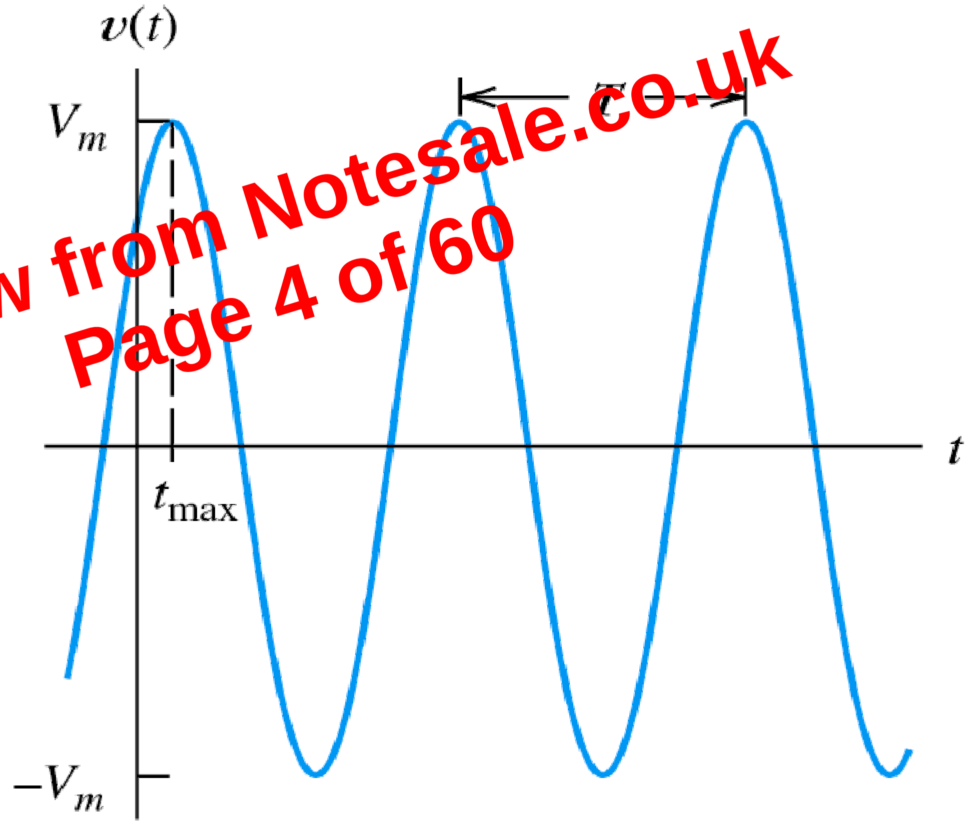
Sinusoidal Currents or Voltages



A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$.
Note: Assuming that θ is in degrees, we have $t_{max} = \frac{-\theta}{360} \times T$.
For the waveform shown, θ is $-45°$.

Real and Complex Signals

A complex number is given by:

$$Z = x + jy$$ ← Rectangular form

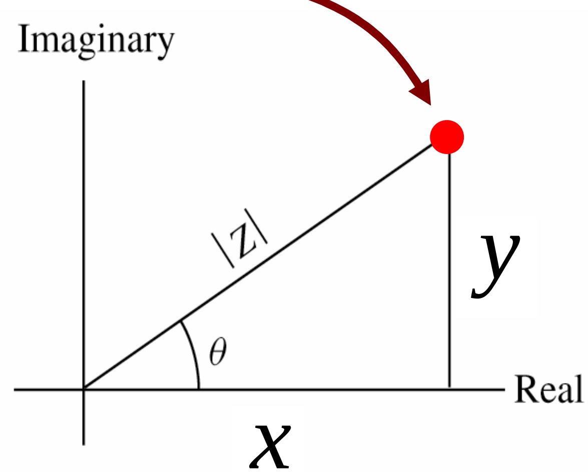
Real part (x) — Imaginary part (y)

$$j = \sqrt{-1}$$

It can also be represented by a **POINT** in the complex Plane

Note that Z can be written as : $|Z| \angle \theta$

Polar form



Complex conjugate of z is:

$$z^* = x - jy$$

Complex Numbers

$$z = x + jy$$ ⟵ Rectangular form

Real part

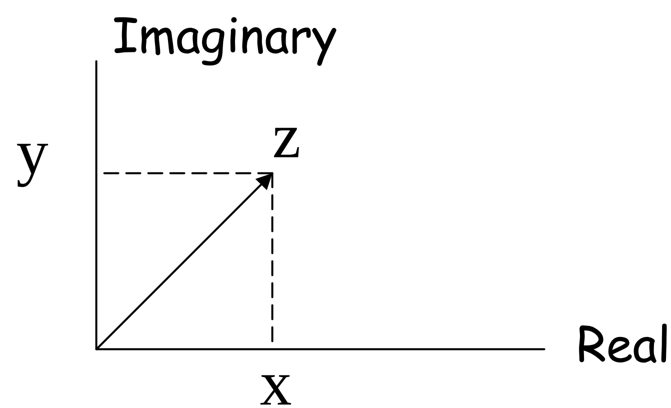
Imaginary part

$$j = \sqrt{-1}$$

A complex number can be represented as a point in the complex Plane

Complex conjugate of z is:

$$z^* = x - jy$$



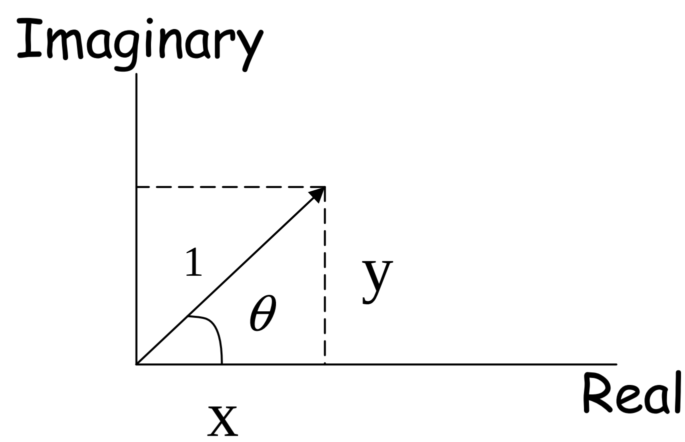
Complex Numbers in Polar Form

- Represent the complex number by the length of the arrow and the angle between the arrow and the positive real axis
- We write complex numbers in polar form as:

$$Z = |Z| \angle \theta$$

$$|z|^2 = x^2 + y^2 \implies |z| = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x} \quad \text{or} \quad \theta = \tan^{-1}\frac{y}{x}$$



Polar to Rectangular:

$$x = |z|\cos\theta$$

$$y = |z|\sin\theta$$

In exponential form: $= |Z|\, e^{j\theta}$

Forms of a Complex Number

$$z_2 = 10 + j5$$ ← Rectangular form

$$z_2 = \sqrt{(10)^2 + (5)^2}\angle\tan^{-1}\left(\frac{5}{10}\right)$$

$$= 11.18\angle 26.57^\circ$$ ← Polar form

$$= 11.18e^{j26.57^\circ}$$ ← Exponential form

Important Note:

$$1\angle 90^\circ = \cos 90 + j\sin 90 = j$$

Complex Impedances: Inductances

$$i_L = I_m \sin(\omega t + \theta) \longrightarrow \mathbf{I_L} = I_m \angle \theta - 90^\circ$$

$$v_L = L\frac{di_L}{dt}$$

$$= \omega L I_m \cos(\omega t + \theta) \longrightarrow \mathbf{V_L} = \omega L I_m \angle \theta = V_m \angle \theta$$

Current through the inductor lags the voltage by 90°

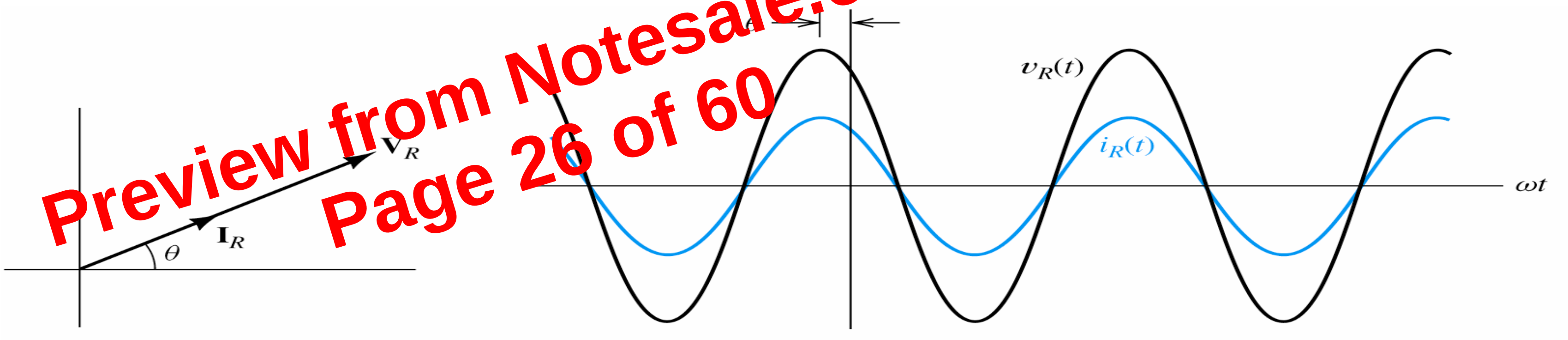
$$V_L = \omega L I_m \angle \theta \angle (90^\circ - 90^\circ)$$
$$= \omega L \angle 90^\circ I_m \angle (\theta - 90^\circ)$$
$$= \omega L \mathrm{I_L} \angle 90^\circ$$
$$= j\omega L \mathrm{I_L}$$

$$V_L = Z_L \mathrm{I_L}$$ ← Ohm's law in phasor form

$Z_L = j\omega L$ is called the impedance of the inductor (measured in ohms)

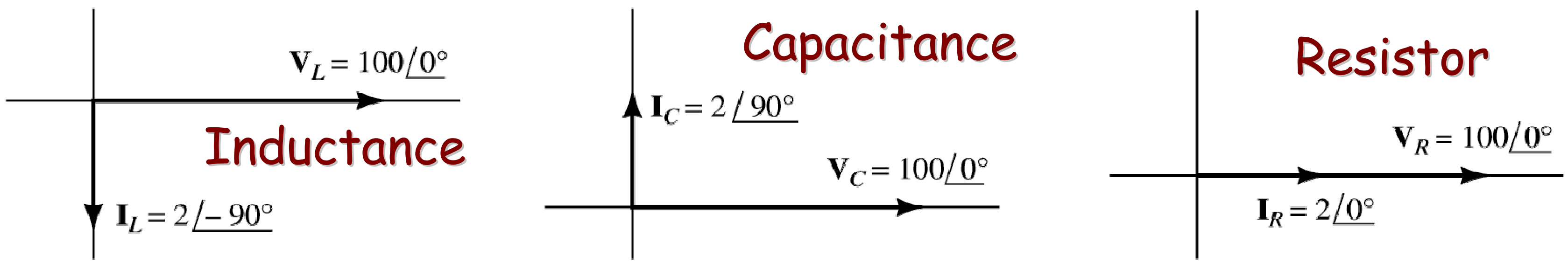
Complex Impedances: Resistance

$$\mathbf{V}_R = R\mathbf{I}_R \Rightarrow Z_R = R$$



(a) Phasor diagram (b) Current and voltage versus time

In a pure resistor, current & voltage are in phase



The impedances that are pure imaginary are called reactance

Example (cont.)

Inductor:

$$Q_L = I_{rms}^2 X_L = (0.1)^2 100 = 1\textbf{VAR}$$

Capacitor:

$$Q_C = I_{Crms}^2 X_C = (\frac{0.1}{\sqrt{2}})^2 (-100) = -0.5\textbf{VAR}$$

Resistor:

$$P_R = I_{Rrms}^2 R = (\frac{0.1}{\sqrt{2}})^2 100 = 0.5\textbf{W}$$

It is important to note that: $\begin{cases} P_L = 0 \\ P_C = 0 \end{cases}$

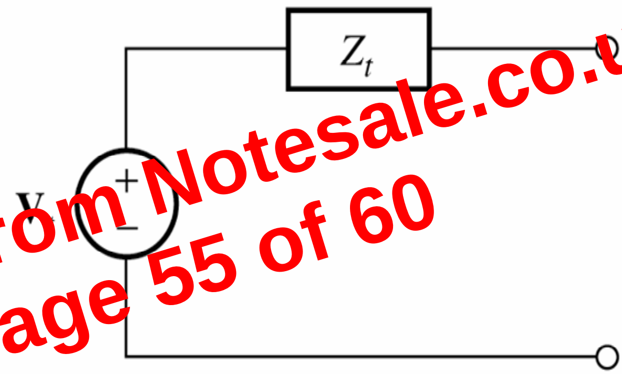
Check:

$$Q = Q_L + Q_C$$

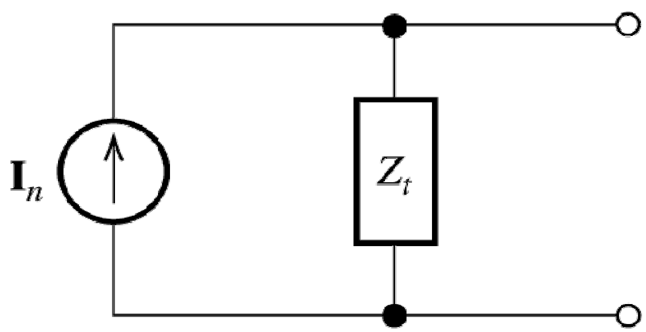
$$P = P_R$$

In real life, the values are much bigger (kW, MW, kVA)

Thévenin and Norton Equivalent Circuits

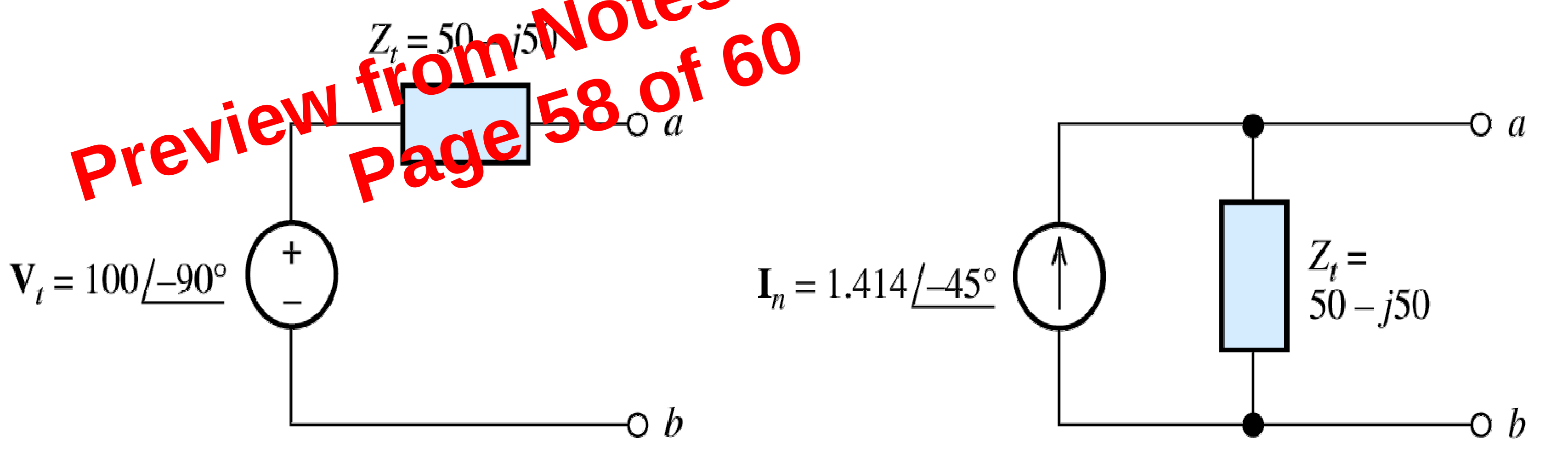


The Thévenin equivalent for an ac circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.



The Norton equivalent circuit consists of a phasor current source $\mathbf{I}_n$ in parallel with the complex impedance Z_t.

Example (cont.)



(a) Thévenin equivalent

(b) Norton equivalent