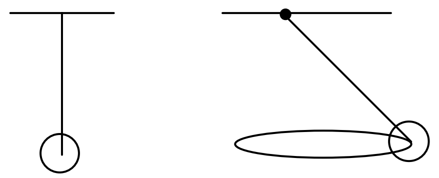
3. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension :
(1) angular momentum changes in direction but not in magnitude.
(2) angular momentum changes both in direction and magnitude.
(3) angular momentum is conserved.
(4) angular momentum changes in magnitude but not in direction.

3. (1)



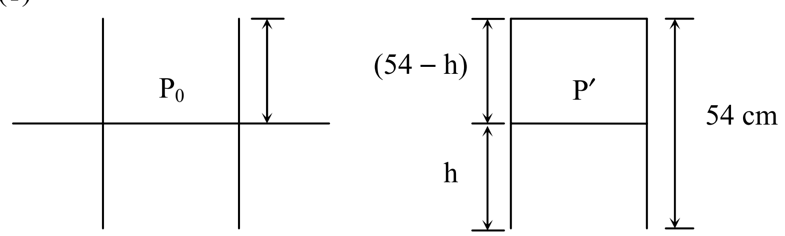
$$\vec{\tau}_{net} = \vec{\tau}_T + \vec{\tau}_{mg}$$
$$= \vec{\tau}'_{mg} \neq 0$$
$\vec{L} \neq$ constant
But as ω is constant
$|\vec{L}|$ = MVL = constant

4. The current voltage relation of diode is given by I = $(e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?
(1) 0.5 mA (2) 0.05 mA (3) 0.2 mA (4) 0.02 mA

4. (3)
$$I = \left(e^{\frac{1000V}{T}} - 1\right)$$
$$\frac{dI}{dV} = \frac{1000}{T} e^{\frac{1000V}{T}}$$
$$dI = \left(\frac{1000}{T} e^{\frac{1000V}{T}}\right) dV$$
I = 5 MA
$$\Rightarrow e^{\frac{1000V}{T}} = 6 \text{ MA}$$
$$dI = \left(\frac{1000}{300}\right)(0.01)(6) = \frac{10}{50} = \frac{1}{5}$$
dI = 0.2 mA

5. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now?
(Atmospheric pressure = 76 cm of Hg)
(1) 38 cm (2) 6 cm (3) 16 cm (4) 22 cm

5. (1)



$$\frac{1}{\lambda_3} \propto \frac{4}{\left(1+\frac{me}{4M_p}\right)} \qquad \frac{1}{\lambda_4} \propto \frac{9}{\left(1+\frac{me}{6M_p}\right)}$$

If $\frac{me}{M_p} << 1$: we have $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

25. The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to :

(1) 0.8 eV (2) 1.6 eV (3) 1.8 eV (4) 1.1 eV

25. (4)

$r = \frac{mv}{eB} = \frac{\sqrt{2mE_p}}{eB}$ [E = electron's K.E.] [m = electron mass, e = electron charge]

$E = E_p - \phi$ E_p = photon energy = 1.9 eV.

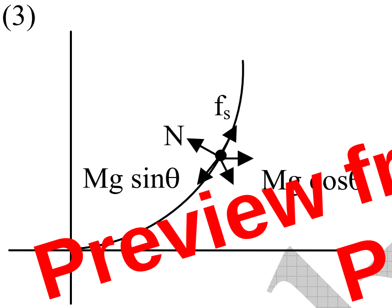
$\Rightarrow (reB)^2 = 2m\,[E_p - \phi] \Rightarrow \phi = E_p - \frac{(reB)^2}{2m}$

$\Rightarrow \phi = 1.9 \text{ eV} - \frac{10^{-4} \times 1.6 \times 10^{-19} \times 9 \times 10^{-8}}{2 \times 9.1 \times 10^{-31}} \simeq 1.9\text{eV} - 0.79\text{V} = 1.1\text{eV}$

26. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :

(1) $\frac{1}{3}$m (2) $\frac{1}{2}$m (3) $\frac{1}{6}$m (4) $\frac{2}{3}$m

26. (3)



$mg \sin\theta = \mu_s mg \cos\theta$ [when partied is just balanced]

$\Rightarrow \tan\theta = \mu_s$

$\tan\theta = \frac{dy}{dx} = \left(\frac{x^2}{2}\right)$

$\therefore \frac{x^2}{2} = 0.5 \Rightarrow x = 1$

$\Rightarrow y = \frac{1}{6}$

27. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is :

(1) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (2) $\frac{1}{2}\left[\frac{aL^2}{2} + \frac{bL^3}{3}\right]$ (3) $aL^2 + bL^3$ (4) $\frac{1}{2}(aL^2 + bL^3)$

27. (1)

$w = \int_o^L F.dx \Rightarrow \omega = \frac{aL^2}{2} + \frac{bL^3}{3}$

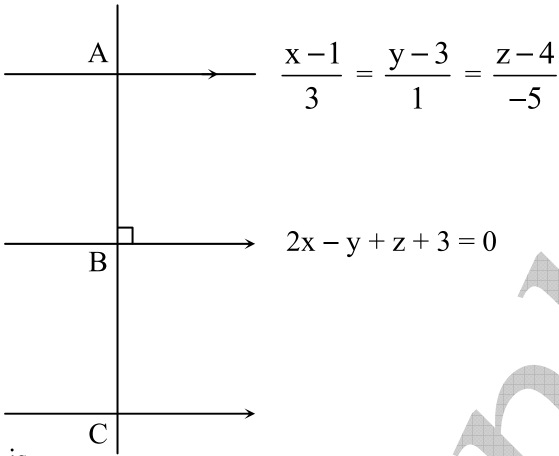
PART- B : MATHEMATICS

31. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line:

(1) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$ (2) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

(3) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$ (4) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

31. (1)



Equation of AB is

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=\lambda$$

Co-ordinate of point B is

$\Rightarrow$ $x = 1 + 2\lambda$ point satisfy the equation of plane

$y = 3 - \lambda$

$z = 4 + \lambda$

$2(1 + 2\lambda) - (3 - \lambda) + (4 + \lambda) + 3 = 0$

$\Rightarrow$ $\lambda = -1$

$\Rightarrow$ Co-ordinate of point B(−1, 4, 3)

$\Rightarrow$ Co-ordinate of point C(−3, 5, 2)

$\Rightarrow$ equation of line passing through 'C' is

$$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$$

32. If the coefficients of x^3 and x^4 in the expansion of $(1+ ax+ bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to:

(1) $\left(16, \frac{251}{3}\right)$ (2) $\left(14, \frac{251}{3}\right)$ (3) $\left(14, \frac{272}{3}\right)$ (4) $\left(16, \frac{272}{3}\right)$

32. (4)

$(1 + ax + bx^2)\,[1 - {}^{18}C_1\, 2x + {}^{18}C_2\,(2x)^2) - {}^{18}C_3(2x)^3 + {}^{18}C_4\,(2x)^4 \ldots]$

Coefficient of x^3 is

$-{}^{18}C_3(2^3) + a({}^{18}C_2 \times 4) - b({}^{18}C_1 \times 2) = 0$ …(i)

Coefficient of x^4 is

${}^{18}C_4\,(2^4) + a(-{}^{18}C_3\, 2^3) + {}^{18}C_2\, b2^2 = 0$ …(ii)

or solving both these equation

a = 16 and b = 272/3.

33. If $a \in R$ and the equation $-3(x-[x])^2+2(x-[x]) + a^2 = 0$

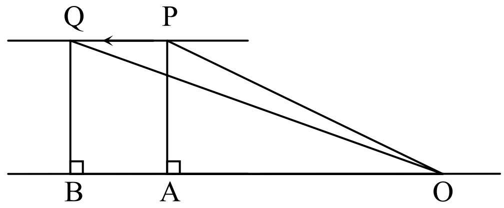
(where [x] denotes the greatest integer $\le$ x) has no integral solution, then all possible values of a lie in the interval:

(1) $(-1, 0) \cup (0, 1)$ (2) $(1, 2)$ (3) $(-2, -1)$ (4) $(-\infty, -2) \cup (2, \infty)$

36. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is:

(1) $40\left(\sqrt{2}-1\right)$ (2) $40\left(\sqrt{3}-\sqrt{2}\right)$ (3) $20\sqrt{2}$ (4) $20\left(\sqrt{3}-1\right)$

36. (4)



Here, AP = QB = 20 m

$\angle POA = 45°,\ \angle QOB = 30°$

$\Rightarrow$ OA = 20; OB = $20\sqrt{3}$

$\Rightarrow$ OB − OA $= 20(\sqrt{3}-1)$

Hence distance covered in one second by the bird is

$$AB = 20(\sqrt{3}-1)$$

Thus, speed of bird = $20(\sqrt{3}-1)$ m/s

37. The integral $\int_0^{\pi}\sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\,dx$ equals:

(1) $\pi - 4$ (2) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$ (3) $4\sqrt{3} - 4$ (4) $4\sqrt{3} - 4 - \frac{\pi}{3}$

37. (4)

$$I = \int_0^{\pi}\sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}$$

$$I = \int_0^{\pi}\left|1 - 2\sin\frac{x}{2}\right|dx$$

$$I = \int_0^{\frac{\pi}{3}}\left(1 - 2\sin\frac{x}{2}\right)dx + \int_{\frac{\pi}{3}}^{\pi} -\left(1 - 2\sin\frac{x}{2}\right)dx$$

$$I = \left[x + 4\cos\frac{x}{2}\right]_0^{\frac{\pi}{3}} + \left[-x - 4\cos\frac{x}{2}\right]_{\frac{\pi}{3}}^{\pi}$$

$$I = \frac{\pi}{3} + 4\left(\frac{\sqrt{3}}{2} - 1\right) - \left(\pi - \frac{\pi}{3}\right) - 4\left(0 - \frac{\sqrt{3}}{2}\right)$$

$$I = \frac{\pi}{3} + 2\sqrt{3} - 4 - \frac{2\pi}{3} + 2\sqrt{3} = 4\sqrt{3} - 4 - \frac{\pi}{3}$$

38. The statement ~ (p ↔ ~ q) is:

(1) equivalent to p ↔ q (2) equivalent to ~p ↔ q

(3) a tautology (4) a fallacy

38. (1)

Truth table for ~(p ↔ ~q) is

p	q	~q	(1) p → ~q	(2) ~q → p	(3) (1) ∧ (2)	~(3)
T	T	F	F	T	F	T
T	F	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	T	F	F	T

Thus ~(p ↔ ~q) is equivalent to p ↔ q.

$$P(B) = \frac{5}{6} - \frac{1}{2}$$

$$P(B) = \frac{1}{3} \qquad \ldots \text{(iv)}$$

A of B are Independent because $P(A \cap B) = P(A)\,P(B)$.A and B has different probability so if is not equally likely.

47. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$:

(1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$

(3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

47. (4)

Given, $f(0) = 2$, $g(1) = 2$, $g(0) = 0$, $f(1) = 6$

Let, $F(x) = f(x) - 2g(x)$

$F(0) = f(0) - 2g(0)$

$F(0) = 2 - 2 \times 0$

$F(0) = 2 \qquad \ldots (1)$

$F(1) = F(1) - 2g(1)$

$F(1) = 6 - 2 \times 2$

$F(1) = 2 \qquad \ldots (2)$

F(x) is continuous and differentiable in [0, 1].

$F(0) = F(1)$

So, according to Rolle's theorem, there is at least are root between 0 and 1. At which $F'(x) = 0$.

$f'(x) - 2g'(x) = 0$

$f'(c) - 2g'(c) = 0$

$f'(c) = 2g'(c)$

48. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$.

If $p(0) = 100$, then $p(t)$ equals :

(1) $400 - 300\,e^{t/2}$ (2) $300 - 200\,e^{-t/2}$

(3) $600 - 500\,e^{t/2}$ (4) $400 - 300\,e^{-t/2}$

48. (1)

Rearranging the equation we get,

$$\frac{dp(t)}{p(t) - 400} = \frac{1}{2}dt \qquad \ldots(1)$$

Integrating (1) on both sides we get

$p(t) = 400 + k\,e^{t/2}$, where k is a constant of integration.

Using $p(0) = 100$, we get

$k = -300$

$\therefore$ the relation is

$p(t) = 400 - 300\,e^{t/2}$

49. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to :

(1) $\frac{\sqrt{3}}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$