

Archimedes' principle

It is named after Archimedes of Syracuse, who first discovered this law. According to Archimedes' principle, "Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Vitruvius (De architectura IX.9–12) recounts the famous story of Archimedes making this discovery while in the bath. He was given the task of finding out if a goldsmith, who worked for the king, was carefully replacing the king's gold with silver. While doing this Archimedes decided he should take a break so went to take a bath. While entering the bath he noticed that when he placed his legs in, water spilled over the edge. Struck by a moment of realisation, he shouted "Eureka!" He informed the king that there was a way to positively tell if the smith was cheating him. Knowing that gold has a higher density than silver, he placed the king's crown and a gold crown of equal weight into a pool. Since the king's crown caused more water to overflow, it was, therefore, less dense, Archimedes concluded that it contained silver, causing the smith to be executed. The actual record of Archimedes' discoveries appears in his two-volume work, *On Floating Bodies*. The ancient Chinese child prodigy Cao Chong (196–208 AD) also applied the principle of buoyancy in order to accurately weigh an elephant, as described in the Sanguo Zhi, also known as the Records of Three Kingdoms.

Archimedes' principle does not consider the surface tension (capillarity) acting on the body.^[1]

The weight of the displaced fluid is directly proportional to the volume of the displaced fluid (if the surrounding fluid is of uniform density). Thus, among completely submerged objects with equal masses, objects with greater volume have greater buoyancy.

Suppose a rock's weight is measured as 10 newtons when suspended by a string in a vacuum. Suppose that when the rock is lowered by the string into water, it displaces water of weight 3 newtons. The force it then exerts on the string from which it hangs would be 10 newtons minus the 3 newtons of buoyant force: $10 - 3 = 7$ newtons. Buoyancy reduces the apparent weight of objects that have sunk completely to the sea floor. It is generally easier to lift an object up through the water than it is to pull it out of the water.

The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes:

$$\frac{\text{Density of object}}{\text{Density of fluid}} = \frac{\text{weight}}{\text{weight} - \text{apparent immersed weight}}$$

<http://en.wikipedia.org/wiki/Buoyancy>

Specific gravity

The specific gravity of a material is defined as the ratio of its density to the density of some standard material, such as water at a specified temperature, for example, 60°F (15°C), or (for gases) air at standard conditions of temperature and pressure. Specific gravity is a convenient concept because it is usually easier to measure than density, and its value is the same in all systems of units.

Specific gravity, the ratio of the weight of a given volume of a substance to the weight of an equal volume of some reference substance, or, equivalently, the ratio of the masses of equal volumes of the two substances.

Relationship Between Specific Gravity and Density

Unlike density, which has units of mass per volume, specific gravity is a pure number, i.e., it has no associated unit of measure. If the densities of the substance of interest and the reference substance are known in the same units (e.g., both in g/cm^3 or lb/ft^3), then the specific gravity of the substance is equal to its density divided by that of the reference substance. Similarly, if the specific gravity of a substance is known and the density of the reference substance is known in some particular units, then the density of the substance of interest, in those units, is equal to the product of its specific gravity and the density of the reference substance.

The most widely used reference substance for determining the specific gravities of solids and liquids is water. Because the density of water is very nearly 1 g/cm^3 , the density of any substance in g/cm^3 is nearly the same numerically as its specific gravity relative to water. In the English system of units the density of water is about 62.4 lb/ft^3 , so the near equality between specific gravity and density is not preserved in this system. Specific gravities of gases are often given with dry air as the reference substance. Because the densities of all substances vary with temperature and pressure, the temperature and (particularly for gases) the pressure for both the reference substance and the substance of interest are often included when precise values of specific gravities are given.

Methods of Determining Specific Gravity

A number of experimental methods for determining the specific gravities of solids, liquids, and gases have been devised. A solid is weighed first in air, then while immersed in water; the difference in the two weights, according to Archimedes' principle, is the weight of the water displaced by the volume of the solid. If the solid is less dense than water, some means must be adopted to fully submerge it, e.g., a system of pulleys or a sinker of known mass and volume. The specific gravity of the solid is the ratio of its weight in air to the difference between its weight in air and its weight immersed in water.

Two methods are commonly used for determining the specific gravities of liquids. One method uses the hydrometer, an instrument that gives a specific gravity reading directly. A second method, called the bottle method, uses a "specific-gravity bottle," i.e., a flask made to hold a known volume of liquid at a specified temperature (usually 20°C). The bottle is weighed, filled with the liquid whose specific gravity is to be found, and weighed again. The difference in weights is divided by the weight of an equal volume of water to give the specific gravity of the liquid. For gases a method essentially the same as the bottle method for liquids is used. Specific gravities of gases are usually converted mathematically to their value at standard temperature and pressure.

Density has many applications in the chemical industry. The relationship between mass and volume is an important aspect of the specification and utilization of both solids and liquids. For example, valuable metals and stones are characterized by their densities. Bulk chemicals are shipped in drum and totes, most often by the pound. Conversion of pounds to gallons or into metric equivalents is a critical aspect of trade. Shipping costs are most often determined by weight. Density can be used to quantify the dissolved solids in liquids. For example, high concentrations of salt in brines increase the density of solutions. The most common units for density are g/mL (g/cm^3), or pounds per gallon.

Specific gravity related to density, but is a unitless quantity, defined as the density of a substance divided by the density of water. Since we often assume the density of water to be 1.0 g/mL, the specific gravity usually agrees closely with density. However, as temperature changes, so does the density of water, so at elevated temperatures, specific gravity can be somewhat different than density of the material being tested. Most often in the lab, a fixed volume container called a pycnometer is used to determine specific gravity. The pycnometer is filled with water and weighed. Then, the pycnometer is filled with the unknown liquid and weighed. The mass of the unknown liquid divided by the mass of the water is the specific gravity.

Measurement of the density of an unknown solid is relatively easy. Determine both the mass and the volume of a substance, and then divide mass by the volume to calculate density. Archimedes discovered that volumes can be measured by displaced volumes of water. He also discovered that a solid mass weighs more in air than when suspended in water. The difference in the two masses is the mass of the displaced liquid. When using water as the liquid, the difference in grams is approximately the same as milliliters. So, by dividing the mass of the solid in air by the *difference* between its mass in air and its mass suspended in water, the density is obtained. Of course, if the liquid is not water, then the density of the liquid must be taken into account and the density of the liquid must be used to convert the displaced mass into displaced volume before the density of the suspended solid can be calculated.

$$\rho_{\text{solid}} = \frac{m_{\text{air}}}{m_{\text{air}} - m_{\text{liquid}}} (\rho_{\text{refliq}})$$

Where ρ_{solid} is the density of the solid to be measured, m_{air} is the mass of the solid in air, and m_{liquid} is the mass of the solid while suspended in liquid. The ρ_{refliq} is the density of the reference liquid at the temperature during the analysis. (*Data available in tables for water and alcohol.*)

An interesting application of this same equation is the determination of the density of a liquid by weighing a suspended solid of known mass and volume in the liquid. (A known, calibrated solid used for this purpose is often called a "sinker." How might this determination of density of the liquid be obtained? Consider this following equation:

$$\rho_{\text{unkliq}} = \frac{m_{\text{displaced liquid}}}{V_{\text{sinker}}}$$

Where ρ_{unkliq} is the density of the liquid to be measured, $m_{\text{displaced liquid}}$ is the mass of the displaced liquid, and V_{sinker} is the volume of the sinker. So, the density of a liquid may be determined by weighing a sinker immersed in the liquid, if the exact volume of the sinker is known.

Question: How would the volume of the sinker affect the reproducibility of the determination? Is it better to use a large sinker or a smaller one? Explain.

Formulae for determining the density of solids with compensation for air density

$$\rho = \frac{A}{A-B} (\rho_0 - \rho_L) + \rho_L$$

$$V = \alpha \frac{A - B}{\rho_0 - \rho_L}$$

ρ = Density of the sample

A = Weight of the sample in air

B = Weight of the sample in the auxiliary liquid

V = Volume of the sample

ρ_0 = Density of the auxiliary liquid

ρ_L = Density of Air (0.0012 g/cm³)

α = Weight correction factor (0.99985), to take the atmospheric buoyancy of the adjustment weight into account

Formula for determining the density of liquids with compensation for air density

$$\rho = \alpha \frac{P}{V} + \rho_L$$

ρ = Density of the liquid

P = Weight of the displaced liquid

V = Volume of the sinker

ρ_L = Density of air (0.0012 g/cm³)

α = Weight correction factor (0.99985), to take the atmospheric buoyancy of the adjustment weight into account

Density Table for Distilled Water

T/°C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10.	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99967	0.99966	0.99965	0.99964
11.	0.99963	0.99962	0.99961	0.99960	0.99959	0.99958	0.99957	0.99956	0.99955	0.99954
12.	0.99953	0.99951	0.99950	0.99949	0.99948	0.99947	0.99946	0.99944	0.99943	0.99942
13.	0.99941	0.99939	0.99938	0.99937	0.99935	0.99934	0.99933	0.99931	0.99930	0.99929
14.	0.99927	0.99926	0.99924	0.99923	0.99922	0.99920	0.99919	0.99917	0.99916	0.99914
15.	0.99913	0.99911	0.99910	0.99908	0.99907	0.99905	0.99904	0.99902	0.99900	0.99899
16.	0.99897	0.99896	0.99894	0.99892	0.99891	0.99889	0.99887	0.99885	0.99884	0.99882
17.	0.99880	0.99879	0.99877	0.99875	0.99873	0.99871	0.99870	0.99868	0.99866	0.99864
18.	0.99862	0.99860	0.99859	0.99857	0.99855	0.99853	0.99851	0.99849	0.99847	0.99845
19.	0.99843	0.99841	0.99839	0.99837	0.99835	0.99833	0.99831	0.99829	0.99827	0.99825
20.	0.99823	0.99821	0.99819	0.99817	0.99815	0.99813	0.99811	0.99808	0.99806	0.99804
21.	0.99802	0.99800	0.99798	0.99795	0.99793	0.99791	0.99789	0.99786	0.99784	0.99782
22.	0.99780	0.99777	0.99775	0.99773	0.99771	0.99768	0.99766	0.99764	0.99761	0.99759
23.	0.99756	0.99754	0.99752	0.99749	0.99747	0.99744	0.99742	0.99740	0.99737	0.99735
24.	0.99732	0.99730	0.99727	0.99725	0.99722	0.99720	0.99717	0.99715	0.99712	0.99710
25.	0.99707	0.99704	0.99702	0.99699	0.99697	0.99694	0.99691	0.99689	0.99686	0.99684
26.	0.99681	0.99678	0.99676	0.99673	0.99670	0.99668	0.99665	0.99662	0.99659	0.99657
27.	0.99654	0.99651	0.99648	0.99646	0.99643	0.99640	0.99637	0.99634	0.99632	0.99629
28.	0.99626	0.99623	0.99620	0.99617	0.99614	0.99612	0.99609	0.99606	0.99603	0.99600
29.	0.99597	0.99594	0.99591	0.99588	0.99585	0.99582	0.99579	0.99576	0.99573	0.99570
30.	0.99567	0.99564	0.99561	0.99558	0.99555	0.99552	0.99549	0.99546	0.99543	0.99540

Density Table for Ethanol

T/°C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10.	0.79784	0.79775	0.79767	0.79758	0.79750	0.79741	0.79733	0.79725	0.79716	0.79708
11.	0.79699	0.79691	0.79682	0.79674	0.79665	0.79657	0.79648	0.79640	0.79631	0.79623
12.	0.79614	0.79606	0.79598	0.79589	0.79581	0.79572	0.79564	0.79555	0.79547	0.79538
13.	0.79530	0.79521	0.79513	0.79504	0.79496	0.79487	0.79479	0.79470	0.79462	0.79453
14.	0.79445	0.79436	0.79428	0.79419	0.79411	0.79402	0.79394	0.79385	0.79377	0.79368
15.	0.79360	0.79352	0.79343	0.79335	0.79326	0.79318	0.79309	0.79301	0.79292	0.79284
16.	0.79275	0.79267	0.79258	0.79250	0.79241	0.79232	0.79224	0.79215	0.79207	0.79198
17.	0.79190	0.79181	0.79173	0.79164	0.79156	0.79147	0.79139	0.79130	0.79122	0.79113
18.	0.79105	0.79096	0.79088	0.79079	0.79071	0.79062	0.79054	0.79045	0.79037	0.79028
19.	0.79020	0.79011	0.79002	0.78994	0.78985	0.78977	0.78968	0.78960	0.78951	0.78943
20.	0.78934	0.78926	0.78917	0.78909	0.78900	0.78892	0.78883	0.78874	0.78866	0.78857
21.	0.78849	0.78840	0.78832	0.78823	0.78815	0.78806	0.78797	0.78789	0.78780	0.78772
22.	0.78763	0.78755	0.78746	0.78738	0.78729	0.78720	0.78712	0.78703	0.78695	0.78686
23.	0.78678	0.78669	0.78660	0.78652	0.78643	0.78635	0.78626	0.78618	0.78609	0.78600
24.	0.78592	0.78583	0.78575	0.78566	0.78558	0.78549	0.78540	0.78532	0.78523	0.78515
25.	0.78506	0.78497	0.78489	0.78480	0.78472	0.78463	0.78454	0.78446	0.78437	0.78429
26.	0.78420	0.78411	0.78403	0.78394	0.78386	0.78377	0.78368	0.78360	0.78351	0.78343
27.	0.78334	0.78325	0.78317	0.78308	0.78299	0.78291	0.78282	0.78274	0.78265	0.78256
28.	0.78248	0.78239	0.78230	0.78222	0.78213	0.78205	0.78196	0.78187	0.78179	0.78170
29.	0.78161	0.78153	0.78144	0.78136	0.78127	0.78118	0.78110	0.78101	0.78092	0.78084
30.	0.78075	0.78066	0.78058	0.78049	0.78040	0.78032	0.78023	0.78014	0.78006	0.77997

Density of C₂H₅OH according to the "American Institute of Physics Handbook".