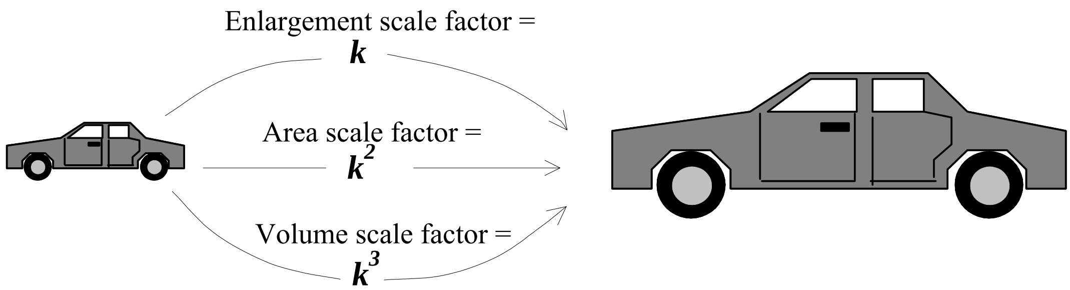
So $2x^2 + 4x - 3x - 6 = 0$

$(2x^2 + 4x) - (3x + 6) = 0$

$2x(x+2) - 3(x+2) = 0$

$(2x-3)(x+2) = 0$, which gives $x = -2, \frac{3}{2}$.

(b) Similarity

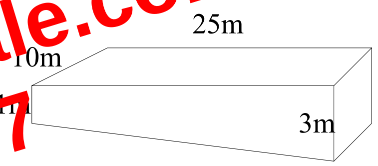


<u>Questions</u>

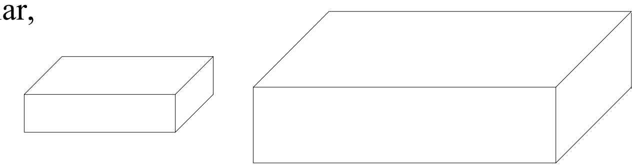
(a) A cylinder has volume $100cm^3$, and height 5cm.
What is its diameter?

(b) A cone of base radius 10cm and height 20cm is sliced parallel to the base half way up into two pieces. What is the volume of the base part? (frustum)

(c) The empty swimming pool shown opposite is to be filled with water. The speed of flow of water in the pipe is 2m/s, and the radius of the pipe is 5cm. How long will the pool take to fill?



(d) Two blocks are geometrically similar, and the big blocks weighs 20 times the small block. What is the ratio of surface areas of the two blocks?



<u>Answers</u>

(a) $\pi r^2 5 = 100$

$\therefore r^2 = \frac{100}{5\pi}$, so $r = \sqrt{\frac{20}{\pi}}$ = 2.52 cm. Whoops! <u>Diameter</u> asked for!

diameter = <u>5.05cm</u> to 3sf

{Note the <u>pre-corrected</u> value was doubled resulting in 5.05 when itself rounded, not 5.04}

14. **Trigonometry**

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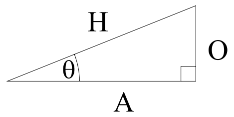
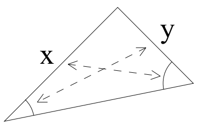


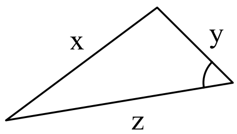
$$\sin\theta = \frac{O}{H}$$
$$\cos\theta = \frac{A}{H}$$
$$\tan\theta = \frac{O}{A}$$

Sine rule: $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

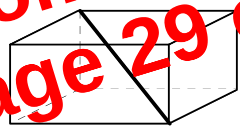
Cosine rule: $\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$



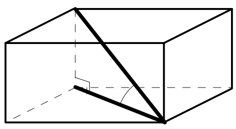
Two opposite pairs: use **sine** rule



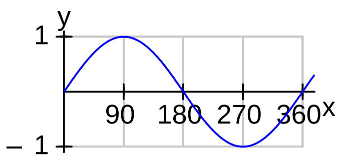
Three sides and one angle: use **cosine** rule



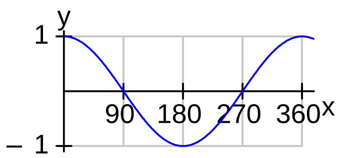
Angle between line and plane is the angle between the line and its projection on the plane: e.g. for the angle between this diagonal and the base, draw the projection, and the angle is shown here:



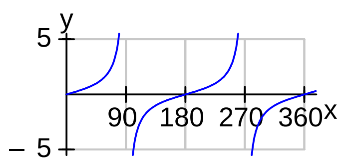
Trigonometric functions for all angles:



sin$x$



cos$x$



tan$x$

Answers

(a) Rearrange: $2x + 6y + 12 = 0$ [-2x, -12]

$6y = -2x - 12$ [$\div 6$]

$y = -\frac{1}{3}x - 2$

so the gradient is $-\frac{1}{3}$ and the y-intercept is –2.

(b) gradient = $-\frac{2}{3}$ and y-intercept = 2.

The equation is $y = -\frac{2}{3}x + 2$ [$+\frac{2}{3}x$]

$\frac{2}{3}x + y = 2$ [$\times 3$]

$2x + 3y = 6$

(c) Solve simultaneously. In this form, substitution would be easier:

sub (1) into (2): $3x + 2(3x - 5) = 6$ which gives $x = \frac{16}{9}$. Sub back into (1) gives $y = \frac{1}{3}$. So the intersection is $(\frac{16}{9}, \frac{1}{3})$.

(d) gradient AB is $\frac{6-3}{5-2} = 1$. Perpendicular gradient is $-\frac{1}{1} = -1$.

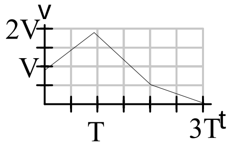
So the required equation is $y = -x + c$ but what is c? Get this by substituting the coordinates of a point on the line, i.e. C.

$\rightarrow 0 = -4 + c$, giving $c = 4$, and the equation is $y = -x + 4$

Questions

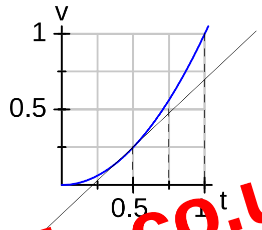
(a) An object moves in a straight line so that its velocity after time t seconds is given by $v = t^2$. Find
(i) its average acceleration over the first second
(ii) its instantaneous acceleration at $t = \frac{1}{2}$
(iii) the distance it covers using the trapezium rule with 4 strips.

(b) In the journey represented in the diagram, the total distance covered was 60m, and the acceleration over the first part was 5 ms^{-2}. Find the values of V and T.



Answers

(a) (i) average acceleration = change in velocity over time taken = 1 m/s per s = 1ms^{-2}.
(ii) acceleration at t = ½ is gradient of tangent there, i.e. 1ms^{-2}.
(iii) Using trapezium rule, distance



$\approx \frac{0.25}{2}\left\{0 + 2\times0.25^2 + 2\times0.5^2 + 2\times0.75^2 + 1^2\right\}$ = 0.34375, or 0.34 m to 2 s.f. {*Note that 0.34375 is an overestimate due to the concave curve*}

(b) Splitting into two trapezia and a triangle, area under curve

$= \frac{1}{2}(V+2V)T + \frac{1}{2}(2V + \frac{1}{2}V)T + \frac{1}{2}T\frac{V}{2}$ which $= 3VT$. So $3VT = 60$

Acceleration on first part = $\frac{V}{T}$ which = 5. Substituting gives

$5T^2 = 20$ which leads to $T = 2$, and $V = 10$.

26. **<u>Statistical calculations, diagrams, data collection</u>** top

(a) calculations

(i) averages:

$$\boxed{\text{mean} = \frac{\sum x_i}{n}}$$

$\boxed{\text{median}}$ = value of the middle item when listed in order

$\boxed{\text{mode}}$ = most commonly occurring value

(ii) measures of spread:

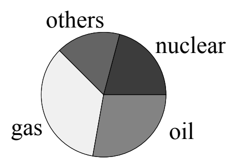
$\boxed{\text{range}}$ = max – min

$\boxed{\text{Interquartile range}}$ = Upper quartile – lower quartile

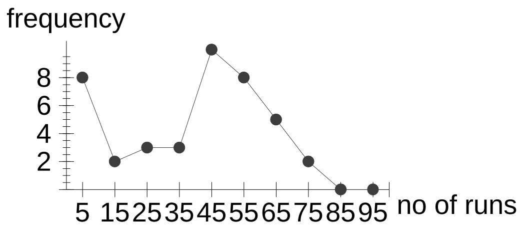
<u>Quartiles in small data sets</u>: fiddly and pointless, but here we go. Median is found. If the number of data was even, split the data into two sets; if the number of data was odd, ignore the median and consider the remaining values as two sets. Then the quartiles are the medians of the two remaining sets.

(b) diagrams

(i) $\boxed{\text{pie chart}}$ for categoric data (non-numerical) e.g. modes of transport used to school

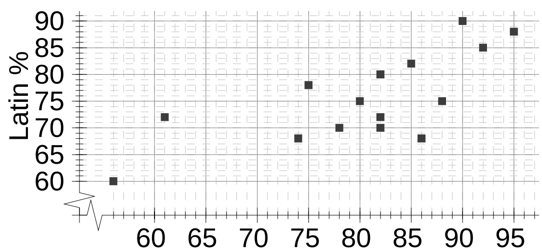


(ii) $\boxed{\text{frequency diagram}}$



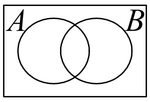
(iii) $\boxed{\text{moving average}}$ in a time sequence, the mean of the last 10 (say) values is calculated then plotted. This smooths out short term fluctuations so that a long term trend may be seen.

(iv) $\boxed{\text{scatter graphs}}$ to see correlation between two variables



<u>Questions</u>

(a) Shade the set $A \cap B'$:



(b) A is the set of animals, B is the set of black objects, and C is the set of cats.

(i) Translate into normal English: $B \cap C \neq \varnothing$

(ii) Describe the set $B \cap A'$

(iii) Is a white mouse a member of the set $A \cap (B \cup C)'$?

(c). In a class of 25, 12 play football, 15 play water polo, but 10 do neither sport. How many play both football and water polo?

(d) ξ is the set of all employed people in England. A is the set of those with a bank account. B is the set of those with a building society account. C is the set of people who work in the catering industry.

(i) Shade the set of those in catering with a bank account but no building society account, and describe this in set notation.

(ii) Shade the set $C \cap (A \cup B)'$, and describe the members of this set.