Binomial theorem

Properties of Binomial Coefficients

1. ${}^{n}\mathrm{C}_{r} = {}^{n}\mathrm{C}_{r-1}$
2. ${}^{n}\mathrm{C}_{r} = {}^{n}\mathrm{C}_{s} \Rightarrow r = s \quad \text{or} \quad r + s = n$
3. ${}^{n}\mathrm{C}_{r} + {}^{n}\mathrm{C}_{r+1} = {}^{n+1}\mathrm{C}_{r+1}$
4. $\dfrac{{}^{n}\mathrm{C}_{r}}{{}^{n+1}\mathrm{C}_{r+1}} = \dfrac{r+1}{n+1}$
5. $\dfrac{{}^{n}\mathrm{C}_{r}}{{}^{n}\mathrm{C}_{r+1}} = \dfrac{r+1}{n-r}$

Series of Binomial Coefficients

$$(1+x)^{n} = {}^{n}\mathrm{C}_{0} + {}^{n}\mathrm{C}_{1}x + {}^{n}\mathrm{C}_{2}x^{2} + \ldots + {}^{n}\mathrm{C}_{n}x^{n} \qquad \ldots (1)$$

1. Sum of the binomial coefficients in the expansion of $(1 + x)^n = 2^n$

Preview from Notesale.co.uk
Page 1 of 3

2. $\sum_{r=0}^{n} (-1)^{r}\, {}^{n}\mathrm{C}_{r} = 0$

3. In the expansion $(1 + x)^n$:

Sum of the binomial coefficients at odd position = Sum of the binomial coefficients at even position

$${}^{n}\mathrm{C}_{0} + {}^{n}\mathrm{C}_{2} + {}^{n}\mathrm{C}_{4} + \ldots = {}^{n}\mathrm{C}_{1} + {}^{n}\mathrm{C}_{3} + {}^{n}\mathrm{C}_{5} + \ldots = 2^{n-1}$$

4. ${}^{n}\mathrm{C}_{1} + 2\,{}^{n}\mathrm{C}_{2} + 3\,{}^{n}\mathrm{C}_{3} + \ldots\ldots. + n\,{}^{n}\mathrm{C}_{n} = n\ 2^{n-1}$

5. ${}^{n}\mathrm{C}_{1} - 2\,{}^{n}\mathrm{C}_{2} + 3\,{}^{n}\mathrm{C}_{3} - \ldots + (-1)^{n-1}\, n\,{}^{n}\mathrm{C}_{n} = 0$

6. ${}^{n}\mathrm{C}_{0}\ {}^{n}\mathrm{C}_{r} + {}^{n}\mathrm{C}_{1}\ {}^{n}\mathrm{C}_{r+1} + {}^{n}\mathrm{C}_{2}\ {}^{n}\mathrm{C}_{r+2} + \ldots\ldots + {}^{n}\mathrm{C}_{n-r}\ {}^{n}\mathrm{C}_{n} = {}^{2n}\mathrm{C}_{n+r}$

7. ${}^{n}\mathrm{C}_{0} + 3\,{}^{n}\mathrm{C}_{1} + 5\,{}^{n}\mathrm{C}_{2} + 7\,{}^{n}\mathrm{C}_{3} + \ldots + (2n+1)\,{}^{n}\mathrm{C}_{n} = (n+1)2^{n}$

- The coefficients of the expansions of a binomial are arranged in an array. This array is called Pascal's triangle. It can be written as

Index	Coefficient(s)
0	${}^{0}\mathrm{C}_{0}$ $(=1)$