$y = -8$

2. **Find the equation of the curve if the slope at pt (2,3) is given by $\frac{2x + 1}{2y - 3}$.**

slope $= \frac{dy}{dx} = \frac{2x + 1}{2y - 3}$ → $(2y – 3)dy = (2x + 1)dx$

$\int (2y – 3)dy = \int (2x + 1)dx$

$y^2 – 3y = x^2 + x + c$

*substitute pt (2,3)

$3^2 – 3(3) = 2^2 + 2 + c$

$c = -6$

Equation: $\mathbf{y^2 – 3y = x^2 + x – 6}$ (hyperbola)

3. **If at any point (x,y) on a curve $\frac{d^3y}{dx^3}$ = 2 and (1,3) is the pt. of inflection at which the slope of the inflectional tangent line is -2, find the equation of the curve.**

$\frac{d^3y}{dx^3} = 2$

$\frac{d}{dx} = 2$

$d = 2dx$

$\int d = \int 2dx$

$\frac{d^2y}{dx^2} = 2x + c_1$

$\frac{d}{dx} = 2x + c_1$

$d = (2x + c_1)dx$

$\int d = \int (2x + c_1)dx$

$\frac{dy}{dx} = x^2 + c_1x + c_2$

$dy = (x^2 + c_1x + c_2)dx$

$\int dy = \int (x^2 + c_1x + c_2)dx$

$y = \frac{x^3}{3} + \frac{c_1x^2}{2} + c_2x + c_3$

SYSTEM OF EQUATIONS:

a. (1,3) is a point on a curve. So we substitute it to the last equation.

$3 = \frac{1}{3} + \frac{c_1}{2} + c_2 + c_3$

$16 = 3c_1 + 6c_2 + 6c_3$

b. slope $= \frac{dy}{dx} = -2$ at $x = 1$. Substitute these values to the second equation.

$-2 = 1^2 + c_1(1) + c_2$

$c_1 + c_2 = -3$

$\frac{d^2y}{dx^2} = 2x + c_1$

$0 = 2 + c_1$

$c_1 = -2$

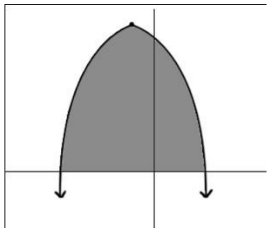
*substitute c1to the other equations to get the other 2 constants

$c_1 = -2$, $c_2 = -1$ and $c_3 = \frac{14}{3}$

*substitute these values to the last equation

$y = \frac{x^3}{3} – x^2 – x + \frac{14}{3}$

4. **Find the area under the parabola $y = 8 – x^2 – 2x$, above the x-axis.**



*complete the square

$x^2 + 2x +$ __ $= 8 – y +$ __

$x^2 + 2x + 1 = 9 – y$

$(x + 1)^2 = –(y – 9)$

*it is a parabola the opens downward

V (-1,9)

$dA = (y_A – y_B)dx$

*dx = xLEFT – xRIGHT

$dA = (8 – x^2 – 2x)dx$

$\int dA = \int (8 – x^2 – 2x)dx$

$A = 8x – \frac{x^3}{3} – x^2 + c$

On the x-axis, y = 0.

*substitute y in $y = 8 – x^2 – 2x$

$0 = 8 – x^2 – 2x$

$x^2 + 2x – 8 = 0$

$(x + 4)(x – 2) = 0$

When x = -4, A = 0

*0 yung area pag x = -4 kasi hindi wala pang area na nabubuo sa point na yun.

*Substitute these values to $A = 8x – \frac{x^3}{3} – x^2 + c$

$0 = 8(-4) – \frac{(-4)^3}{3} – (-4)^2 + c$

$c = \frac{80}{3}$

When x = 2, and $c = \frac{80}{3}$

*Substitute these values to $A = 8x – \frac{x^3}{3} – x^2 + c$. We did this again tapos with x = 2 kasi may area nang macocover sa point na yun .

$A = 8(2) – \frac{2^3}{3} – 2^2 + \frac{80}{3} =$ **36 sq. units**

5. **An art collector purchased for $1000 a painting by an artist whose works are currently increasing with respect to the time according to the formula**

$$\frac{du}{dt} = 5t^{2/3} + 10t + 50$$

$\frac{\infty}{\infty}$ & $\frac{0}{0}$ = 'pag ganyan yung situation, dun sa equation/s kung sa'n naka substitute yung "b" or "a", *derive* both the numerator and the denominator. Then you may start dividing

$\frac{1}{\infty} = 0$

EXAMPLES:

1. $\int_1^\infty \frac{2dy}{y(y+16)}$

$\lim_{b\to\infty} \int_1^b \frac{2dy}{y^2+16y+64-64}$

$2\int_1^b \frac{dy}{(y+8)^2-(8)^2}$ a = 8 u = y + 8 du = dy

$\int \frac{du}{u^2-a^2}$

$2 \cdot \frac{1}{2(8)} \ln \left| \frac{y+8-8}{y+8+8} \right|$

$2 \cdot \frac{1}{2(8)} \ln \left| \frac{y}{y+16} \right| \Big]_1^b$

$\frac{1}{8}$

$\frac{1}{8}$

*so diba infinity over infinity, so bawal yun. Babalik tayo sa equation before this. Yung may b over y + 16 Derive that.

$\frac{1}{8}$

*recall that ln1 = 0

$-\frac{1}{8} \ln \frac{1}{17}$

*recall that $\ln\frac{a}{b}$ = lna - lnb

$-\frac{1}{8}$ [ln1 – ln17]

$-\frac{1}{8}$ [–ln17] = $\mathbf{\frac{1}{8}}$ **[ln17]**

2. $\int_0^\infty xe^{-x^2}dx$

$\lim_{b\to\infty} \int_1^b xe^{-x^2}dx$ $u = -x^2$ $-\frac{du}{2} = xdx$

$-\frac{1}{2}\int_1^b e^u du$

$-\frac{1}{2}\Big]_0^b$

$-\frac{1}{2}$

$-\frac{1}{2}$

$-\frac{1}{2} \cdot -1 = \mathbf{\frac{1}{2}}$

3. $\int_2^\infty \frac{dx}{x^2+1}$

$\lim_{b\to\infty} \int_1^b \frac{du}{u^2+a^2}$

$\frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \Big]_2^b$

$\frac{1}{2}$

*so diba infinity over infinity, so derive the numerator and the denominator

$\frac{1}{2}$

$\frac{1}{2}$

$-\frac{1}{2} \ln \frac{1}{3} = -\frac{1}{2}$ [ln1 – ln3] = $\mathbf{\frac{1}{2}}$ **ln3**

II. Integrals with infinite discontinuities in the integrand

*in other words, isa or both a and b sa formula na $\int_a^b$ f(x)dx, pag sinubstitute sa f(x)dx, UNDEFINED yung lalabas.

a) If f(x) increases numerically without limit as x → a, then

$\int_m^n f(x)dx = \lim_{a\to m^+} \int_a^n f(x)dx$

a) If f(x) increases numerically without limit as x → b, then

$\int_m^n f(x)dx = \lim_{b\to n^-} \int_m^b f(x)dx$

a) If f(x) increases numerically without limit as x → c, a < c < b , (kumbaga yung point of discontinuity, hindi given pero nasa gitna siya ng a and b) then,

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$= \lim_{n\to c^-} \int_a^n f(x)dx + \lim_{m\to c^+} \int_m^b f(x)dx$

EXAMPLES:

1. $\int_0^2 \frac{dx}{\sqrt{x(2-x)}}$

*pag sinubstite both 0 & 2, magiging undefined yung sagot so ii-integrate both limits

$\lim_{a\to 0 \text{ and } b\to 2} \int_a^b \frac{dx}{\sqrt{1-(x^2-2x+1)}}$

$\int_a^b \frac{du}{\sqrt{a^2-u^2}}$

$\sin^{-1}(x-1)\Big]_a^b$

$\sin^{-1}(b-1) - \sin^{-1}(a-1)$

$\sin^{-1}(1) - \sin^{-1}(-1)$

$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \boldsymbol{\pi}$

* –90° yung $\sin^{-1}(-1)$ instead of 180 kasi pag negative yung value tas Arcsin yung hinahanap, clockwise mo siya babasahin