In the last scene of the play, Hamlet has a fencing match against Laertes. At the beginning of the match, Gertrude drinks from the poisoned cup that Claudius had prepared for Hamlet. Hamlet never determines if his mother is guilty of his father's murder. In the third round of the competition, Laertes wounds Hamlet with the poison tipped rapier and after exchanging swords, Hamlet grazes Laertes in return. This is when he discovers that Claudius persuaded Laertes to kill him;

It is a poison tempered by himself.

Exchange forgiveness with me, noble Hamlet.

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360-363 a gle 1).5 of 5 Mine and my father's death come not upon thee, Nor thine on me

In his indignation Hamlet finally kills Claudius by constraining him to drink his own poison.

Some people say that throughout the play Hamlet is just procrastinating and not being productive. In my opinion, Hamlet is doing the right thing by not rushing into action. Hamlet takes his time to investigate who was really at fault regarding the death of his father. This had severe consequences. The problem with Hamlet not taking action sooner in avenging his father's death is that Polonius, Ophelia, Rosencrantz, Guildenstern, Gertrude, Laertes and himself all die in the process. If he had acted faster, all these people may not have died.