

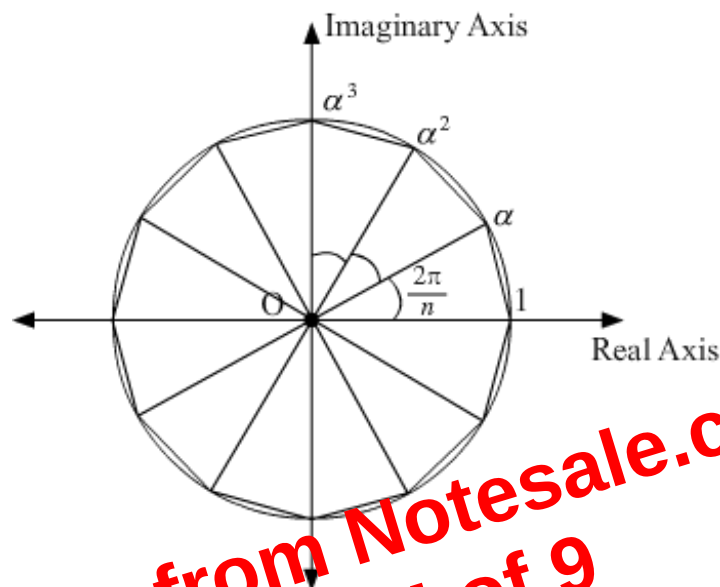
If $r = n - 1$, then $x = \cos \frac{2(n-1)\pi}{n} + i \sin \frac{2(n-1)\pi}{n} = e^{i \frac{2(n-1)\pi}{n}}$

The roots $1, e^{i \frac{2\pi}{n}}, e^{i \frac{4\pi}{n}}, \dots, e^{i \frac{2(n-1)\pi}{n}}$ are the n^{th} roots of unity.

If $e^{i \frac{2\pi}{n}} = \alpha$ then the n^{th} roots of unity will be represented as $1, \alpha, \alpha^2, \dots, \alpha^{n-1}$.

Representation of n^{th} roots of unity on the Argand plane:

The n^{th} roots of unity when plotted on Argand plane represent the vertices of a regular polygon of n sides which are inscribed in the circle $|z| = 1$.



Properties of n^{th} roots of unity:

- Sum of n^{th} roots of unity, $1 + \alpha + \alpha^2 + \dots + \alpha^{n-1} = 0$
- Product of n^{th} roots of unity, $1 \times \alpha \times \alpha^2 \times \dots \times \alpha^{n-1} = -1^{n-1}$
- $1 + \alpha^r + \alpha^{2r} + \dots + \alpha^{(n-1)r} = 0$ if H.C.F $(r, n) = 1$
- A number of the form $a + ib$, where a and b are real numbers and $i = \sqrt{-1}$, is defined as a complex number.
- For the complex numbers $z = a + ib$, a is called the real part (denoted by $\text{Re } z$) and b is called the imaginary part (denoted by $\text{Im } z$) of the complex number z .

Example: For the complex number $z = \frac{-5}{9} + i \frac{\sqrt{3}}{17}$, $\text{Re } z = \frac{-5}{9}$ and $\text{Im } z = \frac{\sqrt{3}}{17}$

- Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are equal if $a = c$ and $b = d$.

• Addition of complex numbers

Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ can be added as,

$$z_1 + z_2 = (a + ib) + (c + id) = (a + c) + i(b + d)$$