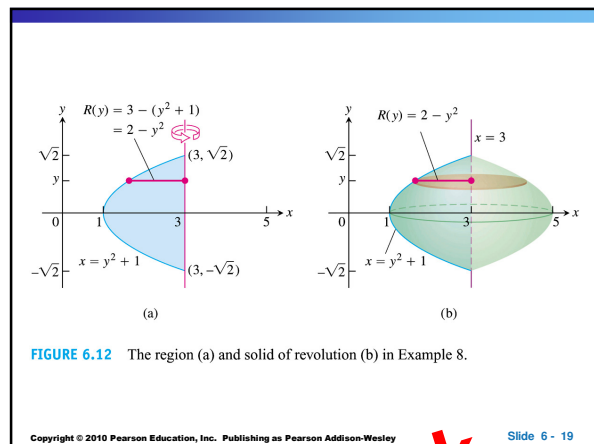
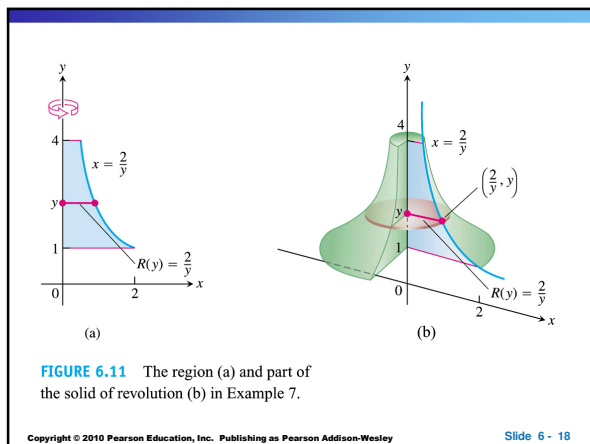
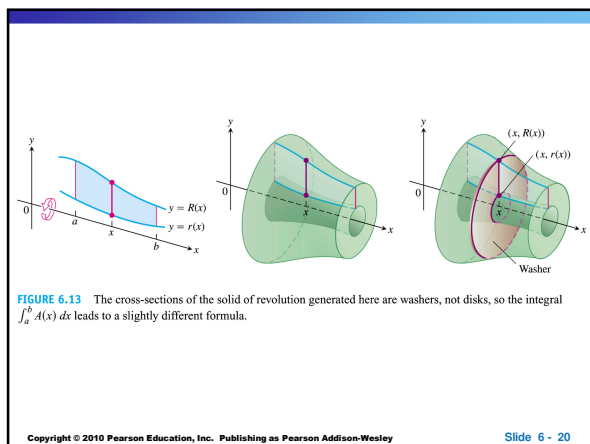




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Solids of Revolution: The Washer Method

If the region we revolve to generate a solid does not border on or cross the axis of revolution, the solid has a hole in it (Figure 6.13). The cross-sections perpendicular to the axis of revolution are *washers* (the purplish circular surface in Figure 6.13) instead of disks. The dimensions of a typical washer are

$$\text{Outer radius: } R(x)$$

$$\text{Inner radius: } r(x)$$

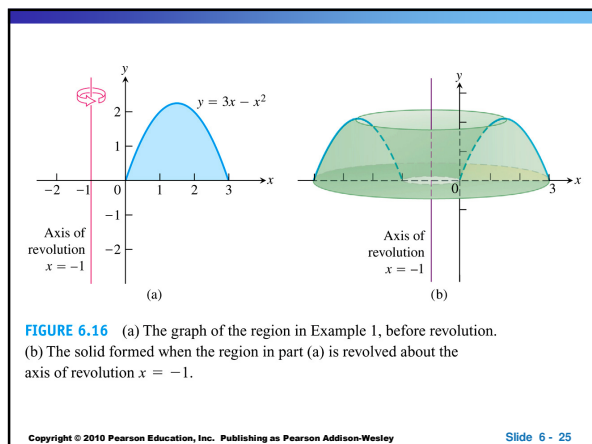
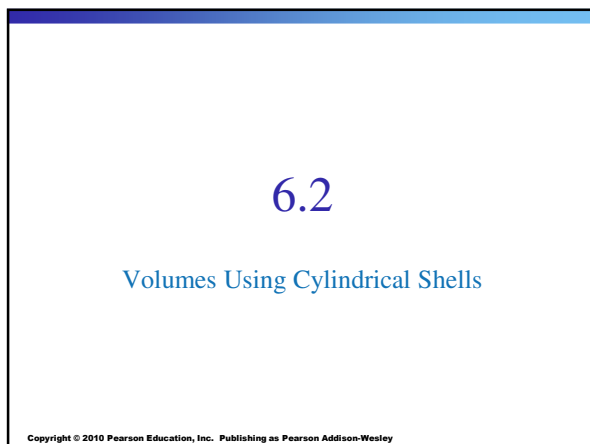
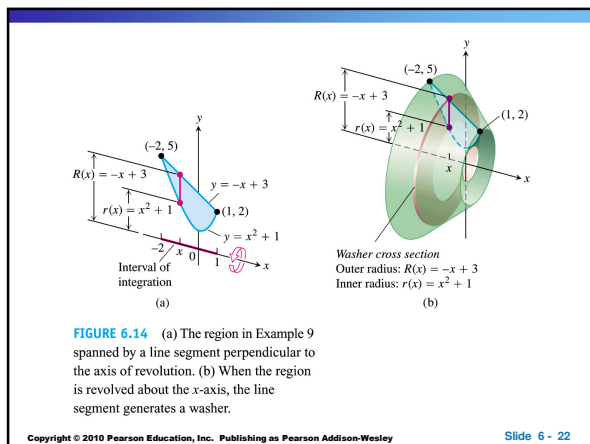
The washer's area is

$$A(x) = \pi[R(x)]^2 - \pi[r(x)]^2 = \pi([R(x)]^2 - [r(x)]^2).$$

Consequently, the definition of volume in this case gives

Volume by Washers for Rotation About the x-axis

$$V = \int_a^b A(x) dx = \int_a^b \pi([R(x)]^2 - [r(x)]^2) dx.$$



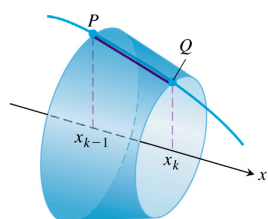


FIGURE 6.31 The line segment joining P and Q sweeps out a frustum of a cone.

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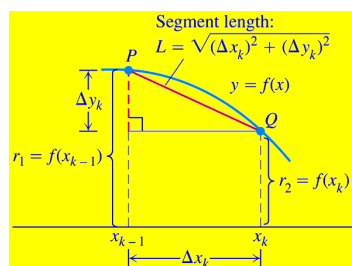


FIGURE 6.32 Dimensions associated with the arc and line segment PQ .

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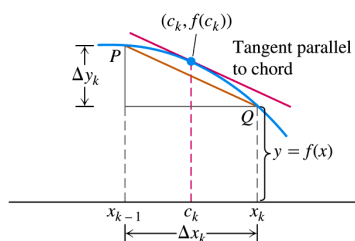


FIGURE 6.33 If f is smooth, the Mean Value Theorem guarantees the existence of a point c_k where the tangent is parallel to segment PQ .

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DEFINITION If the function $f(x) \geq 0$ is continuously differentiable on $[a, b]$, the **area of the surface** generated by revolving the graph of $y = f(x)$ about the x -axis is

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} dx. \quad (3)$$

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