Example

Find v_x using voltage division and then find i_s and use it to find i_3 using current division

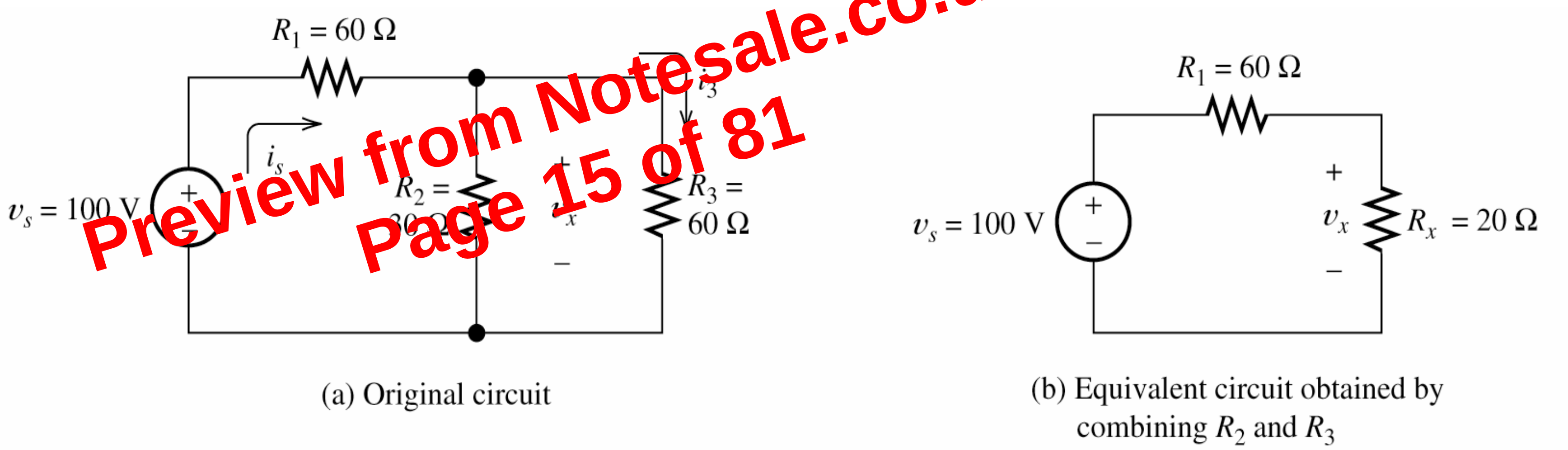


Figure 2.11 Circuit for Example 2.4.

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\Omega \;\rightarrow\; v_x = \frac{R_x}{R_1 + R_x} v_s = \frac{20}{60+20} 100 = 25V$$

$$i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60+20} = 1.25A \;\rightarrow\; i_3 = \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30+60} \times 1.25 = 0.417A$$

Example

Use current division rule to find i_1

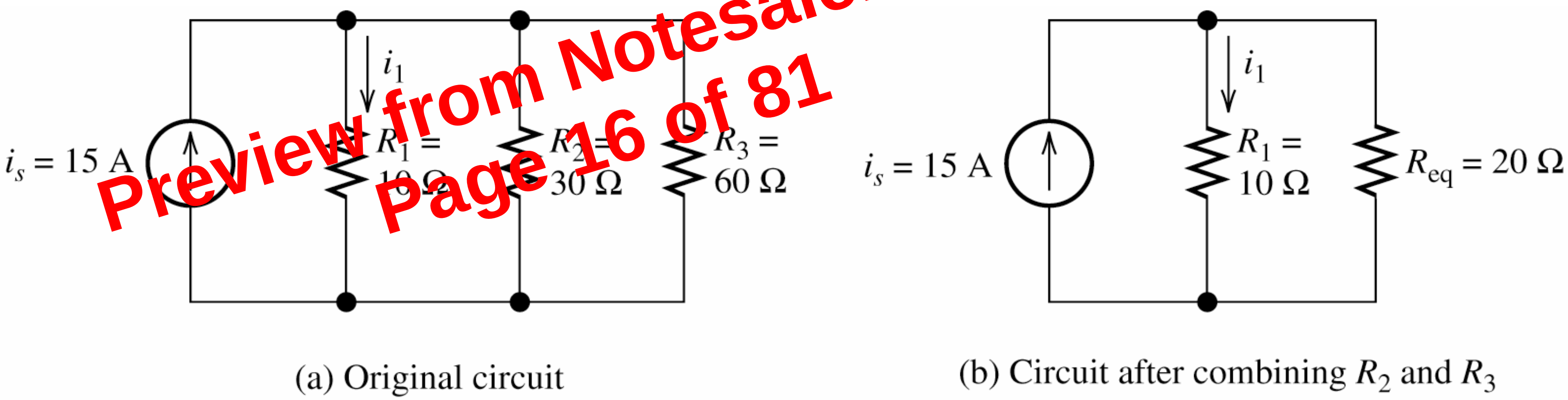


Figure 2.12 Circuit for Example 2.5.

$$i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10+20} 15 = 10\text{A}$$

Note

Although series/parallel equivalents and the current/voltage division principles are very important concepts, yet they are not sufficient to solve all circuits !!

Example

Write the set of equations for node voltages for the circuit shown

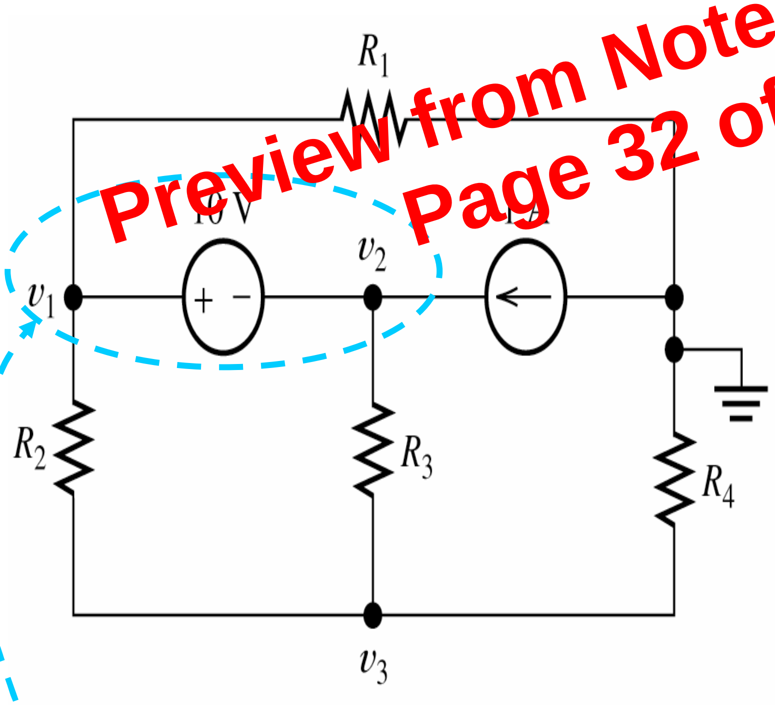


Figure 2.26 Circuit for Exercise 2.11.

$$v_1 - v_2 = 10$$

At node 3:

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

At the super node:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

Example (cont.)

Next, we find an expression for the controlling variable i_x in terms of the node voltages

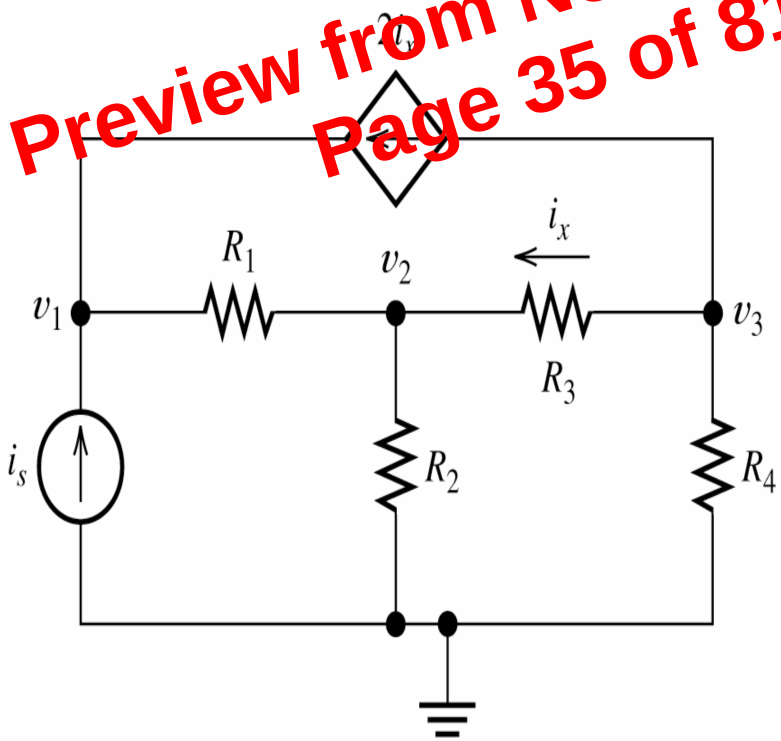


Figure 2.27 Circuit containing a current-controlled current source. See Example 2.9.

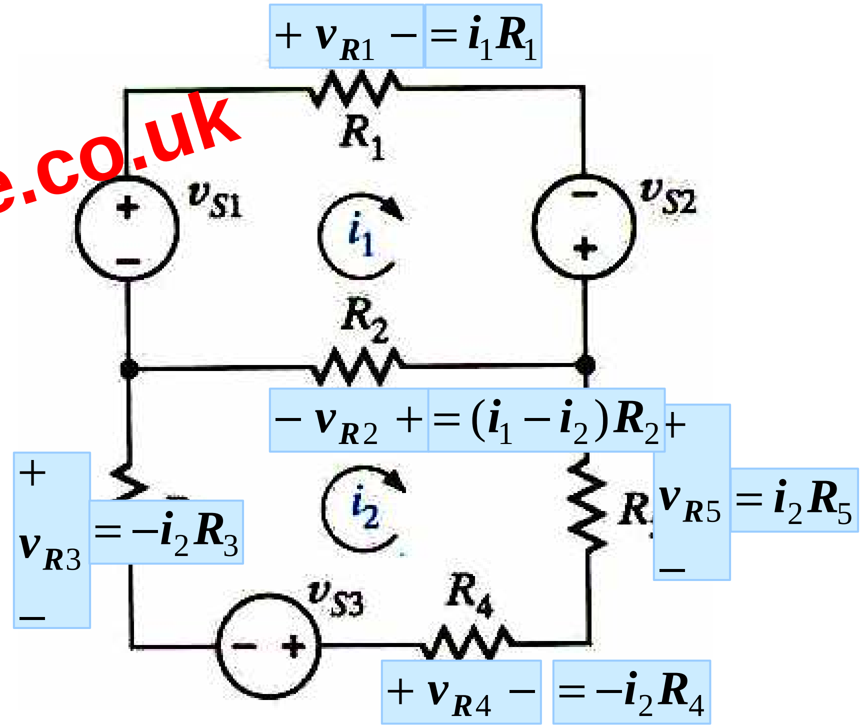
$$i_x = \frac{v_3 - v_2}{R_3}$$

Writing the Mesh Equations

Branches = 8
Nodes = 7

Loop currents needed = 2

And we are told to use mesh currents!
This defines the loop currents to be used



Identify all voltage drops

Write kvl on each mesh

Top mesh : $-v_{S1} + v_{R1} - v_{S2} + v_{R2} = 0$

Bottom : $-v_{R2} + v_{R5} - v_{R4} + v_{S3} - v_{R3} = 0$

Use Ohm's law

$$-v_{S1} + i_1 R_1 - v_{S2} + (i_1 - i_2) R_2 = 0$$

$$i_2 R_3 + (i_2 - i_1) R_2 + i_2 R_5 + i_2 R_4 + V_{S3} = 0$$

Mesh Currents in Circuits Containing Current Sources

Supermesh: Current source common to two meshes

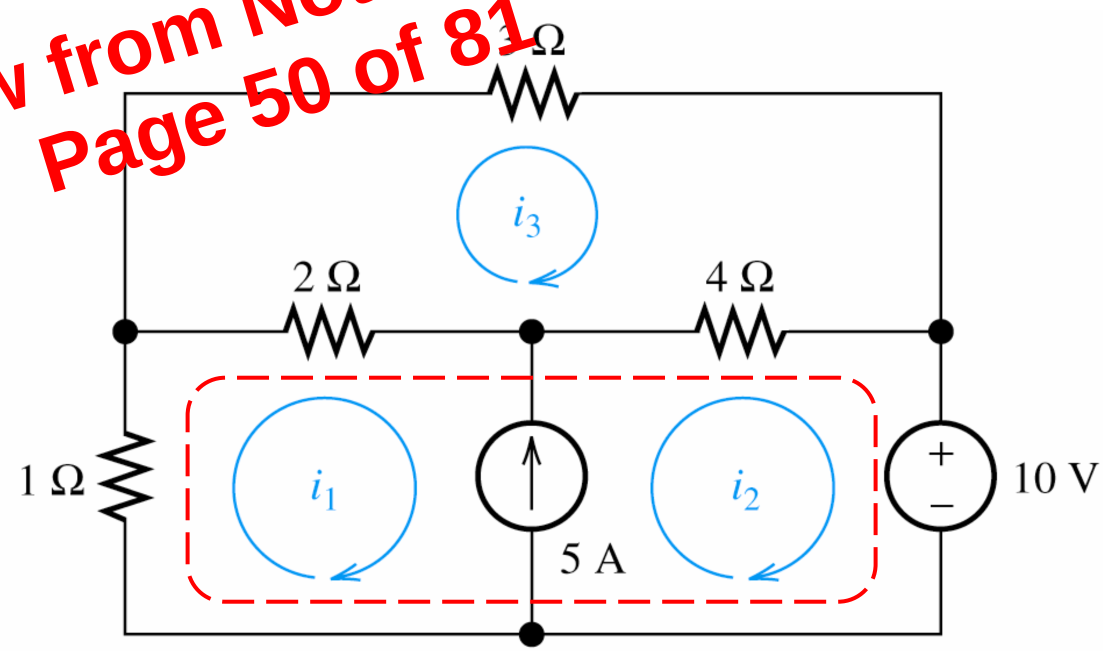


Figure 2.36 A circuit with a current source common to two meshes.

Circuits with Dependent Sources

$$i_x + 2i_x = \frac{v_{oc}}{10}$$

$$i_x = \frac{10 - v_{oc}}{5}$$

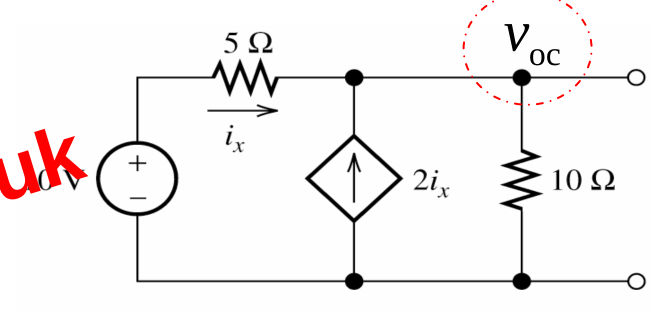
$$3\frac{10 - v_{oc}}{5} = \frac{v_{oc}}{10}$$

$$v_{oc} = 8.57$$

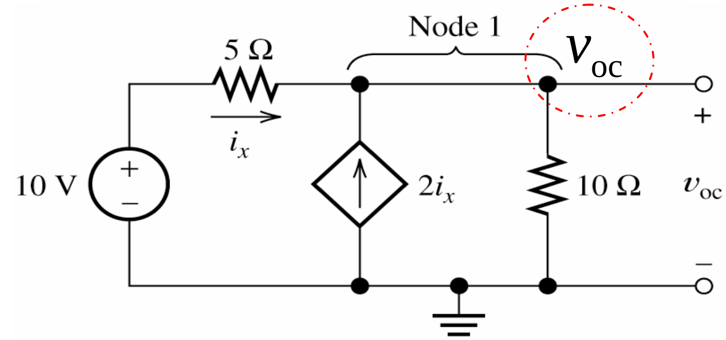
$$i_x = \frac{10\text{V}}{5\Omega} = 2\text{A}$$

$$i_{sc} = 3i_x = 6\text{A}$$

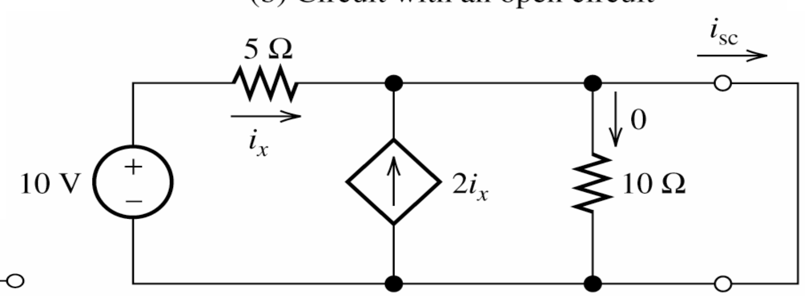
$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{8.57\text{ V}}{6\text{ A}} = 1.43\Omega$$



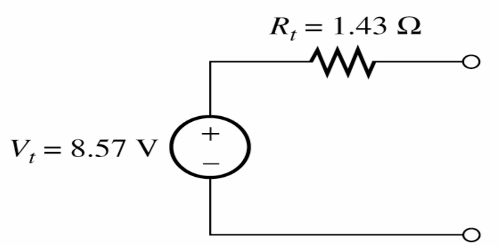
(a) Original circuit



(b) Circuit with an open circuit



(c) Circuit with a short circuit



(d) Thévenin equivalent

Step-by-step Thévenin/Norton-Equivalent-Circuit Analysis

1. Perform two of these:
   - Determine the open-circuit voltage $V_t = v_{oc}$
   - Determine the short-circuit current $I_n = i_{sc}$
   - Zero the sources and find the Thévenin resistance R_t looking back into the terminals

2. Use the equation $V_t = R_t\, I_n$ to compute the remaining value

3. The Thévenin equivalent consists of a voltage source V_t in series with R_t

4. The Norton equivalent consists of a current source I_n in parallel with R_t

Circuits with Dependent Sources

$$\frac{v_x}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

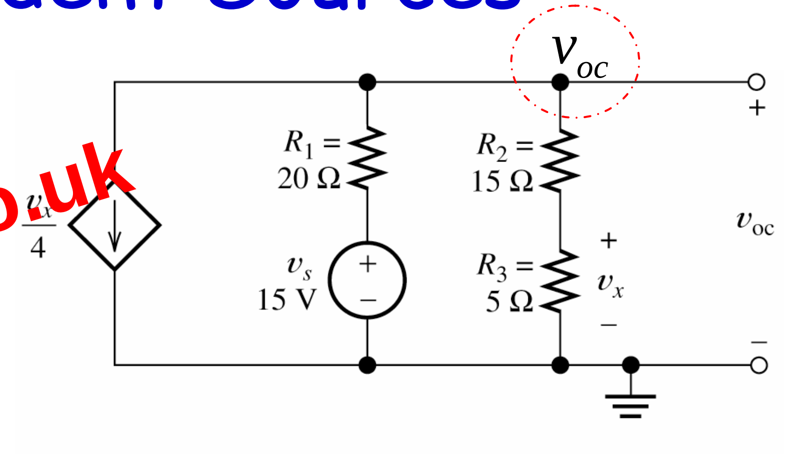
$$v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25 v_{oc}$$

$$\frac{0.25 v_{oc}}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

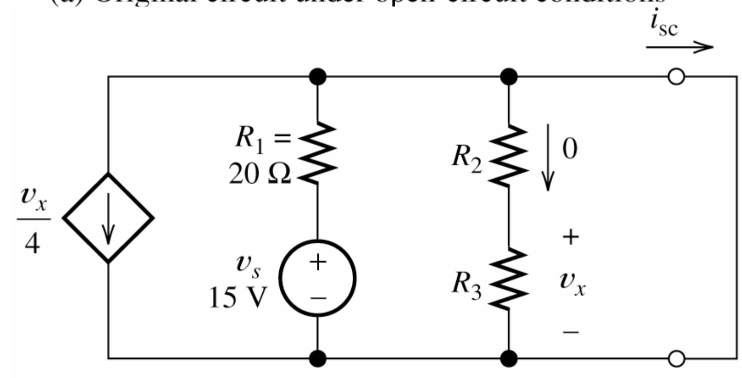
$$\Rightarrow \boxed{v_{oc} = 4.62\text{V}}$$

and we got: $i_{sc} = \frac{v_s}{R_1} = \frac{15\text{V}}{20\Omega} = 0.75\text{A}$

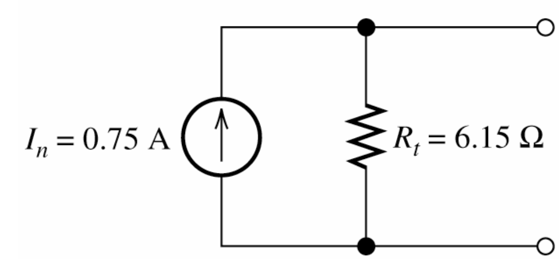
$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{4.62\text{V}}{0.75\text{A}} = 6.15\Omega$$



(a) Original circuit under open-circuit conditions



(b) Circuit with a short circuit



Superposition Principle

Example

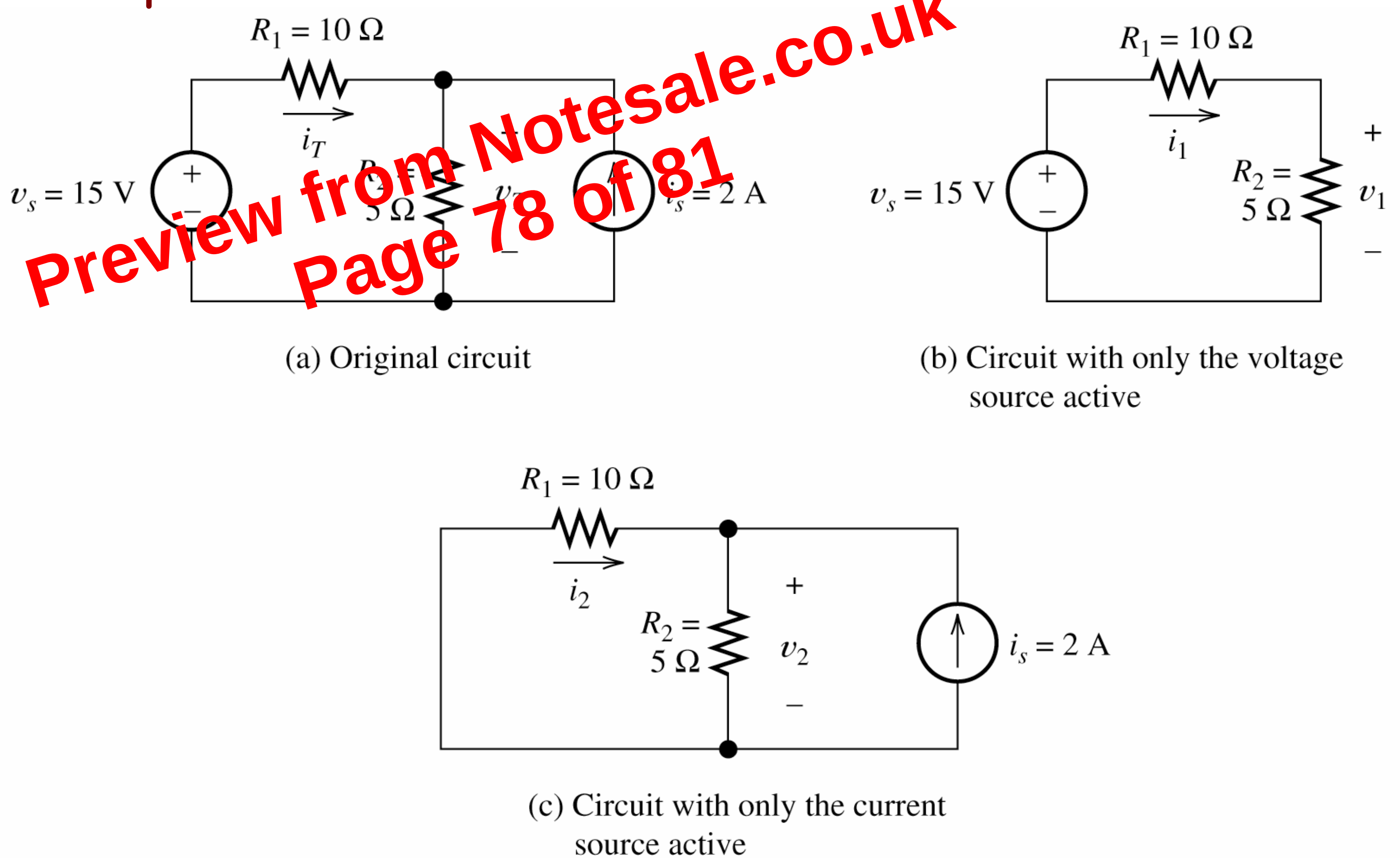


Figure 2.60 Circuit for Example 2.20 and Exercise 2.27.

The Wheatstone bridge

Used to measure unknown resistances

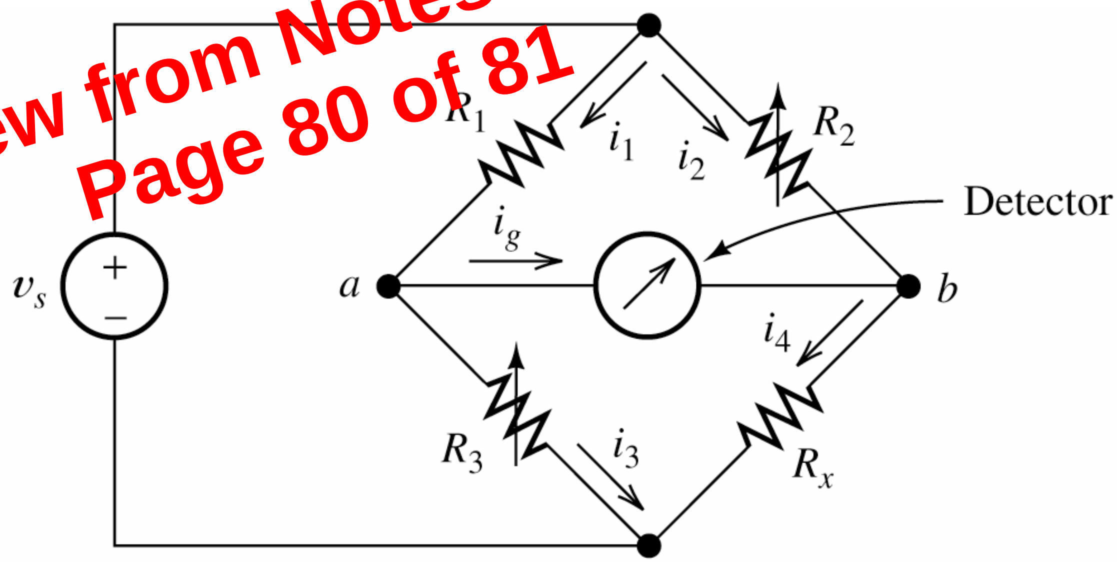


Figure 2.62 The Wheatstone bridge. When the Wheatstone bridge is balanced, $i_g = 0$ and $v_{ab} = 0$.

The Wheatstone bridge

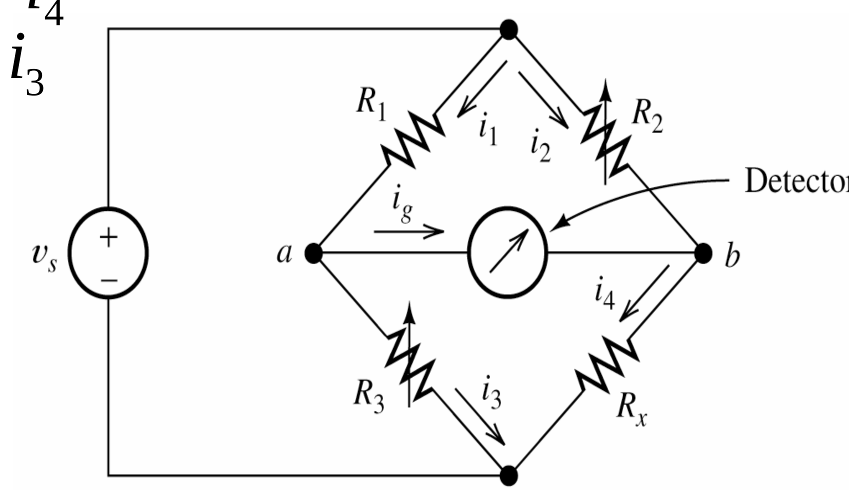
The Wheatstone bridge is used by mechanical and civil engineers to measure the resistances of strain gauges in experimental stress studies of machines and buildings.

Operation:

- Adjust the values of R_1 & R_2 until the detector current $i_g = 0$

Applying KCL at node a, we get: $i_2 = i_4$

Applying KCL at node b, we get: $i_1 = i_3$



When the bridge is balanced, $v_{ab} = 0$

$$\Longrightarrow \quad R_1 i_1 = R_2 i_2 \;\; \& \;\; R_3 i_3 = R_x i_4$$

From which we get:

$$\boxed{R_x = \frac{R_2}{R_1} R_3}$$