$$b_n = \frac{8(-1)^{n+1}}{n\pi(4 - n^2\pi^2)} \qquad (6)$$

Using (6) in (4) we get,

$$x(t) = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin n\pi t}{n(4 - n^2\pi^2)} \qquad (7)$$

Example- 4. Consider an undefined spring-mass system given by:

$$m\frac{d^2x}{dt^2} + k(x) = f(t) \qquad (1)$$

where $m = \frac{1}{16}\,slug,$ **spring constant** $k = 4lb/ft^2$ **and**

$f(t) = 4t, \quad 0 < t < 1.$

Solution. The Fourier series of f(t) is:

$$4t = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\pi t \qquad (2)$$

So, equation (1) can also be written as:

$$\frac{1}{16}\frac{d^2x}{dt^2} + 4x = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\pi t \qquad (3)$$