12.1 INTRODUCTION

The majority of electrical power in the world is generated, distributed, and consumed in the form of 50- or 60-Hz sinusoidal alternating current (AC) and voltage. It is used for household and industrial applications such as television sets, computers, microwave ovens, electric stoves, to the large motors used in the industry.

AC has several advantages over DC. The major advantage of AC is the fact that it can be transformed, however, direct current (DC) cannot. A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of kV) is that less current is required to produce the same amount of power. Less current permits smaller wires to be used for transmission.

In this chapter, we will introduce a sinusoidal signal and its basic mathematical equation. We will discuss and analyze circuits where currents $i(t)$ and voltages $v(t)$ vary with time. The phasor analysis techniques will be used to analyze electric circuits under sinusoidal steady-state operating conditions. Single-phase power will conclude the chapter.

12.2 SINUSOIDAL WAVEFORMS

AC unlike DC flows first in one direction then in the opposite direction. The most common AC waveform is a sine (or sinusoidal) waveform. Sine waves are the signal whose shape neither is nor altered by a linear circuit, therefore, it is ideal as a test signal.

In discussing AC signal, it is necessary to express the current and voltage in terms of maximum or peak values, peak-to-peak values, effective values, average values, or instantaneous values. Each of these values has a different meaning and is used to describe a different amount of current or voltage. Figure 12-1 is a plot of a sinusoidal wave. The correspondence mathematical form is

$$v(t) = V_p \cos(wt + \theta) \tag{12.1}$$

Where V_p is the peak voltage, $\omega = 2\pi f$ is the angular speed expressed in radians per second (rad/s), f is the frequency expressed in Hertz (Hz), t is the time expressed in second (s), and θ is phase of the sinusoid expressed in degrees.

The function (Figure 12-1) starts at a value of 0 at 0^o, and rise smoothly to a maximum of 1 at 90^o. They then fall, just as they rose, back to 0^o at 180^o. The negative peak is reached three quarters of the way at 270^o. The function then returns symmetrically to 0^o at 360^o.

temperature at 100°C. Also assume that the AC in Figure is increased until the temperature of the resistor is 100° C. At this point it is found that a maximum AC value of 1.414 *A* is required in order to have the same heating effect as DC. Therefore, in the AC circuit the maximum current required is 1.414 times the effective current.

When a sinusoidal voltage is applied to a resistance, the resulting current is also a sinusoidal. This follows Ohm's law which states that current is directly proportional to the applied voltage. Ohm's law, Kirchhoff's law, and the various rules that apply to voltage, current, and power in a DC circuit also apply to the AC circuit. Ohm's law formula for an AC circuit may be stated as

$$I_{eff} = \frac{V_{eff}}{R} \qquad (12.8)$$

Importantly, all AC voltage and current values are given as effective values.

12.2.6 Frequency

If the signal in the Figure makes one complete revolution each second, the generator produces one complete cycle of AC during each second (1 Hz). Increasing the number of revolutions to two per second will produce two complete cycles of ac per second (2 Hz). The number of complete cycles of alternating current or voltage completed each second is referred to as the "frequency, *f*" or "event frequency". Event frequency is always measured and expressed in hertz. Because there are 2π radians in a full circle, a cycle, the relationship between ω, f, and period, T, can be expressed as

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ radians/second} \qquad (12.9)$$

Where ω is the angular velocity in radians per second (rad/s). The dimension of frequency is reciprocal second. The frequency is an important term to understand since most AC electrical equipment requires a specific frequency for proper operation.

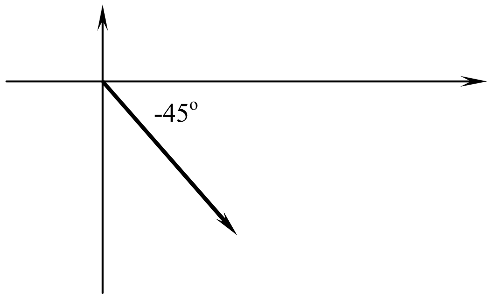

Figure 12-7 Phasor diagram of Example 12-5.

12.3.2 Rectangular Form

The horizontal and vertical components denote a complex number. The angled vector is taken to be the hypotenuse of a right triangle, described by the lengths of the adjacent and opposite sides. These two dimensional figures (horizontal and vertical) are symbolized by two numerical figures. In order to distinguish the horizontal and vertical dimensions from each other, the vertical is prefixed with a lower-case "*i*" (in pure mathematics) or "*j*" (in electronics). Figure 12-8 shows that a point on a complex plane located by a phasor could be described in rectangular form.

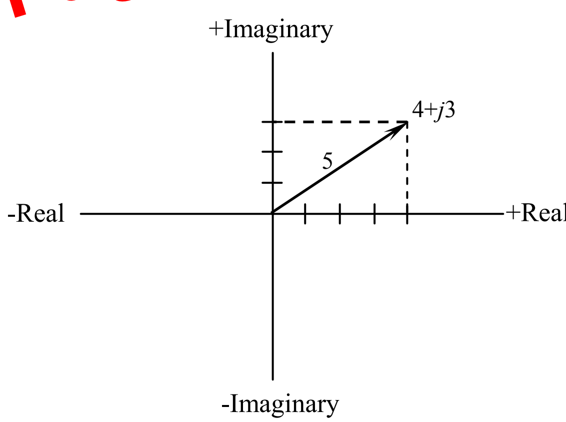

Figure 12-8 A point on the complex plane located by the phasor 4+*j*3 expressed in the rectangular form.

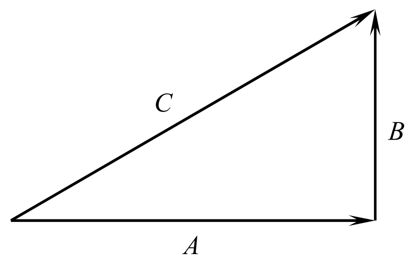

Figure 12-9 Relation between polar and rectangular forms.

To convert from the polar to the rectangular form of a phasor, you must convert $C\angle\theta$ into A and B. From trigonometry, the cosine of an included angle relates the length of the adjacent side and the length of the hypotenuse.

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{A}{C} \qquad \sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{B}{C} \tag{12.16}$$

To convert from rectangular form to polar form requires a different set of trigonometric relationships.

$$C = \sqrt{A^2 + B^2} \qquad \tan\theta = \frac{B}{A} \tag{12.17}$$

Taking the inverse tangent of each side leaves θ as

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) \tag{12.18}$$

In general, any load in rectangular form may be converted into polar form as the following

$$\mathrm{Z} = R + \mathrm{j}X_\mathrm{L} \qquad \mathrm{Z} = \sqrt{R^2 + X_\mathrm{L}{}^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right) \tag{12.19}$$

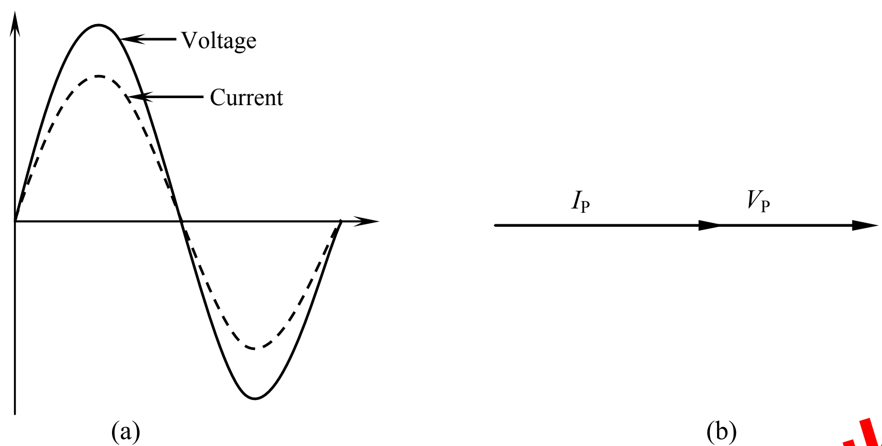

Figure 12-15 (a) Voltage in phase with current. (b) Phase angle between voltage and current is 0^o.

When sinusoidal current flows through the impedance, we have

$$v(t) = i(t) \times R \qquad (12.27)$$

where

$$i(t) = I_P \sin(wt)$$

then

$$v(t) = RI_P \sin(wt)$$

Now convert the above equation from time domain form into phasors

$$V = R \times (I_{rms} \angle 0^o)$$

Substitute into Equation (12.26), we obtain

$$Z = \frac{(R \times I_{rms} \angle 0^o)}{(I_{rms} \angle 0^o)}$$

The impedance diagram of an inductor is shown in Figure 12-19. The length of the phasor X_L lies entirely along the imaginary (+*y*) axis.

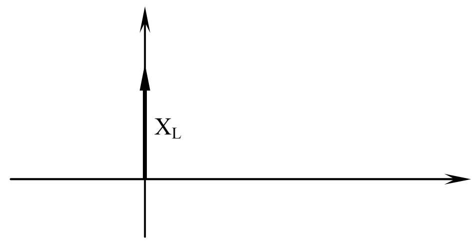

Figure 12-19 Impedance diagram of an inductor.

12.6.3 Power in Inductive Load

In a pure resistive circuit, the true power is equal to the product of the voltage and current. In a pure inductive circuit, however, no true power is produced. In order to produce true power, voltage and current must both be either positive or negative. Since the voltage and current are 90° out of phase with each other in a pure inductive circuit, the current and voltage will be at different polarities 50% of the time and the same polarity 50% of the time.

Example 12-23

The inductor shown in Figure has an inductance of 1 H and is connected to a 120 V 60 Hz line. How much current will flow in this circuit?

Solution:

	$X_L = 2\pi fL$ $X_L = 2\times 3.1416\times 60\times 1$ $X_L = 377\,\Omega$	

X_L may be substituted for *R* in Ohm's law

	$I = \dfrac{V}{X_L} = \dfrac{120}{377} = 0.398\text{ A}$	

12.8 AC CIRCUIT ANALAYSIS

The impedance parameters defined in the previous sections are very useful in solving AC circuit analysis problems, because it makes possible to take advantage of most of the network theorems developed for DC circuits by replacing resistances with complex-valued impedances. Figure 12-21 depicts the impedances of *R*, *L*, and *C* in the complex plane.

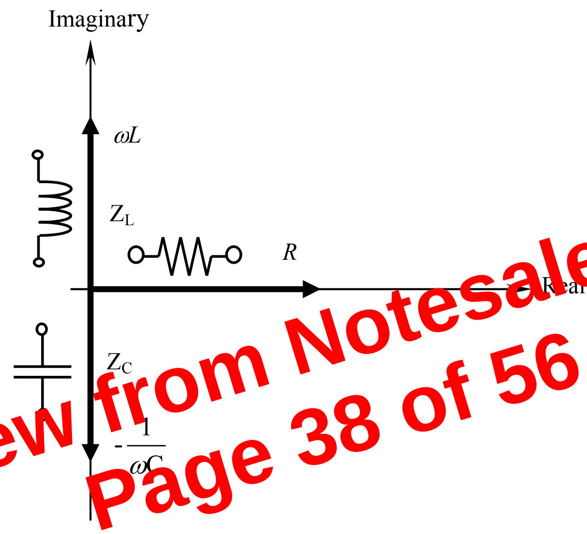

Figure 12-21 Impedances of *R*, *L*, and *C* in the complex plane.

All the rules and laws learned in the study of DC circuits apply to AC circuits including Ohm's law, Kirchhoff's laws, and network analysis methods. The only qualification is that all variables must be expressed in complex form, taking into account phase as well as magnitude, and all voltages and currents must be of the same frequency (in order that their phase relationships remain constant).

It is necessary to emphasize that although the impedance of circuit elements is either purely real (for resistors) or purely imaginary (for inductors and capacitors), the general definition of impedance for an arbitrary circuit should allow for the possibility of having both a real and imaginary part, since practical circuits are made up of more or less complex interconnections of various circuit elements.

REVIEW QUESTIONS

1. What is the difference between AC and DC electricity?
2. Find 5 electrical appliances around the house and determine their voltage, current, and power requirements.
3. Identify an AC electrical device in an automobile.
4. How many degrees are the current and voltage out of phase with each other in a pure resistive circuit?
5. How many degrees are the current and voltage out of phase with each other in a pure inductive circuit?
6. To what is inductive reactance proportional?
7. What two factors determine the capacitive reactance of a capacitor?
8. What is power factor and reactive factor?
9. What is meant by a leading and lagging power factor?
10. What types of connections are possible for three-phase generators and loads?

MULTIPLE CHOICE QUESTIONS

- The peak value of a sine wave occurs

  a. Once each cycle at the positive maximum value.
  b. Once each cycle at the negative maximum value.
  c. Twice each cycle at the positive and negative maximum value.
  d. Twice each cycle at the positive maximum value.

- One of the following is not a right format to express the sinusoid $V \cos \omega t$.

  a. V cos $(2\pi f t)$
  b. V cos $(2\pi t/T)$
  c. V cos $(t - T)$
  d. V sin $(2\pi f t - 80^{o})$

- $\sqrt{-36}$ can be expressed as the following imaginary number

  a. 6
  b. $j6$
  c. –6
  d. $-j6$

- $-\sqrt{-36}$ can be expressed as the following imaginary number

  e. 6
  f. $j6$
  g. –6
  h. $-j6$

- Total opposition to current flow in a circuit with resistance and reactance is

  a. Resistance
  b. Reactance
  c. Impedance
  d. Inductance

- The imaginary part of an impedance is called:

  a. Resistance
  b. Reactance
  c. Admittance
  d. Conductance